

QUEUING THEORY FUZZY APPROACH APPLIED IN OUR DAILY LIFE

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ABSTRACT

Modern society like ours in India, people are very busy in routine works and no one likes to wait. There are several situations in daily life when a queue is formed. Queuing theory is the mathematical study of waiting lines and it is very helpful for analyzing the problem the procedure of queuing of daily life of human being. Queuing theory applies not only in daily life but also in sequence of computer programming, medical field, banking ,railway ticket booking etc, In this paper, we try analyze the basic features of waiting line and its different applications.

INTRODUCTION:

A queuing system which consists of the customers and the servers.

Modern society like ours, people are very busy and no one likes to wait. Every minute waiting at a bus stop, a traffic light, banks, post office, petrol pumps or an elevator , all have Queuing problems. Queues are very familiar in our daily life. Waiting of time is considered to be a waste of time. So we want to reduce the waiting time. Reduction of waiting time usually requires extra investments. To decide whether to invest or not, it is important to know the effect of the investment on the waiting time. A queue is a line or sequence of people or vehicles awaiting their turn to be attended to or to proceed. Queues (or waiting lines) help facilities or businesses provide service in an orderly fashion. Forming a queue being a social phenomenon, it is beneficial to the society if it can be managed so that both the units that wait and the one that serves get the most benefit. A queuing system can be described by the flow of units for services, forming or joining the queue, if service is not available soon, and leaving the system after being served.

Queuing theory is the mathematical study of waiting lines or queues. The theory enables mathematical analysis of several related processes, including arriving at the queue, The theory permits the derivations and calculations of several performance measures including the average waiting time in the queue or the system, the expected number waiting or

receiving service and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait a certain time to be served.

There are many valuable applications of the theory, most of which have been well documented in the literature of probability theory, operations research, management service and industrial engineering.

A queuing situation is basically characterized by the flow of customers at a service station. On arrival the customers may be served immediately if server is idle or may have to wait until the facility is available. Situations of this type occur frequently in everyday life.

Various examples of queuing system are as follows:

- Commercial service system
- Transportation service system
- Industrial service system
- Social service system

Due to limited number of available server, it is natural that the customer has to wait for service. An increase in the number of servers in the system will decrease the expected waiting time but will also increase the cost of the service. Thus, it is possible to select the optimum number of servers that minimize the sum of the costs of service and waiting time. This aspect is theoretically sound but it is difficult in practice to calculate the case per unit of waiting time. This indicates that queuing theory is not an optimization technique. Rather it is an analytical tool that provides more effective information about the problem. Several people can go to a restaurant and obtain service as a batch. This phenomenon is known as queuing. A Poisson stream of customers arriving in groups and served at a counter in batches of varying size under the general rule of bulk service in which the server remain idle until the queue size reaches or exceeds a fixed number (less than or equal to the capacity) where upon he services them together.

Queuing applications in various public utilities are important. The idea of queuing theory is applied in telephone system which makes it convenient to examine **loss** of system. Applications of idea from queuing theory have found use in landing of aircraft, bed allocation in hospitals etc. It is intended here to make an analytical analysis of bulk queue and applications of queuing theory in bed allocation problems of hospitals and also in reservation of railway tickets in India.

The steady state solutions for M/M/S system, where balking and reneging behavior of customers for limited space is taken into consideration, shall be obtained and the

characteristics of waiting time will be calculated. The result of a single server case will also be calculated as a particular case which may tally with the previous results. It is also proposed to study vehicle dispatch strategy for bulk arrival, bulk service queue and busy periods for bulk arrival and bulk service models. The service time of customer is a random variable having a probability distribution function

$$\sum_{r=1}^j c_r \frac{\mu(\mu t)^{r-1}}{r-1!} e^{-\mu t}$$

and is independent of the number of customers in a batch. The Erlangian techniques or its modifications, the generating functions and Laplace transforms will be used.

Allocation of beds among various wards in a hospital is a vital problem. Emergency ambulances and intensive care units (ICUs) need quick service where any delay can lead to death. Queuing theory will be used in the present analysis to allocate the number of beds in different wards of a hospital so as to reduce waiting time of admissions.

The reservation of railway tickets of Indian railways is done through either of the two alternatives. The first one is that the individual himself goes to the counter opened by the Indian railways and book the advance ticket and other method is through online reservation. We propose to use ATM for booking railway tickets besides withdrawing money.

1. REVIEW OF LITERATURE

History of queuing theory goes back nearly 100 years. Johannsen's "waiting time and number of calls" (an article published in 1907 and represented in post office Electrical engineers Journal, London, 1910) seems to be the first paper on the subject. But the method used in this paper was not mathematically exact and therefore, from the point of view of exact treatment, the paper that has historical importance in A. K. Erlang's "The theory of probabilities and telephone conversations" (Nyt tidsskrift for matematik, B, 20 (1909), pg.33). In this paper he lays foundation for the place of poisson (and hence exponential) distribution in queuing theory. His paper written in the next 20 years contain some of the most important concepts and techniques; the notion of statistical equilibrium and the method of writing down balance of state equations (later called Chapman – Kolmogorov equation) are two such examples. Special mention should be made of his paper "on the rational determination of the number of circuits" (Brockmeyer et al.[1960]) in which an optimization problem in queuing theory was tackled for the time. Although

Erlang was preceded slightly by work of Johannsen (1907), Erlang is still considered the father of mathematical queuing theory.

