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CERTAIN CENTRALIZER STRUCTURES ON INTUITIONISTIC FUZZY SOFT SUBGROUP

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Abstract: In this study, a group structure on intuitionistic fuzzy soft set called intuitionistic fuzzy soft subgroup is constructed and some of its properties are investigated. Also (t,s)-level set of an intuitionistic fuzzy soft set is defined and some of its properties are obtained.

Keywords: Soft set, intuitionistic fuzzy soft set, soft int-groupoid, intuitionistic fuzzy soft group, level set, index set, extended parameter set, e-set, pre-image, injective function.

1. Introduction: The concept of an intuitionistic fuzzy set was introduced by K. T. Atanassov [3,4,5] as a generalization to the notion of fuzzy sets by L.A. Zadeh [20]. R. Biswas [6] was the first to introduce the intuitionistic fuzzification of algebraic structure and developed the concept of intuitionistic fuzzy subgroup of a group in [4]. Later on many mathematicians worked on it and introduced the notion of intuitionistic fuzzy subring, intuitionistic fuzzy sub module etc. The theory of fuzzy sets has developed in many directions and is finding applications in a wide variety of fields. Rosenfeld in 1971 used this concept to develop the theory of fuzzy group. Aktas and Cagman[1] studied the basic concepts of soft set theory and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences. They also discussed the notion of soft groups. Cagman and Enginoglu[7] modified the definitions of soft set operations and gave a decision making method called Uni-int decision method. F. Feng et.al [8]defined the concept of soft semi rings. Ali et.al [9] defined some new operations on soft set theory such as extended union and intersection, restriction union and intersection. K.Hayat et.al [13] defined applications of double-framed soft ideals in BE-algebra.In 1999, Molodtsov introduced soft set theory [16] as an alternative approach to fuzzy set theory [20] defined by Zadeh in 1965. After Molodtsov's study, many researchers have studied on set theoretical approaches and decision making applications of soft sets. For example Maji et.al [14,15] defined some new operations of soft sets and gave a decision making method on soft sets. Sezgin and Atagun [17,18] studied on soft set operations defined by Ali et.al [9].

In this study, a group structure on intuitionistic fuzzy soft set called intuitionistic fuzzy soft group is constructed and some of its properties are investigated. Also (t,s)-level set of a intuitionistic fuzzy soft set is defined and some of its properties are obtained.

2. Preliminaries:

In this section , we recall basic definitions of soft set theory that are useful for subsequent sections. For more detail see the papers [11,12].

Through out the paper, U refers to an initial universe, E is a set of parameters and P(U) is the power set of U. $\subset$ and $\supset$ stand for proper subset and super set, respectively.

Definition 2.1[12] For any subset A of E, a soft set λ_A over U is a set, defined by a function λ_A , representing the mapping λ_A: E → P(U). A soft set over U can also be represented by the set of ordered pairs $\lambda_A = \{ (x , \lambda_A(x)) ; x \in E , \lambda_A(x) \in P(U) \}$. Note that the set of all soft sets over U will be denoted by S(U).

Definition 2.2[12] Let $\lambda, \mu \in S(U)$. Then

(i) If $\lambda(e) = \emptyset$ for all $e \in E$, λ is said to be a null soft set, denoted by $\emptyset$.

(ii) If $\lambda(e) = U$ for all $e \in E$, λ is said to be an absolute soft set, denoted by U.

(iii) λ is a soft subset of μ , denoted $\lambda \subseteq \mu$, if $\lambda(e) \subseteq \mu(e)$ for all $e \in E$.

(iv) Soft union of λ and μ , denoted by $\lambda \cup \mu$, is a soft set over U and defined by $\lambda \cup \mu: E \rightarrow P(U)$ such that $(\lambda \cup \mu)(e) = \lambda(e) \cup \mu(e)$ for all $e \in E$.

(v) $\lambda = \mu$, if $\lambda \subseteq \mu$ and $\lambda \supseteq \mu$.

(vi) Soft intersection of λ and μ , denoted by $\lambda \cap \mu$, is a soft set over U and defined by $\lambda \cap \mu: E \rightarrow P(U)$ such that $(\lambda \cap \mu)(e) = \lambda(e) \cap \mu(e)$ for all $e \in E$.

(vii) Soft complement of λ is denoted by λ^C and defined by $\lambda^C : E \rightarrow P(U)$ such that $\lambda^C(e) = U/\lambda(e)$ for all $e \in E$.

Definition 2.3[12]: Let E be a parameter set, $S \subset E$ and $\lambda: S \rightarrow E$ be an injection function. Then

$S \cup \lambda(s)$ is called extended parameter set of S and denoted by ξ_S.

If S=E , then extended parameter set of S will be denoted by ξ.

Definition 2.4[20] : Let $\mu_{\tilde{A}}$: U ⟶ [0,1] be any function and A be a crisp set in the universe 'U'. Then the ordered pairs $\tilde{A}$ = {(x, $\mu_{\tilde{A}}$ (x)) / x ∈ U} is called a fuzzy set and $\mu_{\tilde{A}}$ is called a membership function.

Definition 2.5[3]: An intuitionistic fuzzy pair $\langle(\bar{\alpha},\bar{\lambda});\ G\rangle$ is called an intuitionistic fuzzy soft set (briefly IFS-set) over U where $\bar{\alpha}$ and $\bar{\lambda}$ are mapping from A to P(U).

For an IFS-set $\langle(\bar{\alpha},\bar{\lambda});\ G\rangle$ over U and two subsets γ and δ of U, the γ-inclusive set and the δ-exclusive set of $\langle(\bar{\alpha},\bar{\lambda});\ G\rangle$, denoted by i_A ($\bar{\alpha}$; γ) and $e_A(\bar{\lambda};\delta)$ respectively, are defined as follows.

i_A ($\bar{\alpha}$; γ) = { x ∈ A / γ ⊆ $\bar{\alpha}$ (x) } and $e_A(\bar{\lambda}, \delta)$ = { x ∈ A / δ ⊆ $\bar{\lambda}$(x) } respectively.

The set $IF_A(\bar{\alpha},\bar{\lambda})_{(\gamma,\delta)}$ = { x ∈ A / γ ⊆ $\bar{\alpha}$ (x), δ ⊆ $\bar{\lambda}$(x) } is called an intuitionistic fuzzy including set of $\langle(\bar{\alpha},\bar{\lambda});\ G\rangle$. It is clear that $IF_A(\bar{\alpha},\bar{\lambda})_{(\gamma,\delta)} = i_A(\bar{\alpha};\gamma) \cap e_A(\bar{\lambda};\delta)$.

From now on, we will take G, as set of parameters, which is a group unless otherwise specified.

Note:2.6 Let λ_S = ($\overline{\alpha_S}$, $\overline{\beta_S}$;E) be an intuitionistic fuzzy soft set over U. We will say that λ_S(e)= ($\overline{\alpha_S}$(e) , $\overline{\beta_S}$(e)) is image of parameter e ∈ E.

Definition 2.7: Let λ_A and λ_B ∈ IFS_E(U) .Then

(i) If α_A(e) = ∅ and β_A(e)=U for all e ∈ E , λ_A is said to be a null intuitionistic fuzzy soft set, denoted by $\emptyset_b$ = (∅,U,E).

