

FREE CONVECTIVE FLOW OF A JEFFREY FLUID IN AN INCLINED CIRCULAR PIPE

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ABSTRACT

Free convection flow of a Jeffrey fluid in an inclined circular pipe has been investigated. A non-linear density temperature relationship is taken to express the body force term as buoyancy term. Applying perturbation method, the nonlinear governing equations are solved and the expressions for the velocity field and the temperature distribution are obtained. The rate of heat transfer from the pipe wall to the fluid is determined. It is observed that the velocity decreases with the increase in the parameters γ and δ , whereas the temperature increases with the increase in the parameters γ and δ . The results have been compared with the corresponding case of linear density temperature variation. The Nusselt number increases with the increasing inclination parameter δ .

Key Words: Free Convection flow; Jeffrey fluid; inclined circular pipe.

1. INTRODUCTION

Heat transfer is a phenomenon associated with both Newtonian and non-Newtonian fluids and finds its relevance in various industrial processes. The problems concerning heat transfer have many applications in engineering sciences such as design of cooling systems for motors, condensation, heating and cooling of fluids. An understanding of the laws of heat flow is important to the civil engineers in the construction of dams. In almost every branch of engineering, heat transfer problems arise and many of them can be solved only with the combination of laws of thermodynamics and laws of fluid motion. Heat transfer is associated with the process of transmission of energy from one region to another as a result of temperature differences between them. The three modes of heat transfer are classified as conduction, convection and radiation.

Thermal convection is a process of energy transport affected by the circulation of mixing of a fluid medium (gas, liquid or a powdery substance). Convection is possible only in a fluid medium and is directly linked with the transport of medium itself. Macroscopic particles of a fluid moving in space cause the heat exchange and thus convection constitutes the macroform of the heat transfer. The effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid. In fluids, convection takes place by forced convection and natural convection. Forced convection is a mechanism, or type of transport in which fluid motion is generated by an external force (like a pump, fan, suction device, etc.). So it is used to increase the rate of heat exchange. Free convection flow, on the other hand, results from the action of body forces on the fluid, that is, forces which are proportional to the mass or the density of the fluid. In this case, the flow patterns are determined by the buoyant force on the heated fluid. A common example of natural convection is the rise of smoke from a fire.

Ostrach (1952, 1954) has analyzed the effect of the frictional heating and the heat sources in the fluid, on the fully developed laminar convection flow between two parallel vertical plates when the wall temperatures are either constant or varying linearly along the plate length. He has used the usual linear density-temperature (LDT) variation. Liquids generally expand on heating and contract on cooling. However, water is a liquid which does not behave like normal liquids. The volume of water increases, if we heat it or cool it, provided initially the water is at 4⁰c. This is known as anomalous expansion of water. In such cases, the density and temperature, relationship is modelled as quadratic density temperature (QDT) variation. Goren (1966) has obtained a similarity solution of the boundary layer equations of the free convection flow from a semi-infinite plate of uniform temperature to water at 4°C. In this study he has established the necessity of using QDT variation. Using QDT relationship, Sinha (1969), Agarwal & Upmanyu (1976), and Balakrishan et al. (1984) discussed free convective flows in tubes and channels. Bhargava and Agarwal (1979) have investigated the fully developed free convective flow of a Newtonian fluid in a circular pipe.

A nonlinear density temperature (NDT) relation naturally accommodates the LDT relation and QDT relation to some extent. Further this relation takes care of the linear temperature-dependence of β used by earlier researchers. Gilpin (1975) has used a density-temperature relation, which is similar to the NDT relation and which has been introduced by Vanier and Tien (1968) with a view to predict the heat-transfer results in the case of water for temperatures between 0⁰ and 20°C. Sastri and Vajravelu (1977) considered the problem of free convection between vertical walls. Using NDT relation they investigated the fully developed free convection flow and heat transfer between two long parallel vertical walls kept at constant temperatures. Krishna Gopal Singha (2009) investigated analytical solution to the problem of MHD free convective flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field. Hayat et al. (2009) investigated the effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. Vajravelu et al. (2011) examined the free convection flow of Jeffrey fluid in a vertical porous stratum under peristalsis.

A nonlinear density temperature variation can be defined as

$$\Delta\rho = -\beta_0\rho(T - T_s) - \beta_1\rho(T - T_s)^2 \quad (1)$$

Where β_0, β_1 are the constants and T_s is the temperature in hydrostatic condition,

This reduces to LDT variation when $\beta_1=0$ and QDT variation when $\beta_0=0$.

