

ON THE CONSTRUCTION OF CONFERENCE MATRICES OF ORDER 30

P.K.Manjhi*
Arjun Kumar**

Abstract

In this paper we forward a method of construction of conference matrices of orders 30 by suitable combination of adjacency matrices of suitable coherent configuration.

Keywords:

Coherent Configuration;
Conference matrix;
Symmetric Conference matrix.

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Author correspondence:

Dr. P. K. Manjhi, Assistant Professor, University Department of Mathematics, Vinoba Bhave University, Hazaribag,

1. Introduction:

A weighing matrix M of order m and weight w is an $m \times m$ matrix with entries $(0, \pm 1)$ such that $MM^T = wI_m$, where M^T is transpose of M and I_m is identity matrix of order m . A weighing matrix of order m and weight w is denoted by $M(m, w)$. A $M(m, m)$ is a Hadamard matrix. A $M(m, m-1)$, m even with zeros on the diagonal such that $MM^T = (m-1)I_m$ is conference matrix. If $m \equiv 2 \pmod{4}$ such that $M = M^T$ is symmetric conference matrix. (vide [3]).

$$\text{Exaple: } M(6,5) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

A conference matrix of order m with entries $0, +1$ and -1 is called symmetric conference matrix if $MM^T = M^T M = mI_m$ where M^T is transpose of M and I_m is the identity matrix.

*Assistant Professor, University Department of Mathematics, Vinoba Bhave University, Jharkhand, India.

**Research Scholar, University Department of Mathematics, Vinoba Bhave University, Jharkhand, India.

$$\text{.Example: } M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix} \quad (\text{Vide [5] and [6]})$$

A coherent configuration on a finite set X is a set $C = \{C_1, \dots, C_m\}$ of partition of $X \times X$ satisfying the following four conditions:

- (i) There exist a subset I of $\{1, 2, 3, \dots, m\}$ such that $\bigcup_{i \in I} C_i = \{(x, x) : x \in X\}$
- (ii) $C_i \in C \Rightarrow C_i^{-1} \in C$
- (iii) There exist an integer P_{ij}^k for $1 \leq i, j, k \leq m$ such that for any $(\alpha, \beta) \in C_k$

$$P_{ij}^k = |\{\gamma : (\alpha, \gamma) \in C_i \text{ and } (\gamma, \beta) \in C_j\}|$$
 P_{ij}^k independent of the choice of $(\alpha, \beta) \in C_k$.

Coherent configuration is also defined by adjacency matrices of classes of C . If $\{M_1, \dots, M_m\}$ are adjacency matrices of C_1, \dots, C_m respectively then the axioms take the following form

- (i) $M_1 + \dots + M_m = J$
- (ii) There exist a subset of $\{M_1, \dots, M_m\}$ with sum I =identity matrix;
- (iii) Each element of the set $\{M_1, \dots, M_m\}$ is closed under transposition;
- (iv) $M_i M_j = \sum_{k=1}^m P_{ij}^k M_k$ where P_{ij}^k are non-negative integers.

(Vide[7])

2. MAIN WORK:

In [3],[4] and [5] methods of construction of conference matrices of order 6, 10, 14, 18 and 26 are given, in this paper we forward method of construction of two different symmetric conference matrices each of order 30 by suitable linear combination of coherent configurations:

2.1. CONSTRUCTION OF SYMMETRIC CONFERENCE MATRICES OF ORDER 30

Consider $X = \{1, 2, 3, \dots, 30\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ of $X \times X$ where

$$C_1 = \{(i, i) : i = 1\}, C_2 = \{(1, i) : i = 2, 3, 4, \dots, 30\}, C_3 = \{(i, 1) : i = 2, 3, 4, \dots, 30\}, C_4 = \{(i, i) : i = 2, 3, 4, \dots, 30\},$$

$$C_5 = \{(2, i) : i = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\} \cup \{(3, i) : i = 2, 4, 5, 8, 9, 10, 15, 17, 18, 22, 26, 27, 28, 30\}$$

$$\cup \{(4, i) : i = 2, 3, 5, 6, 9, 14, 16, 17, 21, 23, 24, 27, 28, 29\} \cup \{(5, i) : i = 2, 3, 4, 6, 7, 13, 15, 20, 22, 23, 25, 28, 29, 30\}$$