(ii) If α_A(e) = U and β_A(e)= ∅ for all e ∈ E , λ_A is said to be an absolute intuitionistic fuzzy set, denoted by $\emptyset_b$ = (U, ∅,E).

(iii) λ_A is an intuitionistic fuzzy soft subset of λ_B, denoted by $\lambda_A \subseteq \lambda_B$, if $\alpha_A(e) \subseteq \alpha_B(e)$ and $\beta_A(e) \supseteq \beta_B(e)$for all e ∈ E.

(iv) An intuitionistic fuzzy soft union and intersection of λ_A and λ_B, denoted by ($\alpha_A \cup \alpha_B$) : A∪B → P(U) such that $(\alpha_A \cup \alpha_B)(e) = \alpha_A(e) \cup \alpha_B(e)$ and $(\beta_A \cap \beta_B)(e) = \beta_A(e) \cap \beta_B(e)$ for all e ∈ E.
Also ($\alpha_A \cap \alpha_B$) : A∩B → P(U) such that $(\alpha_A \cap \alpha_B)(e) = \alpha_A(e) \cap \alpha_B(e)$ and $(\beta_A \cup \beta_B)(e) = \beta_A(e) \cup \beta_B(e)$ for all e ∈ E.

(v) An intuitionistic fuzzy soft complement of λ_A is denoted by ${\lambda_A}^C$ and defined by ${\lambda_A}^C$: E→ P(U) × P(U) such that ${\lambda_A}^C(e)$ = {(e, α_A(e), β_A(e)) / e ∈ E }.

3. INTUITIONISTIC FUZZY SOFT ACTION SUBGROUP

In this section, first of all we give the definition of soft intersection group (Soft int-group) defined by Cagman et.al[1]. Then, we define an intuitionistic fuzzy soft action group (IFS-group) structure and investigate some of its properties.

Definition 3.1[14]: Let G be a group and $\emptyset_G \in S(U)$. Then $\emptyset_G$ is called soft intersection groupoid over U if $\emptyset_G(xy) \supseteq \emptyset_G(x) \cap \emptyset_G(y)$ for all x, y ∈ G.

If , for all x ∈ G, the soft intersection groupoid satisfies $\emptyset_G(x^{-1}) = \emptyset_G(x)$, then $\emptyset_G$ is called a soft intersection group over U.

Definition 3.2: Let G be a group. Let λ: G → G be injection function. Then $\lambda_G = (\tilde{\alpha},\tilde{\beta}; G)$ $\in IFS_G(U)$ is called an intuitionistic fuzzy soft action groupoid (IFS-groupoid) over U if $\lambda_G(xy) \supseteq \lambda_G(x) \cap \lambda_G(y)$ for all x, y ∈ G. Here $\lambda_G(xy) \supseteq \lambda_G(x) \cap \lambda_G(y)$ means that $\tilde{\alpha}(xy) \supseteq \tilde{\alpha}(x) \cap \tilde{\alpha}(y)$ and $\tilde{\beta}(xy) \subseteq \tilde{\beta}(x) \cap \tilde{\beta}(y)$.

Definition 3.3: Let λ_G be an intuitionistic fuzzy soft action groupoid over U. If $\lambda_G(x^{-1}) = \lambda_G(x)$, then λ_G is called an intuitionistic fuzzy soft action group (IFS-group) and denoted by λ_G.

Clearly, a IFS-set $\langle(\overline{\alpha_G}, \overline{\beta_G}); G\rangle$ over U is called an intuitionistic fuzzy soft action group

(briefly IFS-group) over U if it satisfies:

(IFS-G_1) $\overline{\alpha_G}(xy) \supseteq \overline{\alpha_G}(x) \cap \overline{\alpha_G}(y)$ and $\overline{\beta_G}(xy) \subseteq \overline{\beta_G}(x) \cup \overline{\beta_G}(y)$,

(IFS-G_2) $\overline{\alpha_G}(x^{-1}) = \overline{\alpha_G}(x)$ and $\overline{\beta_G}(x^{-1}) = \overline{\beta_G}(x)$ for all x, y ∈ G.

Example 3.1: Assume that $U = \{u_1, u_2, u_3,\dots, u_{14}\}$ is a universal set and $G = Z_5$ be the subset of parameters. We define an IFS-set λ_G by

$\overline{\alpha_G}(0) = \{u_1, u_2, u_3,\dots, u_6\}$ and $\overline{\beta_G}(0) = \{u_{10}, u_{12}\}$

$\overline{\alpha_G}(1) = \{u_1, u_2, u_3,\dots, u_5\}$ and $\overline{\beta_G}(1) = \{u_9, u_{12}, u_{13}, u_{14}\}$

$\overline{\alpha_G}(2) = \{u_2, u_3, u_6\}$ and $\overline{\beta_G}(2) = \{u_4, u_5, u_{10}, u_{11}, u_{12}, u_{14}\}$

$\overline{\alpha_G}(3) = \{u_2, u_3, u_6\}$ and $\overline{\beta_G}(3) = \{u_4, u_5, u_{10}, u_{11}, u_{12}, u_{14}\}$

$\overline{\alpha_G}(4) = \{u_1, u_2, u_3,\dots, u_5\}$ and $\overline{\beta_G}(4) = \{u_9, u_{12}, u_{13}, u_{14}\}$

Here λ_G is not an IFS-group over U , because here $\overline{\beta_G}(1.4) \nsubseteq \overline{\beta_G}(1) \cup \overline{\beta_G}(4)$.

Example 3.2: Consider the group $G = \{1,\omega,\omega^2\}$ with respect to the binary operation " complex number multiplication" where ω is the imaginary root of unity. Clearly, an IFS-set

$A = \{(1,0.9,0.1), (\omega, 0.6,0.2), (\omega^2, 0.6,0.2)\}$ is an IFS- group of the group G.

Definition 3.4: Let $\lambda_G = (\overline{\alpha_G}, \overline{\beta_G}; G)$ be an IFS-set over U. If, for all $x \in G$, $\overline{\alpha_G}(x) \cup \overline{\beta_G}(x) = U$, an IFS-set $\lambda_G = (\overline{\alpha_G}, \overline{\beta_G}; G)$ is called full IFS-set.

Example 3.3: In example: 3.1 , if we take as $(\overline{\alpha_G}(x))^C = \overline{\beta_G}(x)$ for all $x \in G$, then λ_G is a full IFS-set and so λ_G is an IFS-group .

Theorem 3.1: Let λ_G be full IFS-set over U. Then λ_G is an IFS action group if and only if $\overline{\alpha_G}$ is soft int-group.

Proof: The proof is clear from Definition: 3.1 and 3.2.

Theorem 3.2: Let λ_G be an IFS action group over U. Then,

(i) $\lambda_G(e) \supseteq \lambda_G(x)$ for all $x \in G$.

(ii) $\lambda_G(xy) \supseteq \lambda_G(y)$ if and only if $\lambda_G(x) = \lambda_G(e)$.