In this paper, we discuss the problem of free convection flow of a Jeffrey fluid in an inclined circular pipe, implementing the NDT relationship define above. The flow and heat transfer both depend upon a new parameter $\gamma = (\beta_1 / \beta_0) \Delta T$ in addition to the heat source parameter α and the free convection parameter K. The velocity field, the temperature distribution and Nusselt number are obtained and the results are discussed through graphs.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the fully developed steady laminar free convection flow of a Jeffrey fluid in a circular pipe of radius a . In the cylindrical coordinate system (r, ϕ, z) , let u, v, w be the velocity components. The motion being rotationally symmetric and assuming that the flow is fully developed, all the physical quantities will be independent of ϕ and z . The radial and tangential components of velocity are zero. The corresponding equations of continuity, motion and energy are

$$\frac{\partial w}{\partial z} = 0 \quad (2)$$

$$0 = -\frac{\partial p}{\partial z} + \frac{\mu}{1 + \lambda_1} \left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right] - \rho g_1 \sin \delta \quad (3)$$

$$0 = K_1 \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] + \mu \left(\frac{\partial w}{\partial r} \right)^2 + Q \quad (4)$$

Where $g_1 = -g$ and Q is a constant, denotes the heat added due to heat sources, $g_1 \sin \delta$ the generating body force, K_1 the coefficient of thermal conductivity p the pressure and δ is the angle of inclination of the pipe with the horizontal.

The boundary conditions are

$$\text{at } r = 0, \frac{dw}{dr} = 0, \frac{dT}{dr} = 0 \quad (5)$$

$$\text{at } r = a, w = 0, T = T_w \quad (6)$$

Following Ostrach (1952), the body force term in (3) can be expressed as buoyancy term. In the hydrostatic condition equation (3) gives

$$-\rho_s g_1 \sin \delta - \frac{\partial p_s}{\partial z} = 0 \quad (7)$$

And hence

$$\begin{aligned} -\rho g_1 \sin \delta - \frac{\partial p}{\partial z} &= -(\rho - \rho_s) g_1 \sin \delta + \rho_s g_1 \sin \delta - \frac{\partial p}{\partial z} \\ &= -(\rho - \rho_s) g_1 \sin \delta + \frac{\partial p_s}{\partial z} - \frac{\partial p}{\partial z} \\ &= -(\rho - \rho_s) g_1 \sin \delta - \frac{\partial p_D}{\partial z} \end{aligned} \quad (8)$$

where

$$p_D = p - p_s. \quad (9)$$

From (1), we get

$$\Delta \rho = -\beta_0 \rho \theta - \beta_1 \rho \theta^2 \quad (10)$$

where $\theta = T - T_s$

Now using relation (9) and (10), equations (2)-(5) lead to

$$\frac{d^2w}{dr^2} + \frac{1}{r} \frac{dw}{dr} + \frac{\rho}{\mu} (1 + \lambda_1) (\beta_0 \theta + \beta_1 \theta^2) g_1 \sin \delta = 0 \quad (11)$$

$$\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} + \frac{\mu}{K_1} \left(\frac{dw}{dr} \right)^2 + \frac{Q}{K_1} = 0 \quad (12)$$

3. NON-DIMENSIONALIZATION OF THE FLOW QUANTITIES

We introduce the following non-dimensional quantities:

$$\eta = \frac{r}{a}, \theta^* = \frac{K\theta}{\theta_w}, w^* = \frac{Kw}{W},$$

where

$$\theta_w = T_w - T_s, W = \frac{f_z \beta^2 a^2 \theta_s^2}{\nu}, K = \frac{f_z^2 \beta^2 \rho^2 a^4 \theta_s^3}{\mu K_1}, Gr = \beta_0 a^3 \theta_w^2 / \nu^2, Pr = \frac{\mu c_p}{k_1} \quad (13)$$

Equations (11) and (12) reduce to

$$\frac{d^2w^*}{d\eta^2} + \frac{1}{\eta} \frac{dw^*}{d\eta} + (1 + \lambda_1) \left(\theta^* + \frac{\theta^{*2}}{K} \right) = 0 \quad (14)$$

$$\frac{d^2\theta^*}{d\eta^2} + \frac{1}{\eta} \frac{d\theta^*}{d\eta} + \left(\frac{dw^*}{d\eta} \right)^2 + \alpha K = 0 \quad (15)$$

where $\alpha (= Qa^2 / \theta_w K_1)$ is the heat source parameter. Gr is the Grashoff number and Pr is the Prandtl number.

