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ANALYTICAL STUDY OF WAVE MOTION OF A FALLING LIQUID FILM PAST A VERTICAL PLATE

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Abstract:

In this article, the existence of gravity-capillary waves travelling down the surface of a falling liquid film past a vertical plate has been considered. Kapitza's scheme of finding an approximate expression of velocity *u* when the film surface assumes an arbitrary shape $y = h(x, t)$, which changes with time, has been emphasized. The expressions for dimensionless wavelength, dimensionless wave number and Weber number have been obtained and are computed for an admissible range of the wave celerity. The stream line pattern has also been studied and presented through graphs.

Key words: Wave motion, liquid film, gravity-capillary waves, stream line pattern

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1. INTRODUCTOION

The flow of a thin liquid is a special case of flow in which both surface tension and viscosity play dominant role and is of fundamental importance in distillation of gases and liquids, having varied applications in engineering operations such as distillation, gas absorption, condensation of vapors, where heat and mass transfer are intimately connected to fluid motion. Extensive reviews by Norman [1], Dukler and Wicks [2], Levich [3] and Fulford [4] bring out the importance of such studies.

The experimental studies of the flow of thin films have revealed that (i) at Reynolds numbers $\mathrm{Re} = \frac{u_o h_o}{\nu}$ that do not exceed 20 to 30 (where u_o is the average velocity over the cross-section of the film of thickness h_o and ν the kinematic coefficient of viscosity), there exists the usual viscous flow regime and the film thickness is constant, (ii) at Reynolds numbers Re > 30 to 50, a so-called wave regime appears, in which the wave motion is superposed in the forward motion of the film and (iii) at Reynolds numbers Re about 1500, the laminar regime breaks down and the turbulent motion sets in. In the case of very thin films, the break-up of the film into separate drops is caused by the action of capillary forces. The direction depends on wetting conditions.

The undulatory character of the flow was experimentally detected by Kirkbridge [5]. Their studies reveal that this motion appears for values of Reynolds number about 20 to 30. These studies show that the law for laminar flow is obeyed only as an average and refers to the mean thickness of the layer. In fact, the character of the flow differs from that of a simple laminar flow. The reason for this difference is sought due to the fact that in deducing the Navier-Stokes equations, one does not take into account surface tension which in a flow of a fluid with a low viscosity and in thin layers, even if the free surface is slightly deformed, acquires a considerable value quite comparable with forces of viscosity. If capillary forces are taken into account, the undulatory flow even at small velocities is indeed more stable than a simple laminar flow. The existence of a more stable undulatory flow is of great interest because it allows one to explain and describe a number of known physical phenomena observed in the flow of thin layers.

The theoretical aspects of these flows fall mainly under three categories, namely, (i) the search for periodic steady states (ii) the stability analysis and (iii) the semi-empirical characterizations. Kapitza [6] was the first to predict the appearance of gravity-capillary wave theoretically and the results of theoretical analysis of the wave regime in the flow of film down a plate were verified, experimentally, by Kapitza and Kapitza [7]. Direct measurements of wave profile made by the shadow graph optical technique confirmed the theoretical predictions, particularly that of the amplitude. It was found that wave motion of the film readily gives way to turbulent motion when there are external disturbances to the liquid flow.

Chaturvedi et al. [8] have considered exponential fluid flow along an infinite porous plate with constant suction. They observed that there is no back flow near the wall. Raju [9] has studied the oscillatory flow past an infinite vertical porous plate. He revealed that in the case of heating of the plate, the mean velocity increases whereas it decreases in the case of cooling of the plate with increase in Prandtl number. Masthanrao et al. [10] have studied chemical reaction and hall effects on MHD convective flow along an infinite vertical porous plate with variable suction and heat absorption. They observed that an increase in the Prandtl number leads to a decrease in the primary and secondary velocities, and also a decrease in the primary and secondary temperatures. Manjulatha et al. [11] have studied the effect of aligned magnetic filed on free convective flow past an infinite vertical flat plate through porous medium with temperature dependent heat source. They found that the temperature and velocity increase with the increase of heat source parameter.

One of the earliest contributions to the analysis of a falling liquid film (Fig. 1) was made by Nusselt [12]. The equation for slow steady state of a film under the action of gravity is given by

$$g + \nu \frac{d^2u}{dy^2} = 0 \tag{1}$$

The boundary conditions are:

(i) at the free surface of the liquid, $y = h_o$, $\mu \frac{du}{dy} = 0$, and

(ii) at the wall $y = 0$, $u = 0$.