Proof: (i) Since λ_G is an IFS action group over U,

$$\overline{\alpha_G}(x\,x^{-1}) \supseteq \overline{\alpha_G}(x) \cap \overline{\alpha_G}(x^{-1}) = \overline{\alpha_G}(x) \cap \overline{\alpha_G}(x) = \overline{\alpha_G}(x)$$

$$\overline{\beta_G}(x\,x^{-1}) \subseteq \overline{\beta_G}(x) \cup \overline{\beta_G}(x^{-1}) = \overline{\beta_G}(x) \cup \overline{\beta_G}(x) = \overline{\beta_G}(x), \text{ for all } x \in G.$$

(ii) Suppose that $\lambda_G(xy) \supseteq \lambda_G(x)$ for all $y \in G$. Then by choosing $y = e$, we have that

$$\overline{\alpha_G}(x) \supseteq \overline{\alpha_G}(e) \text{ and } \overline{\beta_G}(x) \subseteq \overline{\beta_G}(e), \text{ so } \lambda_G(x) \supseteq \lambda_G(e), \text{ by (i) } \lambda_G(x) = \lambda_G(e).$$

Conversely, suppose that $\lambda_G(x) = \lambda_G(e)$. Then

$$\overline{\alpha_G}(xy) \supseteq \overline{\alpha_G}(x) \cap \overline{\alpha_G}(y) = \overline{\alpha_G}(e) \cap \overline{\alpha_G}(y) = \overline{\alpha_G}(y) \text{ and}$$

$$\overline{\beta_G}(xy) \subseteq \overline{\beta_G}(x) \cup \overline{\beta_G}(y) = \overline{\beta_G}(e) \cup \overline{\beta_G}(y) = \overline{\beta_G}(y).$$

Theorem 3.3: An intuitionistic fuzzy soft set λ_G over U is an IFS action group over U if and only if $\lambda_G(x\,y^{-1}) \supseteq \lambda_G(x) \cap \lambda_G(y)$ for all $x, y \in G$.

Proof: Assume that λ_G be an IFS action group over U. Then

$$\overline{\alpha_G}(x\,y^{-1}) \supseteq \overline{\alpha_G}(x) \cap \overline{\alpha_G}(y^{-1}) = \overline{\alpha_G}(x) \cap \overline{\alpha_G}(y) \text{ and}$$

$$\overline{\beta_G}(x\,y^{-1}) \subseteq \overline{\beta_G}(x) \cup \overline{\beta_G}(y^{-1}) = \overline{\beta_G}(x) \cup \overline{\beta_G}(y) \text{ for all } x, y \in G.$$

Conversely, let $\lambda_G(x\,y^{-1}) \supseteq \lambda_G(x) \cap \lambda_G(y)$ for all $x, y \in G$.

If we take $x = e$,

$$\overline{\alpha_G}(y^{-1}) \supseteq \overline{\alpha_G}(y) \text{ and } \overline{\beta_G}(y^{-1}) \subseteq \overline{\beta_G}(y). \text{ Hence,}$$

$$\overline{\alpha_G}(y) = \overline{\alpha_G}((y^{-1})^{-1}) \supseteq \overline{\alpha_G}(y^{-1}) \text{ and } \overline{\beta_G}(y) = \overline{\beta_G}((y^{-1})^{-1}) \subseteq \overline{\beta_G}(y^{-1}).$$

Thus, $\lambda_G(y) = \lambda_G(y^{-1})$.

If we take $x \neq e$,

$$\overline{\alpha_G}(xy) = \overline{\alpha_G}(x\,(y^{-1})^{-1}) \supseteq \overline{\alpha_G}(x) \cap \overline{\alpha_G}(y^{-1}) = \overline{\alpha_G}(x) \cap \overline{\alpha_G}(y) \text{ and}$$

$\overline{\beta_G}$ (xy) = $\overline{\beta_G}$(x ($y^{-1})^{-1}$) ⊆ $\overline{\beta_G}$(x) ∪ $\overline{\beta_G}$(y^{-1}) = $\overline{\beta_G}$(x) ∪ $\overline{\beta_G}$(y) for all x, y ∈ G.

Therefore, λ_G is an IFS action group.

Theorem 3.4: Let λ_G be an IFS action group over U. Then $\lambda_G(x^n) \supseteq \lambda_G(x)$ for all x ∈ G, when n ∈ G.

Proof: Suppose that λ_G is an IFS action group over U. Then

$\overline{\alpha_G}$ $(x^n) \supseteq \overline{\alpha_G}(x) \cap \overline{\alpha_G}(x) \cap \ldots \cap \overline{\alpha_G}(x) = \overline{\alpha_G}(x)$ and

$\overline{\beta_G}$ $(x^n) \subseteq \overline{\beta_G}(x) \cup \overline{\beta_G}(x) \cup \ldots \cup \overline{\beta_G}(x) = \overline{\beta_G}(x)$, for all x ∈ G.

Thus , $\lambda_G(x^n) \supseteq \lambda_G(x)$.

Theorem 3.5: Let λ_G be an IFS action group over U. If for all x, y ∈ G, $\overline{\alpha_G}(x\,y^{-1})$ = U and $\overline{\beta_G}(x\,y^{-1}) = \emptyset$, then $\lambda_G(x) = \lambda_G(y)$.

Proof:

For all x, y ∈ G,

$\overline{\alpha_G}(x) = \overline{\alpha_G}((x\,y^{-1})\,y) \supseteq \overline{\alpha_G}(x\,y^{-1}) \cap \overline{\alpha_G}(y) = U \cap \overline{\alpha_G}(y) = \overline{\alpha_G}(y)$ and

$\overline{\alpha_G}(y) = \overline{\alpha_G}(\,y^{-1}) = \overline{\alpha_G}(\,x^{-1}(xy^{-1})) \supseteq \overline{\alpha_G}(\,x^{-1}) \cap \overline{\alpha_G}(x\,y^{-1}\,) = \overline{\alpha_G}(\,x^{-1}) \cap U$

$= \overline{\alpha_G}(x)$

Thus, $\overline{\alpha_G}(x) = \overline{\alpha_G}(y)$…………… (1)

Also, $\overline{\beta_G}(x) = \overline{\beta_G}((x\,y^{-1})\,y) \subseteq \overline{\beta_G}(x\,y^{-1}) \cup \overline{\beta_G}(y) = \emptyset \cup \overline{\beta_G}(y) = \overline{\beta_G}(y)$ and

$\overline{\beta_G}(y) = \overline{\beta_G}(\,y^{-1}) = \overline{\beta_G}(\,x^{-1}(xy^{-1})) \subseteq \overline{\beta_G}(\,x^{-1}) \cup \overline{\beta_G}(x\,y^{-1}\,) = \overline{\beta_G}(\,x^{-1}) \cup \emptyset$

$= \overline{\beta_G}(x)$

Thus, $\overline{\beta_G}(x) = \overline{\beta_G}(y)$……………(2).From (1) and (2), $\lambda_G(x) = \lambda_G(y)$.

Definition 3.5: Let λ_G be an IFS-set. Then *e*-set of λ_G, denoted by e_{λ_G}, is defined as $e_{\lambda_G} = \{x \in G \ / \ \lambda_G(x) = \lambda_G(e)\}$.

Example 3.4: Let us consider Klein-four group over G={ e, x, y, z } given as in following cayley table.

	e	x	y	z
e	e	x	y	z
x	x	e	z	y
y	y	z	e	x
z	z	y	x	e

And, let λ_G be an IFS action group over $U = \{u_1, u_2, u_3, \ldots, u_7\}$ with $\overline{\alpha_G}$ and $\overline{\beta_G}$ given as follows.