For the sake of convenience, dropping the stars, eqns. (14) and (15) finally are

$$\frac{d^2w}{d\eta^2} + \frac{1}{\eta} \frac{dw}{d\eta} + (1 + \lambda_1) \left(\theta + \frac{\gamma}{K} \theta^2 \right) = 0 \quad (16)$$

$$\frac{d^2\theta}{d\eta^2} + \frac{1}{\eta} \frac{d\theta}{d\eta} + \left(\frac{dw}{d\eta} \right)^2 + \alpha K = 0 \quad (17)$$

The corresponding boundary conditions are

$$\left. \begin{aligned} &at \eta = 0, \frac{dw}{d\eta} = 0, \frac{d\theta}{d\eta} = 0 \\ &at \eta = 1, w = 0, \theta = K. \end{aligned} \right\} \quad (18)$$

4. SOLUTION OF THE PROBLEM

Equations (16) and (17) are coupled nonlinear differential equations which cannot be solved for exact solution. Applying perturbation method, we write

$$w = Kw_0 + K^2w_1 + K^3w_2 + \dots \quad (19)$$

$$\theta = K\theta_0 + K^2\theta_1 + K^3\theta_2 + \dots \quad (20)$$

Substituting (19) and (20) into (16) and (17) and equating the coefficients of like powers of K on either side of the equations thus obtained, we get the following set of equations;

$$\left. \begin{aligned} w_0'' + \frac{1}{\eta} w_0' + (1 + \lambda_1)(\theta_0 + \gamma\theta_0^2) &= 0 \\ w_1'' + \frac{1}{\eta} w_1' + (1 + \lambda_1)(\theta_1 + 2\gamma\theta_0\theta_1) &= 0 \\ w_2'' + \frac{1}{\eta} w_2' + (1 + \lambda_1)(\theta_2 + \gamma(2\theta_0\theta_2 + \theta_1^2)) &= 0 \end{aligned} \right\} \quad (21)$$

$$\left. \begin{aligned} \theta_0'' + \frac{1}{\eta} \theta_0' + \alpha &= 0 \\ \theta_1'' + \frac{1}{\eta} \theta_1' + w_0^2 &= 0 \\ \theta_2'' + \frac{1}{\eta} \theta_2' + 2\gamma w_0' w_1' &= 0 \end{aligned} \right\} \quad (22)$$

The boundary conditions (18) then reduce to

$$\text{at } \left. \begin{aligned} \eta = 0, w_0' = w_1' = w_2' &= 0 \\ \theta_0' = \theta_1' = \theta_2' &= 0 \end{aligned} \right\} \quad (23)$$

$$\text{at } \left. \begin{aligned} \eta = 1, w_0 = w_1 = w_2 &= 0 \\ \theta_0 = 1, \theta_1 = \theta_2 &= 0 \end{aligned} \right\} \quad (24)$$

Solving eqns. (21) and (22) under (23) and (24), we get

$$W_0 = D_0 + A_1\eta^2 - A_2\eta^4 + A_3\eta^6 \quad (25)$$

$$\theta_0 = 1 + \frac{\alpha}{4} - \frac{\alpha}{4}\eta^2 \quad (26)$$

$$\theta_1 = D_1 - A_4\eta^{12} + A_5\eta^{10} - A_6\eta^8 + A_7\eta^6 - A_8\eta^4 \quad (27)$$

$$W_1 = E_1 + E_2\eta^{16} - E_3\eta^{14} + E_4\eta^{12} - E_5\eta^{10} + E_6\eta^8 - E_7\eta^6 - E_8\eta^4 + E_9\eta^2 \quad (28)$$

$$\text{where } A_1 = -\left(\frac{1 + \lambda_1}{4}\right) \left[1 + \frac{\alpha}{4} + \gamma + \frac{\alpha\gamma}{2} + \frac{\alpha^2\gamma}{16} \right] \cdot \sin \delta, \quad A_2 = -\left(\frac{1 + \lambda_1}{16}\right) \left[\frac{\alpha}{4} + \frac{\alpha^2\gamma}{8} + \frac{\alpha\gamma}{2} \right] \cdot \sin \delta$$

$$A_3 = -\left(\frac{1 + \lambda_1}{36}\right) \frac{\alpha^2 v}{16} \sin \delta, \quad D_0 = -A_1 + A_2 - A_3, \quad A_4 = \frac{36A_3^2}{144}, \quad A_5 = \frac{48A_2A_3}{165}$$

$$A_6 = \frac{16A_2^2}{64} + \frac{24A_1A_2}{64}, \quad A_7 = \frac{16A_1A_2}{36}, \quad A_8 = \frac{4A_1^2}{16}, \quad D_1 = A_4 - A_5 + A_6 - A_7 + A_8$$

$$E_2 = -\frac{(1 + \lambda_1)A_4\alpha v}{512}, \quad E_3 = -\frac{(1 + \lambda_1)\left(A_4\left(1 + 2v + \frac{\alpha v}{2}\right) + A_5\frac{\alpha v}{2}\right)}{196}$$

$$E_4 = -\frac{(1+\lambda_1)\left(A_5\left(1+2\nu+\frac{\alpha\nu}{2}\right)+A_6\frac{\alpha\nu}{2}\right)}{144}, \quad E_5 = -\frac{(1+\lambda_1)\left(A_6\left(1+2\nu+\frac{\alpha\nu}{2}\right)+A_7\frac{\alpha\nu}{2}\right)}{100}$$