$$\cup \{(6, i) : i = 2, 4, 5, 7, 8, 12, 14, 19, 21, 22, 24, 26, 29, 30\} \cup \{(7, i) : i = 2, 5, 6, 8, 9, 11, 13, 18, 20, 21, 24, 25, 27, 30\}$$

$$\cup \{(8, i) : i = 2, 3, 6, 7, 9, 10, 12, 17, 19, 20, 24, 25, 26, 28\} \cup \{(9, i) : i = 2, 3, 4, 7, 8, 11, 16, 18, 19, 23, 25, 26, 27, 29\}$$

$$\cup \{(10, i) : i = 2, 3, 8, 11, 12, 15, 16, 18, 20, 21, 22, 24, 28, 29\} \cup \{(11, i) : i = 2, 7, 9, 10, 12, 13, 16, 19, 21, 22, 23, 27, 28, 30\}$$

$$\cup \{(12, i) : i = 2, 6, 8, 10, 11, 13, 14, 17, 20, 22, 23, 26, 27, 29\} \cup \{(13, i) : i = 2, 5, 7, 11, 12, 14, 15, 17, 18, 21, 23, 25, 26, 28\}$$

$$\cup \{(14, i) : i = 2, 4, 6, 12, 13, 15, 16, 17, 18, 19, 22, 24, 25, 27\} \cup \{(15, i) : i = 2, 3, 5, 10, 13, 14, 16, 18, 19, 20, 23, 24, 26, 30\}$$

$$\cup \{(16, i) : i = 2, 4, 9, 10, 11, 14, 15, 17, 19, 20, 21, 25, 29, 30\} \cup \{(17, i) : i = 3, 4, 8, 12, 13, 14, 16, 20, 21, 25, 26, 27, 28, 30\}$$

$$\cup \{(18, i) : i = 3, 7, 9, 10, 13, 14, 15, 21, 22, 24, 25, 26, 27, 29\} \cup \{(19, i) : i = 6, 8, 9, 11, 14, 15, 16, 22, 23, 24, 25, 26, 28, 30\}$$

$$\cup \{(20, i) : i = 5, 7, 8, 10, 12, 15, 16, 17, 23, 24, 25, 27, 29, 30\} \cup \{(21, i) : i = 4, 6, 7, 10, 11, 13, 16, 17, 18, 24, 26, 28, 29, 30\}$$

$$\cup \{(22, i) : i = 3, 5, 6, 10, 11, 12, 14, 18, 19, 25, 27, 28, 29, 30\} \cup \{(23, i) : i = 4, 5, 9, 11, 12, 13, 15, 19, 20, 24, 26, 27, 28, 29\}$$

$$\cup \{(24, i) : i = 4, 6, 7, 8, 10, 14, 15, 18, 19, 20, 21, 23, 27, 28\} \cup \{(25, i) : i = 5, 7, 8, 9, 13, 14, 16, 17, 18, 19, 20, 22, 28, 29\}$$

$$\cup \{(26, i) : i = 3, 6, 8, 9, 12, 13, 15, 17, 18, 19, 21, 23, 29, 30\} \cup \{(27, i) : i = 3, 4, 7, 9, 11, 12, 14, 17, 18, 20, 22, 23, 24, 30\}$$

$$\cup \{(28, i) : i = 3, 4, 5, 8, 10, 11, 13, 17, 19, 21, 22, 23, 24, 25\} \cup \{(29, i) : i = 4, 5, 6, 9, 10, 12, 16, 18, 20, 21, 22, 23, 25, 26\}$$

$$\cup \{(30, i) : i = 3, 5, 6, 7, 11, 15, 16, 17, 19, 20, 21, 22, 26, 27\}.$$

$$C_6 = \{(2, i) : i = 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$$

$$\cup \{(3, i) : i = 6, 7, 11, 12, 13, 14, 16, 19, 20, 21, 23, 24, 25, 29\} \cup \{(4, i) : i = 7, 8, 10, 11, 12, 13, 15, 18, 19, 20, 22, 25, 26, 30\}$$

$$\cup \{(5, i) : i = 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 21, 24, 26, 27, \} \cup \{(6, i) : i = 3, 9, 10, 11, 13, 15, 16, 17, 18, 20, 23, 25, 27, 28\}$$

$$\cup \{(7, i) : i = 3, 4, 10, 12, 14, 15, 16, 17, 19, 22, 23, 26, 28, 29\} \cup \{(8, i) : i = 4, 5, 11, 13, 14, 15, 16, 18, 21, 22, 23, 27, 29, 30\}$$