$\overline{\alpha_G}(e) = \{u_1, u_3, u_7\}$ and $\overline{\beta_G}(e) = \{u_2, u_4, u_6\}$

$\overline{\alpha_G}(x) = \{u_2, u_3, u_4\}$ and $\overline{\beta_G}(x) = \{u_1, u_5\}$

$\overline{\alpha_G}(y) = \{u_1, u_3, u_7\}$ and $\overline{\beta_G}(y) = \{u_2, u_4\}$

$\overline{\alpha_G}(z) = \{u_1, u_3, u_7\}$ and $\overline{\beta_G}(z) = \{u_2, u_4, u_6\}$

Then, $e_{\lambda_G} = \{e, Z\}$

$= \overline{\beta_G}(e) \cup \overline{\beta_G}(e) = \overline{\beta_G}(e)$ ....(1)

From theorem:3.2 , we know that $\overline{\beta_G}(e) \subseteq \overline{\beta_G}(x\, y^{-1})$.

Thus, $\overline{\beta_G}(e) = \overline{\beta_G}(x\, y^{-1})$.............. (2)

From (1) and (2), $\lambda_G(e) = \lambda_G(x\, y^{-1})$ and $x\, y^{-1} \in e_{\lambda_G}$. Hence e_{λ_G} is a subgroup of G.

Theorem 3.6: Let λ_G and μ_G be two IFS action groups over U. Then $\lambda_G \cap \mu_G$ is also an IFS action group over U.

Proof: Let $x, y \in G$. Then ,

$$(\overline{\alpha_G} \cap \mu_G)(x\, y^{-1}) \supseteq \overline{\alpha_G}(x\, y^{-1}) \cap \mu_G(x\, y^{-1})$$
$$\supseteq (\overline{\alpha_G}(x) \cap \overline{\alpha_G}(y)) \cap (\mu_G(x) \cap \mu_G(y))$$
$$= (\overline{\alpha_G}(x) \cap \mu_G(x)) \cap (\overline{\alpha_G}(y) \cap \mu_G(y))$$
$$= (\overline{\alpha_G} \cap \mu_G)(x) \cap (\overline{\alpha_G} \cap \mu_G)(y)$$

And, $(\overline{\beta_G} \cap \mu_G)(x\, y^{-1}) \subseteq \overline{\beta_G}(x\, y^{-1}) \cap \mu_G(x\, y^{-1})$

$$\subseteq (\overline{\beta_G}(x) \cup \overline{\beta_G}(y)) \cap (\mu_G(x) \cup \mu_G(y))$$
$$= (\overline{\beta_G}(x) \cap \mu_G(x)) \cup (\overline{\beta_G}(y) \cap \mu_G(y))$$
$$= (\overline{\beta_G} \cap \mu_G)(x) \cup (\overline{\beta_G} \cap \mu_G)(y)$$

This implies that $(\overline{\alpha_G} \cap \mu_G)(x\, y^{-1}) \supseteq (\overline{\alpha_G} \cap \mu_G)(x) \cap (\overline{\alpha_G} \cap \mu_G)(y)$ and $(\overline{\beta_G} \cap \mu_G)(x\, y^{-1}) \subseteq (\overline{\beta_G} \cap \mu_G)(x) \cup (\overline{\beta_G} \cap \mu_G)(y)$. Hence $(\lambda_G \cap \mu_G)(x\, y^{-1}) \supseteq (\lambda_G)(x\, y^{-1}) \cap (\mu_G)(x\, y^{-1})$ is an IFS action group over U.

Note that $\lambda_G \cup \mu_G$ is not an IFS action group over U in general.

Example 3.5: Let $G = Z_6$ be the set of parameters and U = Z be the universal set. If we construct two IFS action groups λ_G and μ_G over U by

$\overline{\alpha_G}(0) = Z$ $\quad\quad$ $\overline{\beta_G}(0) = \emptyset$

$\overline{\alpha_G}(1) = \{5, 6, 9, 16\}$ $\quad\quad$ $\overline{\beta_G}(1) = \{0, 1, 4, 12, 13, 14, 19\}$

$\overline{\alpha_G}(2) = Z$ $\quad\quad$ $\overline{\beta_G}(2) = \emptyset$

$\overline{\alpha_G}(3) = \{5, 6, 9, 16\}$ $\quad\quad$ $\overline{\beta_G}(3) = \{0, 1, 4, 12, 13, 14, 19\}$

$\overline{\alpha_G}(4) = Z$ $\quad\quad$ $\overline{\beta_G}(4) = \emptyset$

$\overline{\alpha_G}(5) = \{5, 6, 9, 16\}$ $\quad\quad$ $\overline{\beta_G}(5) = \{0, 1, 4, 12, 13, 14, 19\}$

And,

$\overline{\mu_G}(0) = Z$ $\quad\quad$ $\overline{\beta_G}(0) = \emptyset$

$\overline{\mu_G}(1) = \{7, 8, 11, 18\}$ $\quad\quad$ $\overline{\beta_G}(1) = \{2, 3, 6, 14, 18, 19\}$

$\overline{\mu_G}(2) = \{7, 8, 11, 18\}$ $\quad\quad$ $\overline{\beta_G}(2) = \{2, 3, 6, 14, 18, 19\}$

$\overline{\mu_G}(3) = Z$ $\quad\quad$ $\overline{\beta_G}(3) = \emptyset$

$\overline{\mu_G}(4) = \{7, 8, 11, 18\}$ $\quad\quad$ $\overline{\beta_G}(4) = \{2, 3, 6, 14, 18, 19\}$

$\overline{\mu_G}(5) = \{7, 8, 11, 18\}$ $\quad\quad$ $\overline{\beta_G}(5) = \{2, 3, 6, 14, 18, 19\}$

Here, $(\alpha_G \cup \mu_G)(0) = \overline{\alpha_G}(0) \cup \overline{\mu_G}(0) = Z$

$(\alpha_G \cup \mu_G)(1) = \overline{\alpha_G}(1) \cup \overline{\mu_G}(1) = \{5, 6,7, 8, 9, 11, 16, 18\}$

$(\alpha_G \cup \mu_G)(2) = \overline{\alpha_G}(2) \cup \overline{\mu_G}(2) = Z$

$(\alpha_G \cup \mu_G)(3) = \overline{\alpha_G}(3) \cup \overline{\mu_G}(3) = Z$

$(\alpha_G \cup \mu_G)(4) = \overline{\alpha_G}(0) \cup \overline{\mu_G}(4) = Z$

Here since $(\lambda_G \cup \mu_G)(3+4) \not\supseteq (\lambda_G \cup \mu_G)(3) \cap (\lambda_G \cup \mu_G)(4)$.

Hence, $(\lambda_G \cup \mu_G)$ is not an IFS action group over U.

Definition 3.6: Let H be a subgroup of G , λ_G be an IFS action group over U and λ_H be a non-empty IFS set of λ_G over U. If λ_H is an IFS action group over U, then λ_H is called an IFS action subgroup of λ_G over U and denoted by $\lambda_H \leq \lambda_G$.

Example 3.6: Let us consider, an IFS action group λ_G over U in example:3.5, and let H={0, 2, 4}≤ G. If we define a IFS set λ_H by

$\overline{\alpha_H}(0) = \overline{\alpha_H}(2) = \overline{\alpha_H}(4) = Z$ and $\overline{\beta_H}(0) = \overline{\beta_H}(2) = \overline{\beta_H}(4) = \emptyset$

Then, λ_H is an IFS action subgroup of λ_G over U.