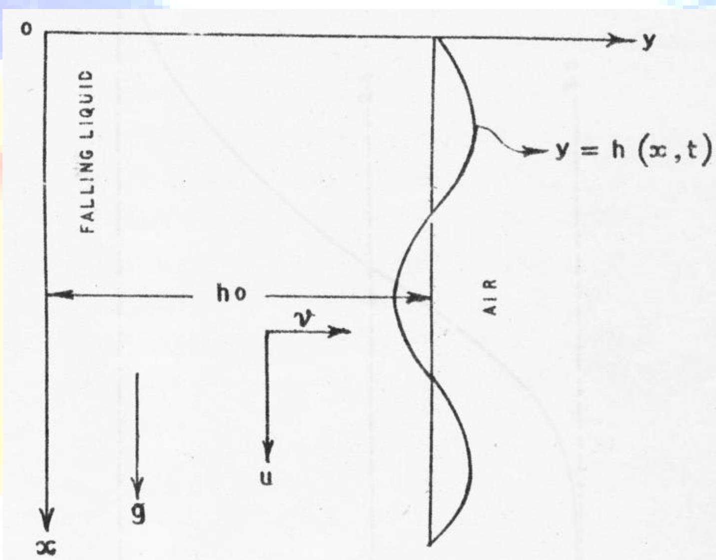

Fig.1 Physical model

The solution of (1) under the above boundary conditions is

$$u = \frac{3u_o}{h_0}\left(y - \frac{y^2}{2h_o}\right), \quad (2)$$

where $$u_o = \frac{1}{h_o}\int_0^{h_o} u\,dy = \frac{gh_o^2}{3\nu} \quad (3)$$

This theory is invalid when waves appear at the free surface. But it is extensively used as a first approximation.

Kapitza [13] made an attempt to describe the wavy flow by postulating an oscillatory steady state solution of the equations of motion. His theory was described in detail by Levich [3] and Bushmanov [14]. All these researchers considered the hydrodynamic equations of motion and boundary conditions for thin liquid film and took into account the effects of surface tension. However, they have neglected the term $\nu\frac{\partial^2 u}{\partial x^2}$ as in boundary-layer theory and variation of the surface coordinate *h* with the longitudinal coordinate *x* at several stages in their derivation of the differential equation, Levich [3]:

$$\frac{\kappa h_o}{\rho}\frac{d^3\psi}{dx^3} + (c-u_o)(c-\frac{9}{10}u_o)\frac{d\psi}{dx} - \frac{3\nu}{h_o^2}(c-3u_o)\psi + g - \frac{3\nu u_o}{h_o^2} = 0 \quad (4)$$

where

$$h = h_o(1+\psi) \quad (5)$$

and ψ the steady state periodic solution of the equation (4) is given by

$$\psi = A\sin\frac{2\pi}{\lambda}(x-ct). \quad (6)$$

They argued that in order that the equation (4) to have undamped periodic solution, both the free term and the coefficient of ψ must be equal to zero.

The first of these requirements is that

$$g - \frac{3\nu u_o}{h^2} = 0 \quad (7)$$

which signifies that in a first approximation, the thickness h_o is close to the thickness of the layer in laminar flow down the plate.

The second requirement takes the form

$$c - 3u_o = 0 \tag{8}$$

and defines the phase velocity of the wave in equation (6) as the first approximation.

The frequency of the wave $\underline{w}$ is given by

$$\underline{w} = \frac{2\pi}{\lambda} c = 3u_o^2 \left(\frac{4.2\,\rho}{\kappa\, h_o} \right)^{1/2}. \tag{9}$$

An important feature of this equation is $\underline{w} \cong u_o^2$ and that both approach zero simultaneously. This is the basic difference between surface waves on a film flowing down a plate and ordinary capillary waves. The undamped nature of waves on a film of viscous liquid flowing down the plate also differentiates them from the ususal capillary waves. Waves on the film surface are constantly maintained by the force of gravity, which moves the film as a whole. Kapitza, next obtained a second order of approximation, a non-linear treatment but lacks justification according to Massot et al. [15]. A quite different periodic solution was obtained by Ishihara et al. [16] who solved the hydraulic equation of motion for unsteady flow in open channel with a constant inclination. Neglecting the surface tension effects, they obtained

$$c = u_o \left[\frac{6}{5} + \sqrt{\frac{6}{25} + \frac{1}{m\,\mathrm{Re}}} \right] \tag{10}$$

where m is the slope of the channel.

Massot et al. [15] elaborated the work of Kapitza by considering the complete x-component equation of motion without neglecting $\frac{\partial^2 u}{\partial x^2}$, the complete y-component equation of motion and took into account the variation of h with x, explicitly at all stages. Their results thus include as a special case of Kapitza and Kapitza [7] and of Ishihara et al. [16] in the frame work of linear approximation.

Yih [17], Benjamin [18] and, Jones and Whitaker [19] investigated the stability of Nusselt's solution in terms of the linearized theory of small perturbations. By postulating that waves are generated by infinitesimal periodic disturbances, and that the most likely to be seen are the fastest growing ones, this theory can be utilised to obtain a partial, if not satisfactory, description of the wave motion.

Philips [20] has examined wavy films from a statistical stand point and Lee [21] used the semi-empirical turbulent model developed for flow in closed conduits (Lee, [21]). However, these works are not detailed further here, since we are considering the apparently laminar range in which regular wave occurs.