$$E_6 = -\frac{(1+\lambda_1)\left(A_7\left(1+2\nu+\frac{\alpha\nu}{2}\right)+A_8\left(\frac{\alpha\nu}{2}\right)\right)}{64}, \quad E_7 = -\frac{(1+\lambda_1)\left(A_8\left(1+2\nu+\frac{\alpha\nu}{2}\right)\right)}{36}$$

$$E_8 = -\frac{(1+\lambda_1)\alpha\gamma D_1}{32}, \quad E_9 = -\frac{(1+\lambda_1)D_1\left(1+2\nu+\frac{\alpha\nu}{2}\right)}{4}$$

Similarly we can obtain the solutions for θ_2 and W_2 . The velocity and temperature functions are then obtained from (19) and (20). The rate of heat transfer from the pipe wall to the fluid per unit area of the pipe surface is given by

$$q = \frac{K_1\theta_w}{aK} \left(\frac{\partial\theta}{\partial\eta} \right)_{\eta=1} = \frac{K_1\theta_w}{a} \left(\frac{-\alpha}{2} + K(-12A_4 + 10A_5 - 8A_6 + 6A_7 - 4A_8) \right) \quad (29)$$

The Nusselt number is therefore

$$Nu = \frac{qa}{\theta_w K_1} = \frac{1}{K} \left(\frac{\partial\theta}{\partial\eta} \right)_{\eta=1} = \frac{-\alpha}{2} + K(-12A_4 + 10A_5 - 8A_6 + 6A_7 - 4A_8) \quad (30)$$

5. RESULTS AND DISCUSSION

In this paper, free convective flow of a Jeffrey fluid in an inclined circular pipe is investigated.

Flow solutions are depicted graphically to study the parameters α , ν , λ_1 , δ and K on the velocity, the temperature and the Nusselt number.

Taking $\nu=0$ in equations (21) and (22) under (23) and (24), we obtain velocity, temperature corresponding to LDT case of Jeffrey fluid flow in a circular pipe and they are given by

$$W_0 = D_0 + A_1\eta^2 - A_2\eta^4 + A_3\eta^6 \quad (31)$$

$$\theta_0 = 1 + \frac{\alpha}{4} - \frac{\alpha}{4}\eta^2 \quad (32)$$

$$\theta_1 = D_1 - A_4\eta^{12} + A_5\eta^{10} - A_6\eta^8 + A_7\eta^6 - A_8\eta^4 \quad (33)$$

$$W_1 = E_1 + E_2\eta^{16} - E_3\eta^{14} + E_4\eta^{12} - E_5\eta^{10} + E_6\eta^8 - E_7\eta^6 - E_8\eta^4 + E_9\eta^2 \quad (34)$$

where $A_1 = -\left(\frac{1+\lambda_1}{4}\right)\left(1+\frac{\alpha}{4}\right)\sin\delta$, $A_2 = -\left(\frac{1+\lambda_1}{16}\right)\frac{\alpha}{4}\sin\delta$, $A_3 = -\left(\frac{1+\lambda_1}{36}\right)\frac{\alpha^2\nu}{16}\sin\delta$

$$D_0 = -A_1 + A_2 - A_3, \quad A_4 = \frac{36A_3^2}{144}, \quad A_5 = \frac{48A_2A_3}{165}, \quad A_6 = \frac{16A_2^2}{64} + \frac{24A_1A_2}{64}$$

$$A_7 = \frac{16A_1A_2}{36}, \quad A_8 = \frac{4A_1^2}{16}, \quad D_1 = A_4 - A_5 + A_6 - A_7 + A_8, \quad E_2 = -\frac{(1+\lambda_1)A_4\alpha\nu}{512}$$

$$E_3 = -\frac{(1+\lambda_1)\left(A_4\left(1+2\nu+\frac{\alpha\nu}{2}\right)+A_5\frac{\alpha\nu}{2}\right)}{196}, E_4 = -\frac{(1+\lambda_1)\left(A_5\left(1+2\nu+\frac{\alpha\nu}{2}\right)+A_6\frac{\alpha\nu}{2}\right)}{144},$$

$$E_5 = -\frac{(1+\lambda_1)\left(A_6\left(1+2\nu+\frac{\alpha\nu}{2}\right)+A_7\frac{\alpha\nu}{2}\right)}{100},$$

$$E_6 = -\frac{(1+\lambda_1)\left(A_7\left(1+2\nu+\frac{\alpha\nu}{2}\right)+A_8\left(\frac{\alpha\nu}{2}\right)\right)}{64}$$

$$E_7 = -\frac{(1+\lambda_1)\left(A_8\left(1+2\nu+\frac{\alpha\nu}{2}\right)\right)}{36}, E_8 = 0, E_9 = -\frac{(1+\lambda_1)D_1\left(1+2\nu+\frac{\alpha\nu}{2}\right)}{4}$$

The variation of velocity with η is calculated, from equations (25) and (28), for different values of γ and λ_1 and is shown in Figures 1-4, for fixed K and δ . We observe that the velocity increases with the increase in the NDT parameter γ , heat source parameter α and Jeffrey parameter λ_1 .