Theorem 3.7: Let λ_G be an IFS action group over U and λ_H , λ_K be two IFS action subgroups of

λ_G over U. Then $\lambda_H \cap \lambda_K \leq \lambda_G$ over U.

Proof: Let x, y ∈ G. Then ,

$$\overline{\alpha_{H\cap K}}(x\,y^{-1}) = \overline{\alpha_H}(x\,y^{-1}) \cap \overline{\alpha_K}(x\,y^{-1})$$
$$\supseteq (\overline{\alpha_H}(x) \cap \overline{\alpha_H}(y)) \cap (\overline{\alpha_K}(x) \cap \overline{\alpha_K}(y))$$
$$= (\overline{\alpha_H}(x) \cap \overline{\alpha_K}(x)) \cap (\overline{\alpha_H}(y) \cap \overline{\alpha_K}(y))$$
$$= \overline{\alpha_{H\cap K}}(x) \cap \overline{\alpha_{H\cap K}}(y)$$

And, $\overline{\beta_{H\cap K}}(x\,y^{-1}) = \overline{\beta_H}(x\,y^{-1}) \cup \overline{\beta_K}(x\,y^{-1})$

$\subseteq \quad (\overline{\beta_H}(x) \cup \overline{\beta_H}(y)) \quad \cup(\overline{\beta_K}(x) \cup \overline{\beta_K}(y)=(\overline{\beta_H}(x) \cup \overline{\beta_K}(x)) \quad \cup$ $(\overline{\beta_H}(y) \cup \overline{\beta_K}(y))$

$= \overline{\beta_{H\cap K}}(x) \cup \overline{\beta_{H\cap K}}(y)$

Therefore, $\lambda_H \cap \lambda_K$ is an IFS action subgroup over U. Note that $\lambda_H \cup \lambda_K$ is not an IFS action subgroup over U in general.

Theorem 3.8: Let λ_{G_i} be a family of an IFS action subgroups over U for all i ∈ I. Then $\bigcap_{i \in I} \lambda_{G_i}$ is an IFS action subgroup over U.

Proof: Let x, y ∈ G. Since λ_{G_i} be an IFS action subgroup over U. This implies that

$\alpha_{G_i}(x y^{-1}) \supseteq \alpha_{G_i}(x) \cap \alpha_{G_i}(y)$ for all i ∈ I. Then

$\bigcap_{i \in I} \alpha_{G_i}(x y^{-1}) \supseteq \bigcap_{i \in I}(\alpha_{G_i}(x) \cap \alpha_{G_i}(y)) = (\bigcap_{i \in I} \alpha_{G_i}(x)) \cap (\bigcap_{i \in I} \alpha_{G_i}(y))$ and

$\beta_{G_i}(x y^{-1}) \subseteq \beta_{G_i}(x) \cup \beta_{G_i}(y)$ for all i ∈ I. Then

$\bigcup_{i \in I} \beta_{G_i}(x y^{-1}) \subseteq \bigcup_{i \in I}(\beta_{G_i}(x) \cup (\beta_{G_i}(y)) = (\bigcup_{i \in I} \beta_{G_i}(x)) \cup(\bigcup_{i \in I} \beta_{G_i}(y))$

Thus, $\bigcap_{i \in I} \lambda_{G_i}$ is an IFS action subgroup over U

Definition 3.7: Let λ_G be an IFS action group over U. For any x ∈ G, centralizer of $(x, \alpha_G(x), \beta_G(x)) \in \lambda_G$ defined as follows:

$$M_{\lambda_G}(x) = \{(h, \alpha_G(h), \beta_G(h)) \in G;\ xh = hx\}.$$

Example 3.7: Let us consider, an IFS action group λ_G over U in example:3.5. The centralizer of

$\{ (u_1, u_2, u_3, u_4), (u_1, u_3, u_5, u_7, u_{10}, u_{14}),(u_{10}, u_{11}, u_{12}, u_{15}) \}$ can be obtained as follows

$M_{\lambda_G}(u_1, u_2, u_3, u_4) = \{ (u_1, U, \emptyset), (u_1, u_2, u_3, u_4),(u_1, u_3, u_5, u_7, u_{10}, u_{14}),(u_7, u_8) \}$.

Theorem 3.9 : Let λ_G be an IFS action subgroup over U and $M_{\lambda_G}(x)$ centralizer of x ∈ G.

Then $M_{\lambda_G}(x)$ is an IFS action subgroup of λ_G.

Proof: The proof is clear from definition 3.7.

4. (t ,s)-level of intuitionistic fuzzy soft set.

Definition 4.1: Let λ_A be an IFS- set over U. Then (t,s)-level of an IFS- set λ_A, denoted by $\lambda_A{}^{(t,s)}$,is defined as follows, $\lambda_A{}^{(t,s)}=\{x \in A / \overline{\alpha_A}(x) \supseteq t \text{ and } \overline{\beta_A}(x) \subseteq s\}$. Here $t \cap s = \emptyset$.

Note that if $t = \emptyset$ or $s = U$, then $\lambda_A{}^{(t,s)}=\{x \in A / \overline{\alpha_A}(x) \neq \emptyset \text{ and } \overline{\beta_A}(x) \neq U\}$ is called support of λ_A , and denoted by supp(λ_A).

Example 4.1: Let the universal set $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ be an initial universe and

$E = \{e_1, e_2, e_3, e_4, e_5\}$ be the parameter set. If we define an intuitionistic fuzzy soft set as follows:

$\overline{\alpha_A}(e_1) = \{\emptyset\}$ and $\overline{\beta_A}(e_1) = \{u_1, u_2, u_3\}$

$\overline{\alpha_A}(e_2) = \{u_2, u_3, u_4, u_5, u_6\}$ and $\overline{\beta_A}(e_2) = \{u_1\}$

$\overline{\alpha_A}(e_3) = \{u_5, u_6, u_7\}$ and $\overline{\beta_A}(e_3) = \{u_2, u_4\}$

$\overline{\alpha_A}(e_4) = \{\emptyset\}$ and $\overline{\beta_A}(e_4) = \{U\}$

$\overline{\alpha_A}(e_5) = \{u_3, u_5, u_7\}$ and $\overline{\beta_A}(e_5) = \{u_2, u_4\}$.

Let $t = \{u_5, u_7\}$ and $s = \{u_2, u_4, u_6\}$. Then $\lambda_A{}^{(t,s)} = \{e_3, e_5\}$.

Proposition 4.1: Let λ_A and λ_B be two IFS-sets over U. A,B $\subseteq$ E. Then the following assertions hold:

1. $\lambda_A \subseteq \lambda_B \Rightarrow \lambda_A{}^{(t,s)} \subseteq \lambda_B{}^{(t\,s)}$, for all t,s $\subseteq$ U such that $t \cap s = \emptyset$.
2. If $t_1 \subseteq t_2$ and $s_2 \subseteq s_1$, then $\lambda_A{}^{(t_2,s_2)} \subseteq \lambda_A{}^{(t_1,s_1)}$, for all $t_1, t_2, s_1, s_2 \subseteq U$ such that $t_1 \cap s_1 = \emptyset$, $t_2 \cap s_2 = \emptyset$.
3. $\lambda_A = \lambda_B \Leftrightarrow \lambda_A{}^{(t,s)} = \lambda_B{}^{(t,s)}$, for all t ,s $\subseteq$ U such that $t \cap s = \emptyset$.