2. THE MATHEMATICAL METHOD

2.1 The basic equations

The development of this method starts with the equations of continuity and motion for two-dimensional vertical flow of an incompressible Newtonian liquid of constant viscosity and is based on the search for an oscillatory steady state solution.

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0. \tag{11}$$

The *x*-component of Navier-Stokes equation is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+g-\frac{1}{\rho}\frac{\partial p}{\partial x}. \tag{12}$$

We propose to solve these equations (11) and (12) subject to the boundary conditions

$$\frac{\partial u}{\partial y}=0 \text{ at } y=h(x,t) \text{ for all } x \text{ and } t, \tag{13}$$

$$u=v=0 \text{ at } y=0 \text{ for all } x \text{ and } t, \tag{14}$$

and

$$p=p_{ext}-\kappa\frac{\partial^2 h}{\partial x^2} \text{ at } y=h(x,t) \tag{15}$$

where κ is the coefficient of surface tension. The radius of the curvature of the film is approximated by $\frac{\partial^2 h}{\partial x^2}$, since the surface slope is expected to be so small that its cube can be neglected.

In addition to the equations (11) and (12), we add the boundary-layer approximation of hydrostatic equilibrium across the film to be valid, following Kapitza [13] and Levich [3]:

$$\frac{\partial p}{\partial y}=0.$$

i.e., $p=p(x,h(x,t),t)$. (16)

As a matter of fact, this is a consequence of the *y*-component of Navier-Stokes equations of motion Schlichting, [22].

Therefore,

$$\frac{\partial p}{\partial x} = -\kappa \frac{\partial^3 h}{\partial x^3}.$$

With this, the equation (12) becomes

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + g + \frac{\kappa}{\rho}\frac{\partial^3 h}{\partial x^3}. \qquad (17)$$

The surface coordinate h is related to u by the mass balance equation

$$\frac{\partial h}{\partial t} = -\frac{\partial}{\partial x}\int_0^h u\,dy = -\frac{\partial}{\partial x}(\bar{u}h) \qquad (18)$$

where $\bar{u}$ is the average velocity over a cross-section defined by

$$\bar{u} = \frac{1}{h}\int_o^h u\,dy. \qquad (19)$$

The equations (11), (17) and (18) are the general equations of motion of the liquid in a thin film.

2.2 The derivation of the wavy equation and its compatibility

We follow the Kapitza's scheme of finding an approximate expression of u when the film surface assumes an arbitrary shape $y = h(x, t)$, which changes with time. We note that u must satisfy the conditions (13) and (14). These conditions led him to pattern the approximate expression after the Nusselt flow. In the same spirit, we write the following expression for u for the flow under consideration.

$$u = \frac{3\bar{u}(x,t)}{h(x,t)}\left[y - \frac{y^2}{2h(x,t)}\right]. \qquad (20)$$

We substitute (20) in the equation of continuity (11) and integrate under the boundary conditions (14) and $v(x,0,t) = 0$. Then

$$v = -\frac{3}{h(x,t)}\frac{\partial \bar{u}}{\partial x}\left[\frac{y^3}{6h(x,t)} - \frac{y^2}{2}\right] + \frac{3\bar{u}(x,t)}{h^2(x,t)}\frac{\partial h(x,t)}{\partial x}\left[\frac{y^2}{2} - \frac{y^3}{3h(x,t)}\right]. \qquad (21)$$

Substituting (20) and (21) in the equation (17) and integrating the result with respect to y from 0 to $h(x, t)$, we get

$$\frac{\partial \bar{u}}{\partial t}-\frac{1}{2}\frac{\bar{u}}{h}\frac{\partial h}{\partial t}+\frac{9}{10}\bar{u}\frac{\partial \bar{u}}{\partial t}-\frac{3}{10}\frac{\bar{u}^2}{h}\frac{\partial h}{\partial x}=g+\frac{\kappa}{\rho}\frac{\partial^3 h}{\partial x^3}+\nu\left(\frac{\partial^2 \bar{u}}{\partial x^2}-\frac{1}{2}\frac{\bar{u}}{h}\frac{\partial^2 h}{\partial x^2}-\frac{3\bar{u}}{h^2}-\frac{5}{4}\frac{1}{h}\frac{\partial \bar{u}}{\partial x}\frac{\partial h}{\partial x}\right). \quad (22)$$

The principle here is to determine $\bar{u}$ so that equation (20) becomes the best representation of u in the sense that it satisfies (22) in an averaged manner. It may be recalled that in this respect, it is like the Kerman-Polhausen technique in solving boundary-layer flow problems, Schlichting [22]. If a solution of equation (22) exists such that it represents a steady travelling wave in the x-direction (i.e., periodic in x and t), it must be a function of a simple variable $x - ct$, where c is the phase velocity of the progressive undamped wave train. To search for such a solution, we can replace $\partial/\partial t$ in equation (22) by $-c(\partial/\partial x)$ and linearize the equation (22) on the assumption that the wavy surface is but a small perturbation on the flat surface $y = h_o$, i.e.,

$$h = h_o[1+\psi(x,t)] \quad (23)$$

where $|\psi| < 1$.