The variation of velocity with η is calculated, from equations (25) and (.28), for different values of γ and δ and is shown in Figures 5 and 6 for fixed α , λ_1 and K . We observe that the velocity decreases with the increase in the parameters γ and δ .

From the equations (26) and (27), we have calculated the temperature as a function of η , for fixed K and δ for different values of γ , λ_1 and α and is shown in Figures 7-10. We observe that the temperature decreases with the increase in the parameters γ , α and λ_1 .

We have calculated the temperature as a function of η , for fixed α , λ_1 and K , and for different values of γ and δ and is shown in Figures 11 and 12. We observe that the temperature increases with the increase in the parameters γ and δ .

From the equation (1.30) we have calculated the Nusselt number as a function of K for fixed λ_1 and δ for different values of NDT parameter γ and α and is shown in Figures 13 and 14. We observe that the Nusselt number increases with the increase in the NDT parameter γ and α .

We have calculated the Nusselt number as a function of K for fixed δ and for different values of Jeffrey parameter λ_1 and γ and is shown in Figures 15 and 16. We observe that the Nusselt number increases with the increase in the parameters λ_1 and γ .

We have calculated the Nusselt number as a function of K for fixed α and λ_1 and for different values of γ and δ and is shown in Figures 17 and 18. We observe that the Nusselt number increases with the increase in the parameters δ and γ .

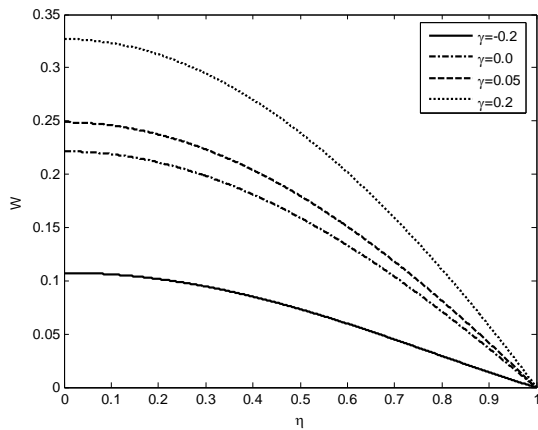


Fig.1 Velocity distribution for various values of γ for fixed $\alpha=5$, $K=0.5$, $\delta = \pi/4$ and $\lambda_1=0.1$.

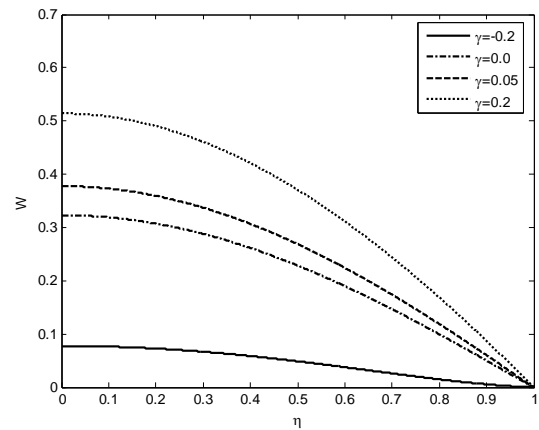


Fig.2 Velocity distribution for various values of γ for fixed $\alpha=10$, $K=0.5$, $\delta = \pi/4$ and $\lambda_1=0.1$.

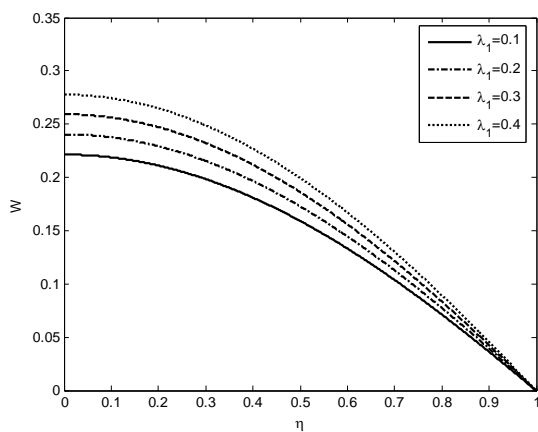


Fig.3 Velocity distribution for various values of λ_1 for fixed $\alpha=5$, $K=0.5$, $\delta = \pi/4$ and $\gamma=0$.