Proof: Suppose that λ_A and λ_B are two IFS-sets over U.

1. Let $x \in \lambda_A{}^{(t,s)}$. Then $\overline{\alpha_A}(x) \supseteq t$ and $\overline{\beta_A}(x) \subseteq s$. Since $\lambda_A \subseteq \lambda_B$, $t \subseteq \overline{\alpha_A}(x) \subseteq \overline{\alpha_B}(x)$ and

   $\overline{\beta_A}(x) \supseteq \overline{\beta_B}(x) \supseteq s$, for all $x \in G$. This implies that $x \in \lambda_B{}^{(t,s)}$. Hence $\lambda_A{}^{(t,s)} \subseteq \lambda_B{}^{(t,s)}$.
2. Let $t_1 \subseteq t_2$, $s_2 \subseteq s_1$ and $x \in \lambda_A{}^{(t_2,s_2)}$. Then $\overline{\alpha_A}(x) \supseteq t_2$ and $\overline{\beta_A}(x) \subseteq s_2$.

   Since $t_1 \subseteq t_2$ and $s_2 \subseteq s_1$, $\overline{\alpha_A}(x) \supseteq t_1$ and $\overline{\beta_A}(x) \subseteq s_1$. This implies that $x \in \lambda_A{}^{(t_1,s_1)}$.

   Hence $\lambda_A{}^{(t_2,s_2)} \subseteq \lambda_A{}^{(t_1,s_1)}$.
3. The proof is clear .

Theorem 4.1 : Let λ_A and λ_B be two IFS-sets over U. A,B $\subseteq$ E and t,s $\subseteq$ E such that $t \cap s = \emptyset$.

Then, (1) $\lambda_A{}^{(t,s)} \cup \lambda_B{}^{(t,s)} \subseteq (\lambda_A \cup \lambda_B)^{(t,s)}$ and (2) $\lambda_A{}^{(t,s)} \cap \lambda_B{}^{(t,s)} = (\lambda_A \cap \lambda_B)^{(t,s)}$.

Proof:

1. For all $x \in E$, let $x \in \lambda_A{}^{(t,s)} \cup \lambda_B{}^{(t,s)}$

   $\Rightarrow (\overline{\alpha_A}(x) \supseteq t$ and $\overline{\beta_A}(x) \subseteq s) \cup (\overline{\alpha_B}(x) \supseteq t$ and $\overline{\beta_B}(x) \subseteq s)$.

$\Rightarrow(\overline{\alpha_A}(x) \cup \overline{\alpha_B}(x) \supseteq t)$ or $(\overline{\beta_A}(x) \cap \overline{\beta_B}(x) \subseteq s)$.

$\Rightarrow x \in (\lambda_A \cup \lambda_B)^{(t,s)}$.

2. Similar to proof (1).

Theorem 4.2: Let I be an index set and λ_{A_i} be a family of an IFS-sets over U. Then, for any

t,s $\subseteq$ U such that t$\cap$s = $\emptyset$. Then,

1. $\bigcup_{i\in I}(\lambda_{A_i})^{(t,s)} \subseteq (\bigcup_{i\in I} \lambda_{A_i})^{(t,s)}$
2. $\bigcap_{i\in I}(\lambda_{A_i})^{(t,s)} = (\bigcap_{i\in I} \lambda_{A_i})^{(t,s)}$

Theorem 4.3: Let λ_A be an IFS-set over U and $\{t_i : i \in I\}$ and $\{s_j : j \in I\}$ be two non-empty family of subsets of U. If $t = \cap\{t_i : i \in I\}$, $\bar{t} = \cup\{t_i : i \in I\}$ and

$s = \cap\{s_j : j \in I\}$, $\underline{s} = \cup\{s_j : j \in I\}$, then

1. $\bigcup_{i\in I} \lambda_A^{(t_i,s_j)} \subseteq \lambda_A^{(\underline{t},\ \bar{s})}$
2. $\bigcap_{i\in I} \lambda_A^{(t_i,s_j)} = \lambda_A^{(\bar{t},\ \underline{s})}$

Proof: The proof is clear from Definition 4.1.

Theorem 4.4: Let λ_G be an IFS action group over U and t,s $\subseteq$ E such that t$\cap$s = $\emptyset$. Then, $\lambda_G^{(t,s)}$ is a subgroup of G whenever it is nonempty.

Proof: It is clear that $\lambda_G^{(t,s)} \neq \emptyset$. Suppose $x,y \in \lambda_G^{(t,s)}$.Then $\overline{\alpha_G}(x) \supseteq t$ and $\overline{\alpha_G}(y) \supseteq t$, $\overline{\beta_G}(x) \subseteq s$ and $\overline{\beta_G}(y) \subseteq s$.

Now, $\overline{\alpha_G}(x y^{-1}) \supseteq \overline{\alpha_G}(x) \cap \overline{\alpha_G}(y^{-1}) = \overline{\alpha_G}(x) \cap \overline{\alpha_G}(y) \supseteq t$.

Also, $\overline{\beta_G}(x y^{-1}) \subseteq \overline{\beta_G}(x) \cup \overline{\beta_G}(y^{-1}) = \overline{\beta_G}(x) \cup \overline{\beta_G}(y) \subseteq s$.

Therefore, $x y^{-1} \in \lambda_G^{(t,s)}$ and $\lambda_G^{(t,s)}$ is a subgroup of G.

Definition 4.2: Let $\hbar$ be a function from A to B and λ_A , $\lambda_B \in$ IFS(U). Then, an intuitionistic fuzzy soft image of λ_A under $\hbar$ and intuitionistic fuzzy soft pre image (or intuitionistic fuzzy soft inverse image) of λ_B under $\hbar$ are the intuitionistic fuzzy soft sets $\hbar(\lambda_A)$ and $\hbar^{-1}(\lambda_B)$ such that

$$\hbar(\lambda_A)(y) = \begin{cases} \{(x, \cup\overline{\alpha_A}(x), \cap\overline{\beta_A}(x)) ; x \in A\}, & \hbar(a) = b \\ (\emptyset, U), & \text{otherwise} \end{cases}$$

for all $y \in B$ and $\hbar^{-1}(\lambda_B)(x) = \lambda_B(\hbar(x))$, for all $x \in A$, respectively. Here $\hbar(\lambda_A)$ is called the image of λ_A under $\hbar$ and $\hbar^{-1}(\lambda_B)$is called the pre image (or inverse image) of λ_B under $\hbar$.