The mass balance equation (18) gives us

$$c(h-h_o)=\bar{u}h-u_o h_o$$

where u_o is the average velocity of the liquid film in equation (3) corresponding to Nusselt solution.

In terms of ψ, this gives us

$$\bar{u}=u_o+(c-u_o)\psi .$$

In a similar manner,

$$\frac{\partial \bar{u}}{\partial t}=-c(c-u_o)\frac{\partial \psi}{\partial x},$$

$$\frac{\partial \bar{u}}{\partial x}=(c-u_o)\frac{\partial \psi}{\partial x},$$

$$\frac{\bar{u}}{h}=\frac{1}{h_o}[u_o+(c-2u_o)\psi],$$

$$\frac{\bar{u}}{h^2}=\frac{1}{h_o^2}[u_o+(c-3u_o)\psi],$$

$$\frac{\partial h}{\partial t} = -c\, h_o \frac{\partial \psi}{\partial x}$$

and

$$\frac{\partial h}{\partial x} = h_o \frac{\partial \psi}{\partial x}.$$

Substituting these equations in the equation (22), we arrive at a linear equation, which is expressed in a non-dimensional form using

$$\mathrm{Re} = \frac{u_o h_o}{\nu} \text{ (Reynolds number)}, \tag{24}$$

$$Fr = \frac{u_o^2}{g h_o} \text{ (Froude number)}, \tag{25}$$

$$P = \frac{\lambda}{2\pi}\sqrt{\frac{\rho\, g}{\kappa}} \text{ (dimensionless wavelength)}, \tag{26}$$

$$N = \frac{2\pi\, h_o}{\lambda} \text{ (dimensionless wave number)} \tag{27}$$

$$We = \frac{u_o^2 h_o}{\kappa / \rho} = P^2 N^2 Fr \text{ (Weber number)} \tag{28}$$

$$\xi = \frac{2\pi x}{\lambda}, \tag{29}$$

and

$$\eta = \frac{c}{u_o} \text{ (Wave celerity)} \tag{30}$$

where λ is wavelength. Thus, we get

$$\frac{N}{P^2}\psi''' + \frac{1}{3}\left(\eta - \frac{3}{2}\right)N^2\psi'' + \left(\eta^2 - \frac{12}{5}\eta + \frac{6}{5}\right)N\,Fr\,\psi' - (\eta - 3)\psi = 0 \tag{31}$$

where the primes represent differentiation with respect to ξ. Equation (31), being homogeneous, will admit a sinusoidal solution, only if

$$P^2 Fr = \left(\eta^2 - \frac{12}{5}\eta + \frac{6}{5}\right)^{-1} \tag{32}$$

and

$$N^2 = -\frac{3(\eta-3)}{\eta-\frac{3}{2}}. \quad (33)$$

P^2 in equation (32) is real, if

$$\eta > \eta_2 \text{ or } \eta < \eta_1 \quad (34)$$

where $\eta_1 < \eta_2$ are the roots of the quadratic equation

$$\eta^2 - \frac{12}{5}\eta + \frac{6}{5} = 0$$

$$\eta_1 = \left(\frac{6-\sqrt{6}}{5}\right) \text{ and } \eta_2 = \left(\frac{6+\sqrt{6}}{5}\right).$$

On the other hand N in equation (33) is real, only if

$$-\frac{3}{2} < \eta < 3. \quad (35)$$

Thus an undamped, steadily travelling wave in the form of a sinusoidal function is compatible with the physical situation if positive values of η lie within the common range as indicated in equations (34) and (35). No solution in the particular form under consideration exists if the two regions do not overlap.

It is clear from equations (34) and (35), that the common range for η is

$$\frac{6+\sqrt{6}}{5} = 1.689 < \eta < 3.$$

For each such admissible η, we can find the wavelength of the corresponding wavy flows

$$\lambda = 2\pi h_o \left[\frac{2\eta-3}{6(\eta-3)}\right] \quad (36)$$

and the Weber number is

$$We = \frac{30(3-\eta)}{(2\eta-3)(5\eta^2-12\eta+6)}. \quad (37)$$

3. THE DESCRIPTION OF THE WAVY FLOW

3.1 Dimensionless wave celerity

At low Weber number (thin films), we have η = 3 which is the value predicted by the theories of Kapitza [13], Yih [17], Benjamin [18], Hanratty and Hershman [23]. Thus, these theories are asymptotic to the present solution at zero Weber number.

Also when $\eta \rightarrow 1.689$, $We \rightarrow \infty$. This result is the limiting value of Ishihara et al. [16] for large Weber number (thick films). Their analysis is therefore asymptotic to the present solution at high Weber number.