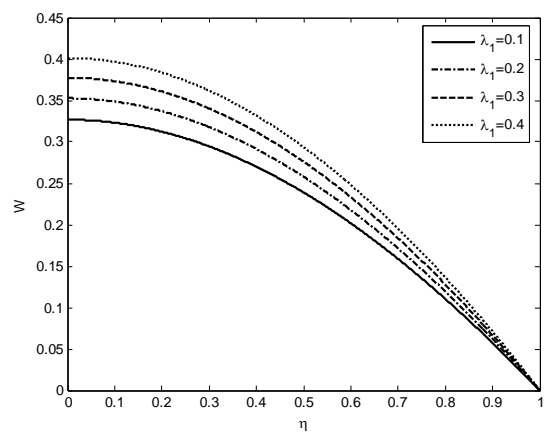


Fig.4 Velocity distribution for various values of λ_1 for fixed $\alpha=5$, $K=0.5$, $\delta = \pi/4$ and $\gamma=0.2$.

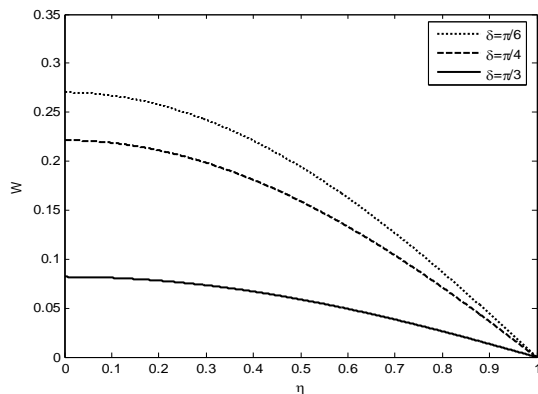


Fig. 5 Velocity distribution for various values of δ for fixed $\alpha=5$, $K=0.5$, $\gamma=0$ and $\lambda_1=0.1$

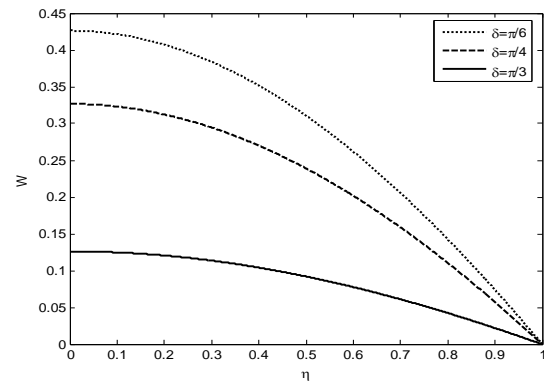


Fig. 6 Velocity distribution for various values of δ for fixed $\alpha=5$, $K=0.5$, $\gamma=0.2$ and $\lambda_1=0.1$

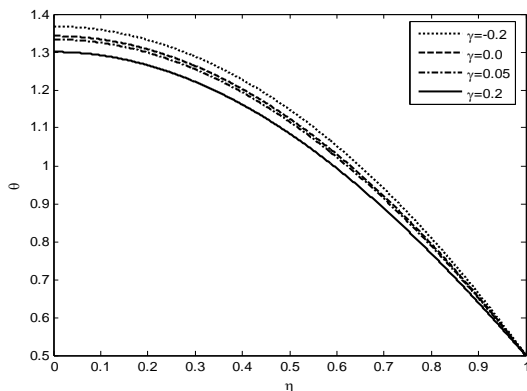


Fig. 7 Temperature distribution for various values of γ for fixed $\alpha=7$, $K=0.5$, $\delta = \pi/4$ and $\lambda_1=0.1$.

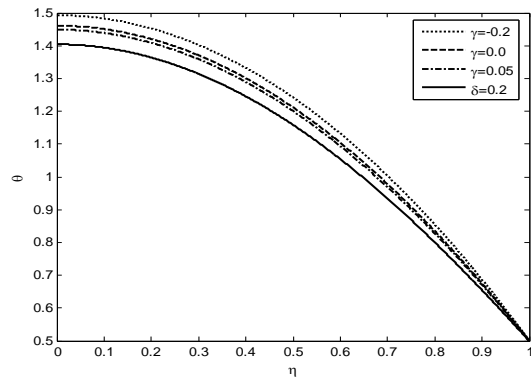


Fig. 8 Temperature distribution for various values of γ for fixed $\alpha=8$, $K=0.5$, $\delta = \pi/4$ and $\lambda_1=0.1$.