Example 4.2: Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$be a universal set. Let A = {a, b, c, d} and

B = {e, a, b, c, d} be two subsets of set of parameters, and $\hbar : A \to B$, $\hbar(x) = x^2$. We define an IFS-set over U by

$\overline{\alpha_A}$ (a) = $\{u_1, u_3\}$ and $\overline{\beta_A}$(a) = $\{u_2, u_4, u_7\}$

$\overline{\alpha_A}$ (b) = $\emptyset$ and $\overline{\beta_A}$ (b) = $\{u_1, u_2, u_3\}$

$\overline{\alpha_A}$(c) = $\{u_2, u_3, u_4, u_5, u_6\}$ and $\overline{\beta_A}$ (c) = $\{u_1\}$

$\overline{\alpha_A}$ (d) = $\{u_5, u_6, u_7\}$ and $\overline{\beta_A}$(d) = $\{u_1, u_2\}$

And,

$\overline{\alpha_A}$(e) = $\{u_3\ u_4, u_5, u_6\}$ and $\overline{\beta_A}$ (e) = $\{u_1, u_2, u_3\}$

$\overline{\alpha_A}$(b) = $\{u_5, u_6, u_7\}$ and $\overline{\beta_A}$ (b) = $\{u_1\}$

$\overline{\alpha_A}$(c) = $\{u_5, u_6, u_7\}$ and $\overline{\beta_A}$ (b) = $\{u_1, u_2\}$

$\overline{\alpha_A}$(d) = $\{u_5, u_6, u_7\}$ and $\overline{\beta_A}$ (d) = $\{u_2, u_2\}$

Then,

$\hbar(\lambda_A)$ = $\{(e, \emptyset, U), (a, (u_2, u_3, u_4, u_5, u_6), (u_1)), (b, \emptyset, U), (c, \emptyset, U), (d, (u_1, u_3, u_5, u_6, u_7), (u_2))\}$.

$\hbar^{-1}(\lambda_B)$={
(a, $(u_2, u_3, u_4, u_5, u_6), (u_1)), (b, (u_5, u_6, u_7), (u_1, u_2)), (c, (u_2, u_3, u_4, u_5, u_6), (u_1))$,
(d,$(u_5, u_6, u_7), (u_1, u_2))\}$.

Theorem 4.5: Let $\hbar$ be a function from A to B, $A_i \subseteq A$, $B_i \subseteq B$ and λ_{A_i}, λ_{B_i} be two IFS-sets over U for all $i \in I$. Then,

1. $\hbar(\cup_{i\in I} \lambda_{A_i}) = \cup_{i\in I} \hbar(\lambda_{A_i})$.
2. $\lambda_{A_1} \subseteq \lambda_{A_2} \Rightarrow \hbar(\lambda_{A_1}) \subseteq \hbar(\lambda_{A_2})$.
3. $\lambda_{B_1} \subseteq \lambda_{B_2} \Rightarrow \hbar^{-1}(\lambda_{B_1}) \subseteq \hbar^{-1}(\lambda_{B_2})$.

Proof:

1. For all $i \in I$, an IFS-set λ_{A_i} and $y \in B$,

$$\hbar(\cup_{i\in I} \lambda_{A_i})(y) = \left\{(x, \cup_{i\in I} \cup_{x\in A_i} \overline{\alpha_{A_i}}(x), \cap_{i\in I} \cap_{x\in A_i} \overline{\beta_{A_i}}(x));\ x \in A_i, \hbar(x) = y\right\}$$

$$= \cup_{i\in I}\left\{(x, \cup_{x\in A_i} \overline{\alpha_{A_i}}(x), \cap_{x\in A_i} \overline{\beta_{A_i}}(x));\ x \in A_i, \hbar(x) = y\right\}$$

$$= \cup_{i\in I} \hbar(\lambda_{A_i})(y).$$

2. Let $\lambda_{A_1} \subseteq \lambda_{A_2}$. So $A_1 \subseteq A_2$, then

$$\hbar(\lambda_{A_1})(y) = \{(x, \cup_{x\in A_1} \overline{\alpha_{A_1}}(x), \cap_{x\in A_1} \overline{\beta_{A_1}}(x));\ \hbar(x) = y\}$$

$$\subseteq \{(x, \cup_{x\in A_2} \overline{\alpha_{A_2}}(x), \cap_{x\in A_2} \overline{\beta_{A_2}}(x));\ \hbar(x) = y\}$$

$$= \hbar(\lambda_{A_2})(y).$$

3. Let $\lambda_{B_1} \subseteq \lambda_{B_2}$ .Then, for all x ∈ A ,

$\hbar^{-1}(\lambda_{B_1})(x) = \lambda_{B_1}(\hbar(x)) = \{(x, \overline{\alpha_{B_1}}(\hbar(x)), \overline{\beta_{B_1}}(\hbar(x))); x \in A\}$.

Since $\lambda_{B_1} \subseteq \lambda_{B_2}$, $\overline{\alpha_{B_1}}(\hbar(x)) \subseteq \overline{\alpha_{B_2}}(\hbar(x))$ and $\overline{\beta_{B_1}}(\hbar(x)) \supseteq \overline{\beta_{B_2}}(\hbar(x))$.

Therefore, $\{(x, \overline{\alpha_{B_1}}(\hbar(x)), \overline{\beta_{B_1}}(\hbar(x))); x \in A\} \subseteq \{(x, \overline{\alpha_{B_2}}(\hbar(x)), \overline{\beta_{B_2}}(\hbar(x))); x \in A\}$ and

$\lambda_{B_1}(\hbar(x)) \subseteq \lambda_{B_2}(\hbar(x)) = \hbar^{-1}(\lambda_{B_2})(x)$.

Theorem 4.6: Let $\hbar$ be a function from A to B , I be a non-empty index set , $B_i \subseteq B$ and λ_{B_i} be IFS-set over U for i ∈I. Then,

1. $\hbar^{-1}(\bigcup_{i\in I} \lambda_{B_i}) = \bigcup_{i\in I} \hbar^{-1}(\lambda_{B_i})$.
2. $\hbar^{-1}(\bigcap_{i\in I} \lambda_{B_i}) = \bigcap_{i\in I} \hbar^{-1}(\lambda_{B_i})$.

Proof: For all x ∈ A,

1. $\hbar^{-1}(\bigcup_{i\in I} \lambda_{B_i})(x) = \bigcup_{i\in I} \lambda_{B_i}(\hbar(x)) = \bigcup_{i\in I} \hbar^{-1}(\lambda_{B_i})(x)$.
2. $\hbar^{-1}(\bigcap_{i\in I} \lambda_{B_i})(x) = \bigcap_{i\in I} \lambda_{B_i}(\hbar(x)) = \bigcap_{i\in I} \hbar^{-1}(\lambda_{B_i})(x)$.

Theorem 4.7: Let $\hbar$ be a function from A to B. Then $\hbar^{-1}(\hbar(\lambda_A)) \supseteq \lambda_A$ for all $\lambda_A \in$ IFS(U).

In particular, if $\hbar$ is an injective function, then $\hbar^{-1}(\hbar(\lambda_A)) = \lambda_A$.

Proof: For all x ∈ A,

$\hbar^{-1}(\hbar(\lambda_A))(x) = \hbar(\lambda_A)(\hbar(x)) = \{(x, \cup \alpha_A(\hbar(x)), \cap \beta_A(\hbar(x)); \hbar(x') = \hbar(x)\}$

$\supseteq \lambda_A$.Thus, $\hbar^{-1}(\hbar(\lambda_A)) \supseteq \lambda_A$.

Corollary 4.8 : If $\hbar$ is one to one function, then $\hbar(x') = \hbar(x)$ implies $x' = x$ and the last inclusion is reduced to equality.