The solution in equation (37) gives the variation of the Weber number *We* with the dimensionless celerity η. Fig. 2 represents a comparison of all the existing theories. Further, at medium Weber number, where the other theories fail, there is a qualitative agreement between experimental results and the present predictions.

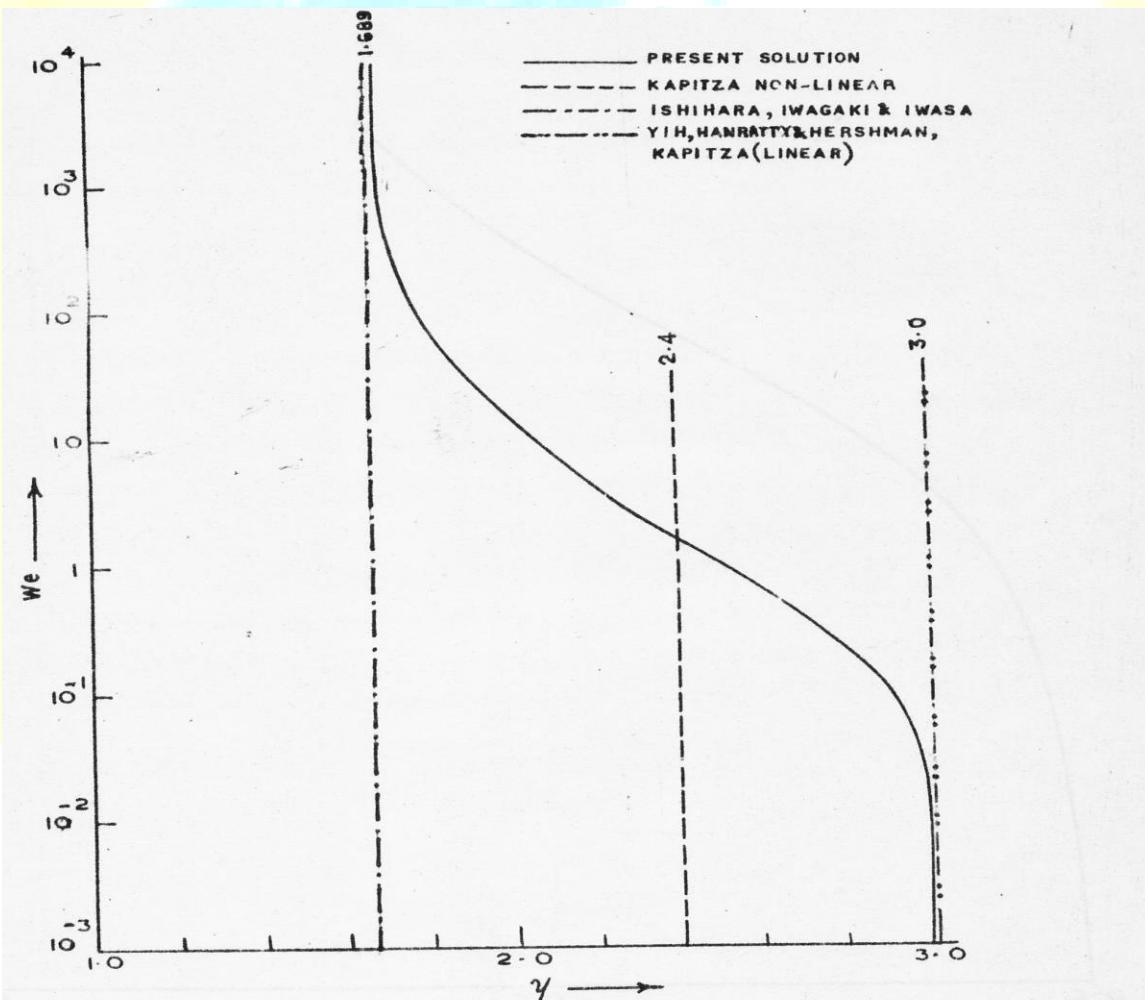

Fig. 2 Weber number *We* as a function of wave celerity η

It is remarkable that all experimental observations of η lie between 3 and 1.689, which is the predicted range. Table 1 gives the computed values of *We* and *N* for the admissible range of η. The values of Weber number *We* and wave number *N* vary from zero to infinity as the wave celerity η varies from 3 to 1.689.

TABLE 1

Computed values of Weber number dimensionless wave number for admissible values of wave celerity

η	We	N
3.0	0.0000	0.0000
2.9	0.0809	0.4269
2.8	0.1989	0.6793
2.7	0.3731	0.8660
2.6	0.6342	1.0445
2.5	1.0345	1.2248
2.4	1.6666	1.4142
2.3	2.7062	1.6202
2.2	4.5113	1.8516
2.1	7.8948	2.1213
2.0	15.0001	2.4495
1.9	33.0006	2.8723
1.8	100.0059	3.4641
1.7	1950.7978	4.4159
1.689	∞	∞

3.2 Dimension wavelength

The dimensionless wavelength *P* comes out as a function of the Weber number through the Reynolds number

$$\mathrm{Re} = We^{3/5}\left(\frac{3\rho\kappa^{3}}{g\mu^{4}}\right)^{1/5} \qquad (38)$$

for assumed values of fluid parameters such as ρ, κ and μ.