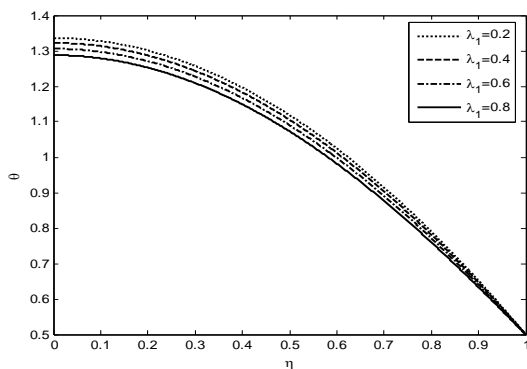


Fig. 9 Temperature distribution for various values of λ_1 for fixed $\alpha=7$, $K=0.5$, $\delta = \pi/4$ and $\gamma=0$.

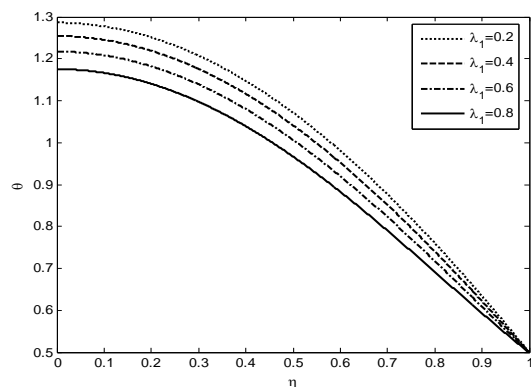


Fig.10 Temperature distribution for various values of λ_1 for fixed $\alpha=7$, $K=0.5$, $\delta = \pi/4$ and $\gamma=0.2$.

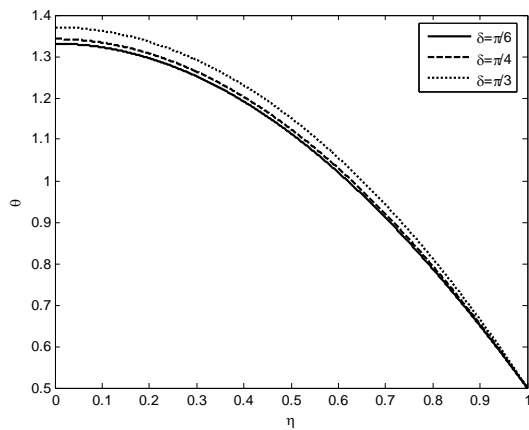


Fig. 11 Temperature distribution for various values of δ for fixed $\alpha=7$, $K=0.5$, $\lambda_1=0.1$ and $\gamma =0$

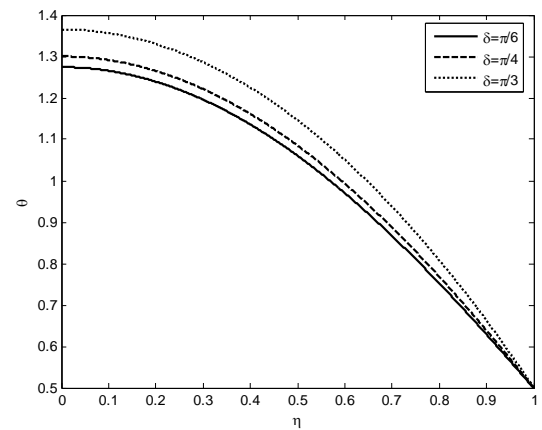


Fig. 12 Temperature distribution for various values of δ for fixed $\alpha=7$, $K=0.5$, $\lambda_1=0.1$ and $\gamma =0.2$.

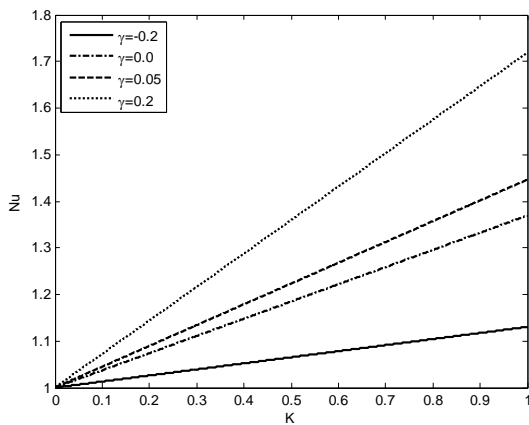


Fig. 13 Nusselt number distribution for various values of γ for fixed $\alpha=5$, $\delta = \pi / 4$ and $\lambda_1=0.1$.