Theorem 4.9: Let $\hbar$ be a function from A to B. For all $\lambda_B \in$ IFS(U), $\hbar^{-1}(\hbar(\lambda_B)) \supseteq \lambda_B$. In particular, if $\hbar$ is a surjective function, then $\hbar(\hbar^{-1}(\lambda_B)) = \lambda_B$.

Proof: For all x ∈ A,

$$\begin{aligned}\hbar(\hbar^{-1}(\lambda_B))(y) &= \cup\{\hbar^{-1}(\lambda_B)(x); x \in A, \hbar(x) = y\}\\ &= \cup\{(\lambda_B)\hbar(x); \text{for all } x \in A, \hbar(x) = y\}\\ &= \begin{cases}\lambda_B(y), & \text{if } y \in \hbar(A)\\ (\emptyset, U), & \text{otherwise}\end{cases}\\ &\subseteq \lambda_B(y)\end{aligned}$$

Therefore, $\hbar(\hbar^{-1}(\lambda_B)) \subseteq \lambda_B$. If λ is one to one function, then $y \in \hbar(A)$ for all y ∈ B and so

$\hbar(\hbar^{-1}(\lambda_B)) = \lambda_B$.

Theorem 4.10: Let $\hbar$ be a function from A to B. Then $\hbar(\lambda_A) \subseteq \lambda_B \Leftrightarrow \lambda_A \subseteq \hbar^{-1}(\lambda_B)$ for all $\lambda_A, \lambda_B \in$ IFS(U) .

Proof: By theorem 4.5, we know that, $\hbar(\lambda_A) \subseteq \lambda_B \Rightarrow \hbar^{-1}(\hbar(\lambda_A)) \subseteq \hbar^{-1}(\lambda_B)$ and from theorem 4.7, $\lambda_A \subseteq \hbar^{-1}(\hbar(\lambda_A))$, so $\lambda_A \subseteq \hbar^{-1}(\lambda_B)$.

Conversely, assume that $\lambda_A \subseteq \hbar^{-1}(\lambda_B)$.Then from theorem:4.5 and 4.9 , $\hbar(\lambda_A) \subseteq \hbar(\hbar^{-1}(\lambda_B)) \subseteq \lambda_B$.

Theorem 4.11: Let $\hbar$ be a function from A to B and ψ be a function from B to C. Then

1. $\psi(\hbar(\lambda_A)) = (\psi\, o\, \hbar)(\lambda_A)$, for all $\lambda_A \in$ IFS(U).
2. $\hbar^{-1}(\psi^{-1}(\lambda_c)) = (\psi \text{ o } \hbar)^{-1}(\lambda_c)$, for all $\lambda_c \in$ IFS(U).

Proof: Consider any $\lambda_A \in$ IFS(U) and any $z \in C$, then ,

1.$\psi(\hbar(\lambda_A))(z) = \cup\{(\hbar(\lambda_A)(y);\ y \in B,\ \psi(y)=z\}$

$= \cup\{\{(x, \cup\overline{\alpha_A}(x), \cap\overline{\beta_A}(x)); x \in A, \hbar(x) = y\}; y \in B, \psi(y)=z\}$

$= \cup\{(x,\ \overline{\alpha_A}(x), \overline{\beta_A}(x));\ x \in A, (\psi\, o\, \hbar)(x) = z\}$

$= (\psi\, o\, \hbar)(\lambda_A)(z)$

2. For any $\lambda_c \in$ IFS(U) and for all $x \in A$,then

$(\psi \text{ o } \hbar)^{-1}(\lambda_c)(x) = (\lambda_c)\,\psi(\hbar(x))$

$= \psi^{-1}(\lambda_c)(\hbar(x)) = \hbar^{-1}\big(\psi^{-1}(\lambda_c)\big)(x)$

Definition 4.3: Let G be a group and λ_G , ψ_G be two IFS-sets over U. Then, product of λ_G and ψ_G is defined as follow, for all $x \in G$,

$(\lambda_G * \psi_G)(x) = \cup\{\ \lambda_G(y) \cap \psi_G(z);\ y, z \in G \text{ and } yz = x\}$

and inverse of λ_G is $\lambda_G{}^{-1}(x) = \lambda_G(x^{-1})$.

Theorem 4.12: Let λ_G , ψ_G and $\hbar_G$ be three IFS-sets over U. Then

$(\lambda_G * \psi_G) * \hbar_G = \lambda_G * (\psi_G * \hbar_G)$.

Proof: Let G be a group and λ_G ,ψ_G , $\hbar_G$ $\in$ IFS-sets over U. Then,

$((\lambda_G * \psi_G) * \hbar_G)(x) = \cup\{(\lambda_G * \psi_G)(y) \cap \hbar_G(z);\ y, z \in G \text{ and } yz = x\}$

$= \cup\{\cup\{(\lambda_G(u) \cap \psi_G(v)): uv = y\} \cap \hbar_G(z);\ y, z \in G \text{ and } yz = x\}$

$= \cup\{(\lambda_G(u) \cap \psi_G(v)) \cap \hbar_G(z);\ uvz = x,\ u,v,z \in G\}$

$= \cup\{\lambda_G(u) \cap (\psi_G(v) \cap \hbar_G(z));\ uvz = x,\ u,v,z \in G\}$

$= \cup\{\{\lambda_G(u) \cap (\psi_G(v) \cap \hbar_G(z));\ vz = t,\ v,z \in G\};\ ut=x,\ u,t \in G\}$

$= \cup\{\lambda_G(u) \cap (\psi_G * \hbar_G)(t);\ ut = x,\ u,t \in G\}$

$= [\lambda_G * (\psi_G * \hbar_G)](x).$

So it is associative.

Theorem 4.13: Let λ_G , ψ_G, $\hbar_{iG}$ $\in$ IFS(U) for all i $\in$ I. Then the following assertions hold,

(1) $(\lambda_G * \psi_G)(x) = \cup_{y\in G}\{\lambda_G(y) \cap \psi_G(y^{-1})\} = \cup_{y\in G}\{\lambda_G(xy^{-1}) \cap \psi_G(y)\}$.

(2) $({\lambda_G}^{-1})^{-1} = \lambda_G$.

(3) $\lambda_G \subseteq {\lambda_G}^{-1} \Leftrightarrow {\lambda_G}^{-1} \subseteq \lambda_G$.

(4) $\lambda_G \subseteq \psi_G \Leftrightarrow {\lambda_G}^{-1} \subseteq {\psi_G}^{-1}$.

(5) $(\cup_{i\in I} \hbar_{iG})^{-1} = \cup_{i\in I} {\hbar_{iG}}^{-1}$.

(6) $(\cap_{i\in I} \hbar_{iG})^{-1} = \cap_{i\in I} {\hbar_{iG}}^{-1}$.

(7) $(\lambda_G * \psi_G)^{-1} = {\psi_G}^{-1} * {\lambda_G}^{-1}$.

Conclusion: In this study, some concepts are defined such as an intuitionistic fuzzy soft action group, (t,s)-level set of an intuitionistic fuzzy soft set, image and pre-image of an intuitionistic fuzzy soft set. Then, in group theory some properties are extended to an intuitionistic fuzzy soft group and some results are obtained about an intuitionistic fuzzy soft action group and (t,s)-level set of an intuitionistic fuzzy soft set.

Future work: I hope that researchers may study the properties of an intuitionistic fuzzy soft action group in other algebraic structures such as ideals, rings, fields and normal subgroups.

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