From equation (32), we have

$$P=\left(\eta^2-\frac{12}{5}\eta+\frac{6}{5}\right)^{-1/2}Fr^{-1/2}\quad=\left[3\left(\eta^2-\frac{12}{5}\eta+\frac{6}{5}\right)\right]^{-1/2}We^{-3/10}\left(\frac{3\rho\kappa^3}{g\mu^4}\right)^{-1/10}\qquad(39)$$

when $\eta\to 3$, $We\to 0$, $P\to\infty$ and when $\eta\to\eta_2 = 1.689$, $We\to\infty$, $P\to\infty$.

The equation (39) is used to plot the dimensionless wavelength P against the Weber number We in Fig. 3 for alcohol and water.

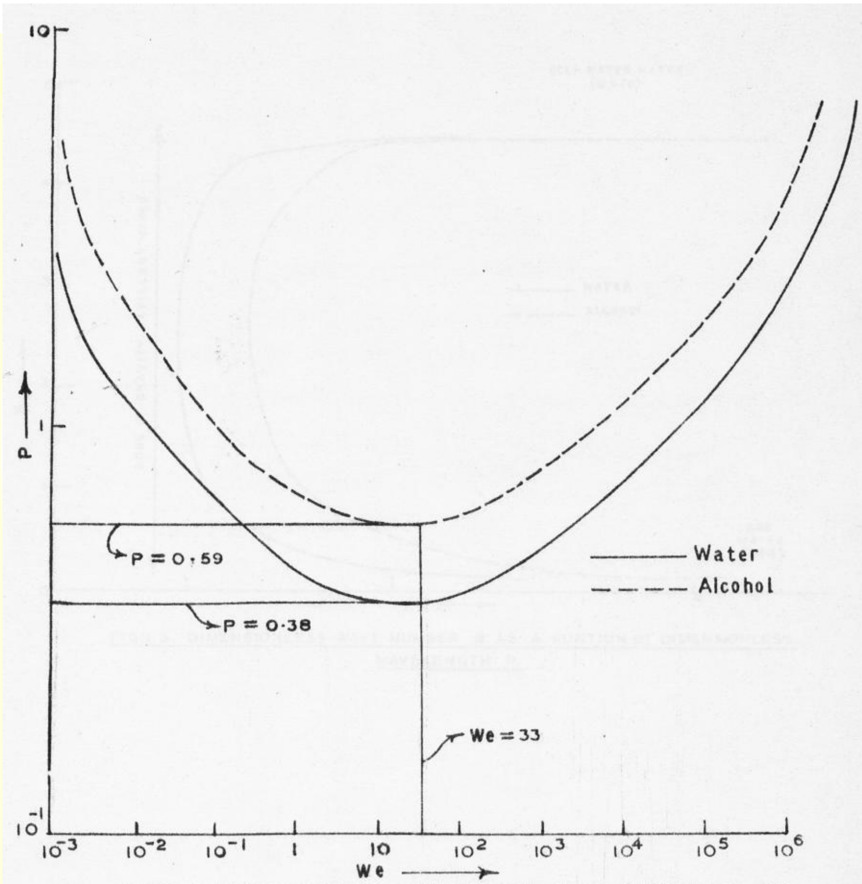

Fig. 3 Dimensionless wave length P as a function of Weber number We

It is observed that the wavelength passes by a minimum (this minimum occurs when $\eta = 1.9$ for any liquid this value corresponds to the Weber number 33 and the dimensionless wavelength P = 0.38 (for water) and P = 0.59 (for alcohol) and becomes very large when $We\to 0$ and $We\to\infty$. Such a behaviour has been observed experimentally by Kapitza and Kapitza [7] whose experimental results seem consistently higher than the theoretical predictions.

3.3 Dimensionless wave number

In a similar way, the dimensionless wave number N is plotted against the dimensionless wavelength P in Fig. 4. The path of the system for increasing Weber number goes through three regions related to Robertson [24] to different known types of waves namely: long waves, gravity-capillary waves and deepwater waves. The transition between the gravity and capillary waves is usually set in the literature at $P = 1$, because this value corresponds to the minimum of celerity of an ideal liquid.