$$\cup \{(9, i) : i = 5, 6, 10, 12, 13, 14, 15, 17, 20, 21, 22, 24, 28, 30\} \cup \{(10, i) : i = 4, 5, 6, 7, 9, 13, 14, 17, 19, 23, 25, 26, 27, 30\}$$

$$\cup \{(11, i) : i = 3, 4, 5, 6, 8, 14, 15, 17, 18, 20, 24, 25, 26, 29\} \cup \{(12, i) : i = 3, 4, 5, 7, 9, 15, 16, 18, 19, 21, 24, 25, 28, 30\}$$

$$\cup \{(13, i) : i = 3, 4, 6, 8, 9, 10, 16, 19, 20, 22, 24, 27, 29, 30\} \cup \{(14, i) : i = 3, 5, 7, 8, 9, 10, 11, 20, 21, 23, 26, 28, 29, 30\}$$

$$\cup \{(15, i) : i = 4, 6, 7, 8, 9, 11, 12, 17, 21, 22, 25, 27, 28, 29\} \cup \{(16, i) : i = 3, 5, 6, 7, 8, 12, 13, 18, 22, 23, 24, 26, 27, 28, \}$$

$$\cup \{(17, i) : i = 2, 5, 6, 7, 9, 10, 11, 15, 18, 19, 22, 23, 24, 29\} \cup \{(18, i) : i = 2, 4, 5, 6, 8, 11, 12, 16, 17, 19, 20, 23, 28, 30\}$$

$$\cup \{(19, i) : i = 2, 3, 4, 5, 7, 10, 12, 13, 17, 18, 20, 21, 27, 29\} \cup \{(20, i) : i = 2, 3, 4, 6, 9, 11, 13, 14, 18, 19, 21, 22, 26, 28, \}$$

$$\cup \{(21, i) : i = 2, 3, 5, 8, 9, 12, 14, 15, 19, 20, 22, 23, 25, 27\} \cup \{(22, i) : i = 2, 4, 7, 8, 9, 13, 15, 16, 17, 20, 21, 23, 24, 26, \}$$

$$\cup \{(23, i) : i = 2, 3, 6, 7, 8, 10, 14, 16, 17, 18, 21, 22, 25, 30\} \cup \{(24, i) : i = 2, 3, 5, 9, 11, 12, 13, 16, 17, 22, 25, 26, 29, 30\}$$

$$\cup \{(25, i) : i = 2, 3, 4, 6, 10, 11, 12, 15, 21, 23, 24, 26, 27, 30\} \cup \{(26, i) : i = 2, 4, 5, 7, 10, 11, 14, 16, 20, 22, 24, 25, 27, 28, \}$$

$$\cup \{(27, i) : i = 2, 5, 6, 8, 10, 13, 15, 16, 19, 21, 25, 26, 28, 29\} \cup \{(28, i) : i = 2, 6, 7, 9, 12, 14, 15, 16, 18, 20, 26, 27, 29, 30\}$$

$$\cup \{(29, i) : i = 2, 3, 7, 8, 11, 13, 14, 15, 17, 19, 24, 27, 28, 30\} \cup \{(30, i) : i = 2, 4, 8, 9, 10, 12, 13, 14, 18, 23, 24, 25, 28, 29\}.$$

Then adjacency matrices M_1, M_2, M_3, M_4, M_5 and M_6 of C_1, C_2, C_3, C_4, C_5 , and C_6 respectively

.Then by the properties of matrix multiplication we can obtain the following results:

$$(1) M_1 + M_2 + M_3 + M_4 + M_5 + M_6 = J_{30}$$

$$(2) M_1 + M_4 = I_{30} \quad (3) M_1' = M_1, M_2' = M_2, M_3' = M_2, M_4' = M_4, M_5' = M_5, M_6' = M_6.$$

$$\Rightarrow MM^T = M^T M = 29I_{30} = (30-1)I_{30}.$$

Which show that M is a symmetric conference matrix of order 30.