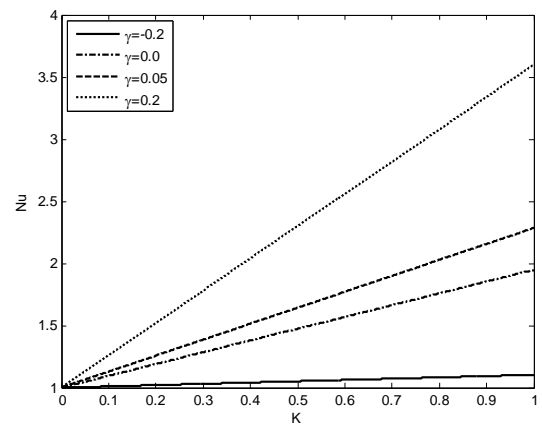


Fig. 14 Nusselt number distribution for various values of γ for fixed $\alpha=10$, $\delta = \pi / 4$ and $\lambda_1=0.1$.

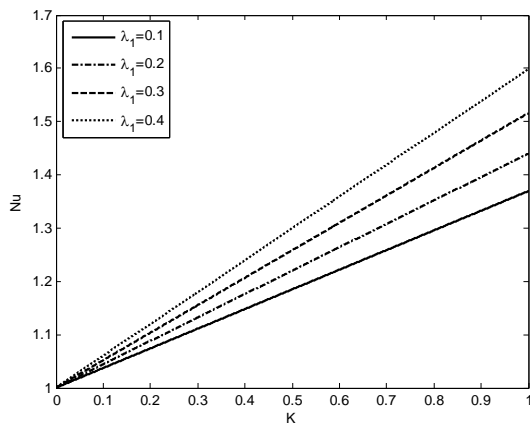


Fig. 15 Nusselt number distribution for various values of λ_1 for fixed $\alpha=5$, $\delta = \pi / 4$ and $\gamma = 0$.

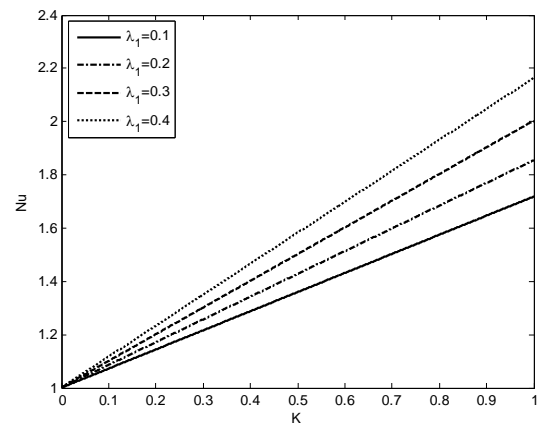


Fig. 16 Nusselt number distribution for various values of λ_1 for fixed $\alpha=10$, $\delta = \pi / 4$ and $\gamma = 0.2$.

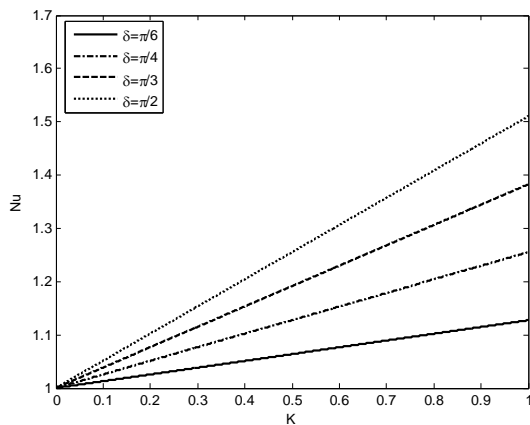


Fig. 17 Nusselt number distribution for various values of δ for fixed $\alpha=5$, $\lambda_1 = 0.1$ and $\gamma = 0$.

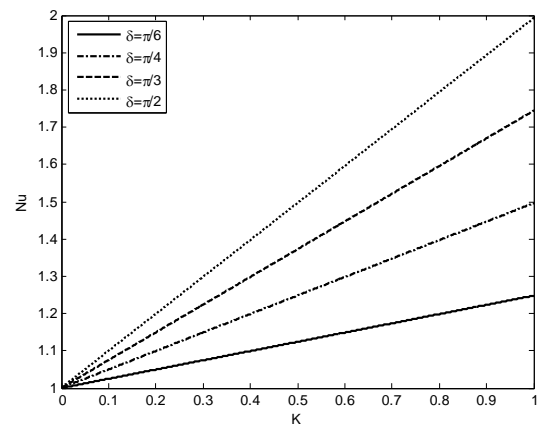


Fig. 18 Nusselt number distribution for various values of δ for fixed $\alpha=5$, $\lambda_1 = 0.1$ and $\gamma = 0.2$.

CONCLUSIONS

- The velocity increases with the increase in the NDT parameter γ and heat source parameter α . Jeffrey parameter λ_1
- The velocity decreases with the increase in the inclination parameter δ .
- The temperature decreases with the increase in the NDT parameter γ and heat source parameter α and Jeffrey parameter λ_1 .
- The temperature increases with the increase in the inclination parameter δ .
- The Nusselt number increases with the increase in the parameters γ , λ_1 , α and δ .

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