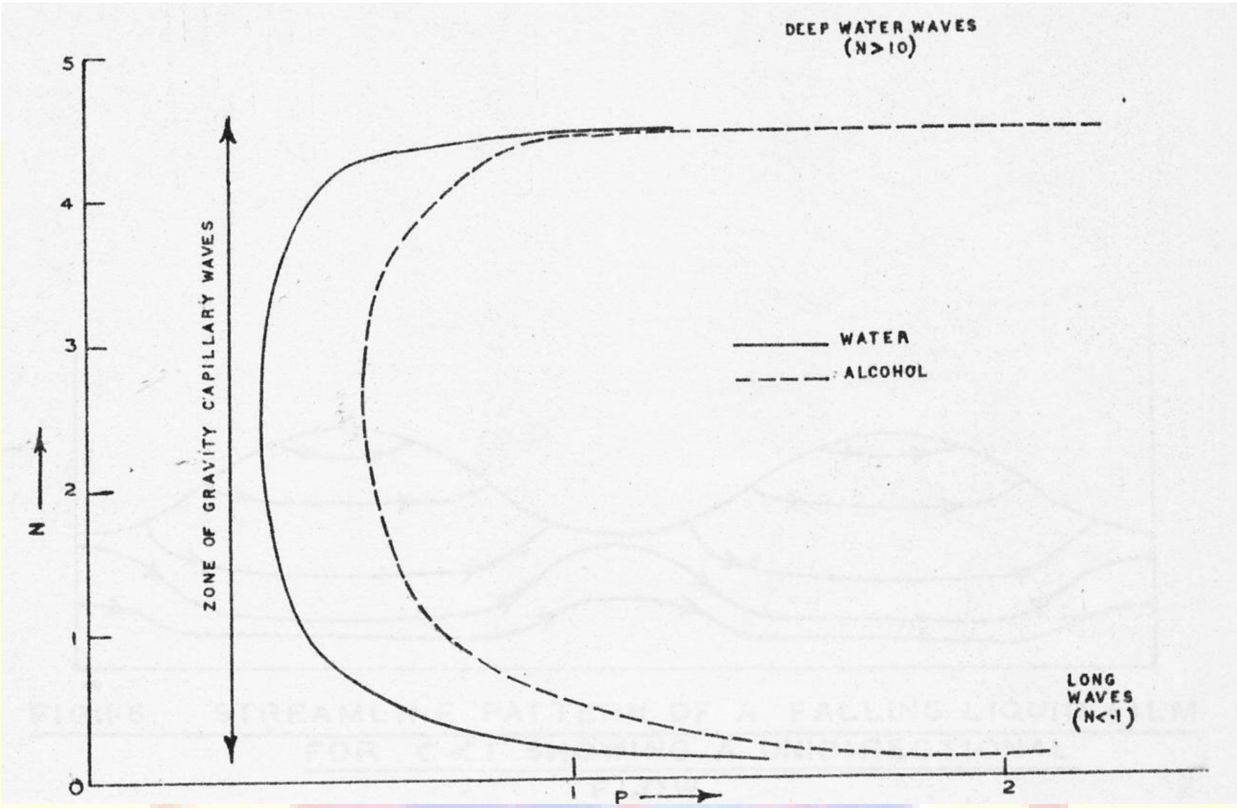

Fig. 4 Dimensionless wave number N as a function of dimensionless wavelength p

4. STREAM LINE PATTERN

The stream lines are defined by

$$\frac{dx}{u} = \frac{dy}{v}. \qquad (40)$$

Let $\delta = y/h$ denote the dimensionless transverse coordinate for stream line calculation. At a fixed instant t ($t = 0$ say), using the equations (20) and (21) for u and v in (40), we obtain

$$\frac{2\delta'}{\delta} + \frac{(3-\delta)'}{3-\delta} = -\frac{(\bar{u}h)'}{\bar{u}h} \qquad (41)$$

where the primes represent differentiation with respect to x. On integration, we find that the dimensionless stream function Ψ (with respect to $u_o h_o$) is

$$\Psi = \frac{3}{2}\delta^2(3-\delta)\left(1+\eta A\sin\frac{2\pi x}{\lambda}\right) \tag{42}$$

in which the form of ψ in equation (23) is

$$\psi = A\sin\frac{2\pi x}{\lambda}. \tag{43}$$

Two families of stream lines appear depending upon the dimensionless celerity – amplitude number $C = \eta A$ being greater or less than unity. When $C < 1$, we obtain the unidirectional stream lines. Fig. 5 gives a general view of the instantaneous stream lines of a falling liquid film corresponding to a slightly deformed parallel flow and to positive values of the stream function ψ.