2.2. CONSTRUCTION OF ANOTHER SYMMETRIC CONFERENCE MATRIX OF ORDERS 30:

Consider $X = \{1,2,3,\dots,30\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ of $X \times X$ where

$$\begin{aligned} C_1 = \{(i, i) : i = 1\}, C_2 = \{(1, i) : i = 2, 3, 4, \dots, 30\}, C_3 = \{(i, 1) : i = 2, 3, 4, \dots, 30\}, C_4 = \{(i, i) : i = 2, 3, 4, \dots, 30\}, \\ C_5 = \{(2, i) : i = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\} \cup \{(3, i) : i = 2, 4, 5, 8, 9, 10, 15, 17, 18, 22, 26, 27, 28, 30\} \\ \cup \{(4, i) : i = 2, 3, 5, 6, 9, 14, 16, 17, 21, 23, 24, 27, 28, 29\} \cup \{(5, i) : i = 2, 3, 4, 6, 7, 13, 15, 20, 22, 23, 25, 28, 29, 30\} \\ \cup \{(6, i) : i = 2, 4, 5, 7, 8, 12, 14, 19, 21, 22, 24, 26, 29, 30\} \cup \{(7, i) : i = 2, 5, 6, 8, 9, 11, 13, 18, 20, 21, 24, 25, 27, 30\} \\ \cup \{(8, i) : i = 2, 3, 6, 7, 9, 10, 12, 17, 19, 20, 24, 25, 26, 28\} \cup \{(9, i) : i = 2, 3, 4, 7, 8, 11, 16, 18, 19, 23, 25, 26, 27, 29\} \\ \cup \{(10, i) : i = 2, 3, 8, 11, 12, 15, 16, 18, 20, 21, 22, 24, 28, 29\} \cup \{(11, i) : i = 2, 7, 9, 10, 12, 13, 16, 19, 21, 22, 23, 27, 28, 30\} \\ \cup \{(12, i) : i = 2, 6, 8, 10, 11, 13, 14, 17, 20, 22, 23, 26, 27, 29\} \cup \{(13, i) : i = 2, 5, 7, 11, 12, 14, 15, 17, 18, 21, 23, 25, 26, 28\} \\ \cup \{(14, i) : i = 2, 4, 6, 12, 13, 15, 16, 17, 18, 19, 22, 24, 25, 27\} \cup \{(15, i) : i = 2, 3, 5, 10, 13, 14, 16, 18, 19, 20, 23, 24, 26, 30\} \\ \cup \{(16, i) : i = 2, 4, 9, 10, 11, 14, 15, 17, 19, 20, 21, 25, 29, 30\} \cup \{(17, i) : i = 3, 4, 8, 12, 13, 14, 16, 20, 21, 25, 26, 27, 28, 30\} \\ \cup \{(18, i) : i = 3, 7, 9, 10, 13, 14, 15, 21, 22, 24, 25, 26, 27, 29\} \cup \{(19, i) : i = 6, 8, 9, 11, 14, 15, 16, 22, 23, 24, 25, 26, 28, 30\} \\ \cup \{(20, i) : i = 5, 7, 8, 10, 12, 15, 16, 17, 23, 24, 25, 27, 29, 30\} \cup \{(21, i) : i = 4, 6, 7, 10, 11, 13, 16, 17, 18, 24, 26, 28, 29, 30\} \\ \cup \{(22, i) : i = 3, 5, 6, 10, 11, 12, 14, 18, 19, 25, 27, 28, 29, 30\} \cup \{(23, i) : i = 4, 5, 9, 11, 12, 13, 15, 19, 20, 24, 26, 27, 28, 29\} \\ \cup \{(24, i) : i = 4, 6, 7, 8, 10, 14, 15, 18, 19, 20, 21, 23, 27, 28\} \cup \{(25, i) : i = 5, 7, 8, 9, 13, 14, 16, 17, 18, 19, 20, 22, 28, 29\} \\ \cup \{(26, i) : i = 3, 6, 8, 9, 12, 13, 15, 17, 18, 19, 21, 23, 29, 30\} \cup \{(27, i) : i = 3, 4, 7, 9, 11, 12, 14, 17, 18, 20, 22, 23, 24, 30\} \\ \cup \{(28, i) : i = 3, 4, 5, 8, 10, 11, 13, 17, 19, 21, 22, 23, 24, 25\} \cup \{(29, i) : i = 4, 5, 6, 9, 10, 12, 16, 18, 20, 21, 22, 23, 25, 26\} \\ \cup \{(30, i) : i = 3, 5, 6, 7, 11, 15, 16, 17, 19, 20, 21, 22, 26, 27\}. \end{aligned}$$