It should be noted that in Erlang's work as well as the works done by others in the twenties and thirties of last century, the motivation has been the practical problem of congestion. During the next two decades several theoreticians became interested in these problems and developed general models which could be used in more complex situations. Some of the authors with important contributions are Crommelin, Feller, Jensen, Kalmogorov, Palm and Pollaczek . A detailed account of the investigations model by those authors may be found in books of Syski (1960) and Saaty (1961). Kolmogorov's and Feller's study of finding discontinuous processes laid the foundation for the theory of Markov processes and it was developed in later years.

Queuing theory as an identifiable body of literature was essentially defined by the foundational research of 1950s and 1960s. Hiller's (1963) paper on economic model for industrial waiting line problem is perhaps the first paper to introduce standard optimization techniques to queuing problem where Hiller considered a M/M/1 queue, Heyman (1968) derived an optimal theory for turning the server on and off in an M/G/1 queue, depending on the state of the system.

Since then, operations researchers trained in mathematical optimization techniques have explained their use in much greater complexity to a large number of queuing systems.

Queuing system with impatient customers have been discussed for simple process such as M/M/1 systems which are only confined to steady state conditions. Most of the works have been done in the systems with balking or reneging. A combined analysis of balking and reneging has been made for particular cases. The studies on impatient customers who balk as well as renege in M/M/1 systems were first developed by Udagawa and Nakamura(1957) who improved the work of Kawata (1955) and considered a M/M/1 system where balking and reneging probabilities depend on 'N'. Ancker and Gafarian (1962, 1963) considered the combined occurrence of balking and reneging in M/M/1 and M/M/R system. Ancker and Gafarian(1963) considered the M/M/1 system with further assumptions about the balking and reneging behavior of customers where an arriving customer balks with probability n/N and therefore joins the system with probability,

$$e_n = 1 - n/N \quad (n=0, 1, 2, 3, \dots, N)$$

where n is the number in the system and N is the maximum number allowed in the system.

After joining the queue each customer waits for service for sometime which is a random variable with density function

$$d(t) = \alpha e^{-\alpha t}$$

and if service does not begin by then he departs and is lost to the system.

Ancker and Gafarian (1963) considered a different model in which the same assumptions about renegeing behavior were made, but balking behavior is different in the sense that an arriving customer joins the system with probability

$$e_n = \{\beta/n\} \quad \text{when } (0 \leq \beta \leq 1, n = 1, 2, 3, \dots)$$
$$e_n = 1 \quad \text{when } (n=0)$$

where 'n' is the number with system and β is a measure of customers willingness to join the queue. In this problem there is no upper limit on the number of customers in the system. The above works on the subject are confined to a special case when balking and renegeing are dependent on the number of customers in the system.

For an excellent overview, a valuable reference is a special issue of the journal queuing system edited by Stidham (1995), which includes several review type articles on special topics. Bauerle (2002) considers an optimal control problem in a queuing network. Adam et al. (2001) gives a broad treatment of queues with multiple waiting lines and Oshalalov (1997) considers systems with state dependent parameters. We have thus outlined the growth of queuing theory identifying major developments and directions.

2. OBJECTIVE

The main two objectives of the study are

- (i) To study the patients waiting time in hospitals and to investigate the possible operational problems that may lead to excessive patients waiting time in a hospital. A patient's experience in waiting time will radically influence his/her perceptions on quality of service.
- (ii) To use ATM for booking railway tickets besides withdrawing money.

3. METHODOLOGY

For waiting time distribution, the Erlangian techniques or its modifications, the generating functions and Laplace transform will be used. Queuing theory will be used to allocate the number of beds in different wards in hospitals so as reduce the waiting time of admissions. Queuing applications in various public utilities are important. The idea of queuing theory is applied in telephone system which makes it convenient to examine **loss** of system. Applications of idea from queuing theory have found use in landing of aircraft, bed allocation in hospitals etc. It is intended here to make an analytical analysis of bulk queue

and applications of queuing theory in bed allocation problems of hospitals and also in reservation of railway tickets in India.

The steady state solutions for M/M/S system, where balking and renege behavior of customers for limited space is taken into consideration, shall be obtained and the characteristics of waiting time will be calculated. The result of a single server case will also be calculated as a particular case which may tally with the previous results. It is also proposed to study vehicle dispatch strategy for bulk arrival, bulk service queue and busy periods for bulk arrival and bulk service models. The service time of customer is a random variable having a probability distribution function

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4. CONCLUSION

It is obvious that long waiting time affects patient's satisfaction towards the services offered. Improvements should be made to fulfill customer's satisfaction. Three main causes have been identified as the contributing factors towards excessive patients waiting time, namely: the registration of waiting time, insufficient number of counter service staff, insufficient number of doctors and insufficient number of beds. Attempts shall therefore be made to reduce the registration time as well as to improve the doctor-patient ratio and the number of beds so as to reduce the waiting time of admission.

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