$$C_6 = \{(2, i) : i = 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$$

$$\begin{aligned} \cup \{(3, i) : i = 6, 7, 11, 12, 13, 14, 16, 19, 20, 21, 23, 24, 25, 29\} \cup \{(4, i) : i = 7, 8, 10, 11, 12, 13, 15, 18, 19, 20, 22, 25, 26, 30\} \\ \cup \{(5, i) : i = 8, 9, 10, 11, 12, 14, 16, 17, 18, 19, 21, 24, 26, 27\} \cup \{(6, i) : i = 3, 9, 10, 11, 13, 15, 16, 17, 18, 20, 23, 25, 27, 28\} \\ \cup \{(7, i) : i = 3, 4, 10, 12, 14, 15, 16, 17, 19, 22, 23, 26, 28, 29\} \cup \{(8, i) : i = 4, 5, 11, 13, 14, 15, 16, 18, 21, 22, 23, 27, 29, 30\} \\ \cup \{(9, i) : i = 5, 6, 10, 12, 13, 14, 15, 17, 20, 21, 22, 24, 28, 30\} \cup \{(10, i) : i = 4, 5, 6, 7, 9, 13, 14, 17, 19, 23, 25, 26, 27, 30\} \\ \cup \{(11, i) : i = 3, 4, 5, 6, 8, 14, 15, 17, 18, 20, 24, 25, 26, 29\} \cup \{(12, i) : i = 3, 4, 5, 7, 9, 15, 16, 18, 19, 21, 24, 25, 28, 30\} \\ \cup \{(13, i) : i = 3, 4, 6, 8, 9, 10, 16, 19, 20, 22, 24, 27, 29, 30\} \cup \{(14, i) : i = 3, 5, 7, 8, 9, 10, 11, 20, 21, 23, 26, 28, 29, 30\} \\ \cup \{(15, i) : i = 4, 6, 7, 8, 9, 11, 12, 17, 21, 22, 25, 27, 28, 29\} \cup \{(16, i) : i = 3, 5, 6, 7, 8, 12, 13, 18, 22, 23, 24, 26, 27, 28, \} \\ \cup \{(17, i) : i = 2, 5, 6, 7, 9, 10, 11, 15, 18, 19, 22, 23, 24, 29\} \cup \{(18, i) : i = 2, 4, 5, 6, 8, 11, 12, 16, 17, 19, 20, 23, 28, 30\} \\ \cup \{(19, i) : i = 2, 3, 4, 5, 7, 10, 12, 13, 17, 18, 20, 21, 27, 29\} \cup \{(20, i) : i = 2, 3, 4, 6, 9, 11, 13, 14, 18, 19, 21, 22, 26, 28, \} \\ \cup \{(21, i) : i = 2, 3, 5, 8, 9, 12, 14, 15, 19, 20, 22, 23, 25, 27\} \cup \{(22, i) : i = 2, 4, 7, 8, 9, 13, 15, 16, 17, 20, 21, 23, 24, 26, \} \\ \cup \{(23, i) : i = 2, 3, 6, 7, 8, 10, 14, 16, 17, 18, 21, 22, 25, 30\} \cup \{(24, i) : i = 2, 3, 5, 9, 11, 12, 13, 16, 17, 22, 25, 26, 29, 30\} \\ \cup \{(25, i) : i = 2, 3, 4, 6, 10, 11, 12, 15, 21, 23, 24, 26, 27, 30\} \cup \{(26, i) : i = 2, 4, 5, 7, 10, 11, 14, 16, 20, 22, 24, 25, 27, 28, \} \\ \cup \{(27, i) : i = 2, 5, 6, 8, 10, 13, 15, 16, 19, 21, 25, 26, 28, 29\} \cup \{(28, i) : i = 2, 6, 7, 9, 12, 14, 15, 16, 18, 20, 26, 27, 29, 30\} \\ \cup \{(29, i) : i = 2, 3, 7, 8, 11, 13, 14, 15, 17, 19, 24, 27, 28, 30\} \cup \{(30, i) : i = 2, 4, 8, 9, 10, 12, 13, 14, 18, 23, 24, 25, 28, 29\}. \end{aligned}$$

Then adjacency matrices M_1, M_2, M_3, M_4, M_5 and M_6 of $C_1, C_2, C_3, C_4, C_5,$ and C_6 respectively.

Then by the elementary properties of matrices we can find the following results:

[- is written in place of -1]

$\Rightarrow MM^T = M^T M = 29I_{30} = (30-1)I_{30}$. Which show that M is another symmetric conference matrix of order 30.

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