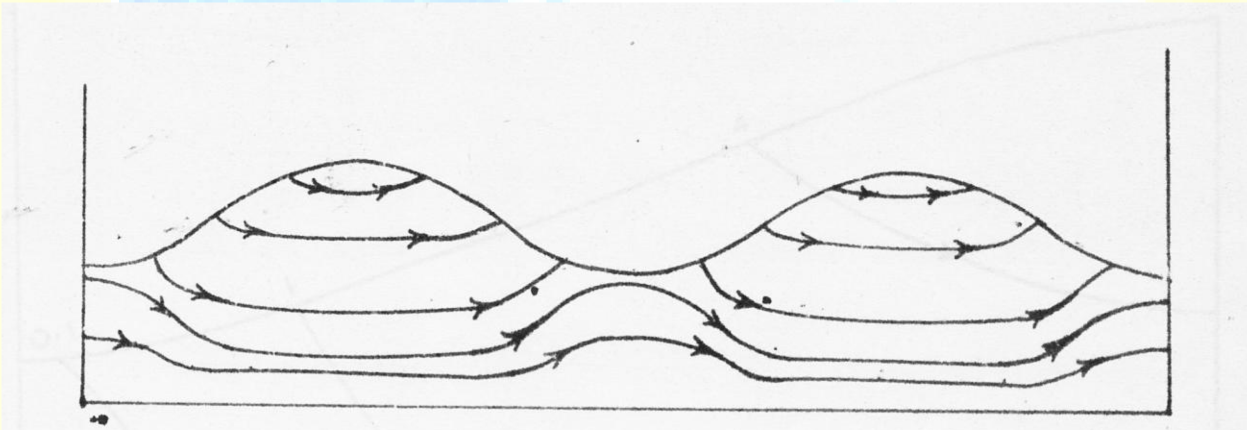

Fig 5. Streamline pattern of a falling liquid film for C < 1 showing a unidirectional flow

A detailed stream line when $\psi = 0.1$, 1.0 and 4.0 is plotted in Fig. 6 for $C = 0.9$.

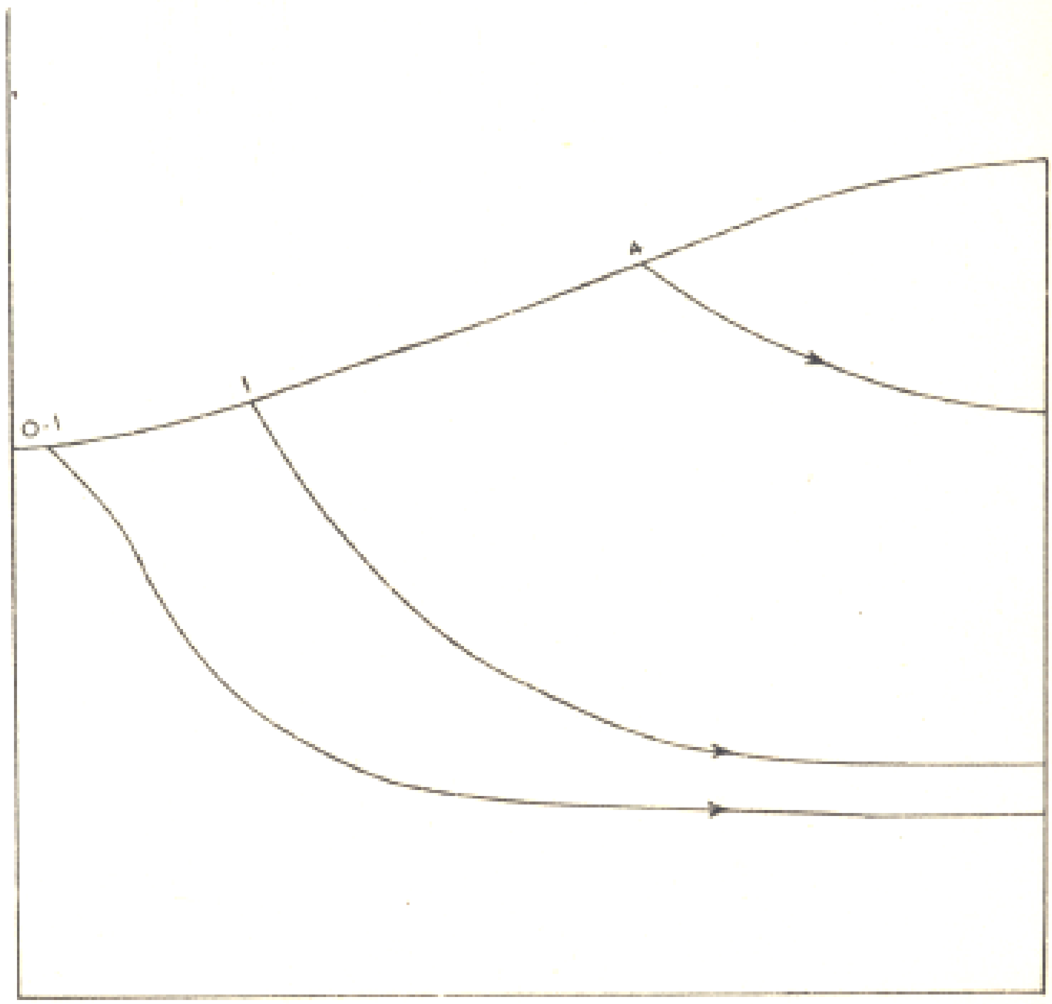

Fig. 6 Streamlines for C = 0.9 < 1

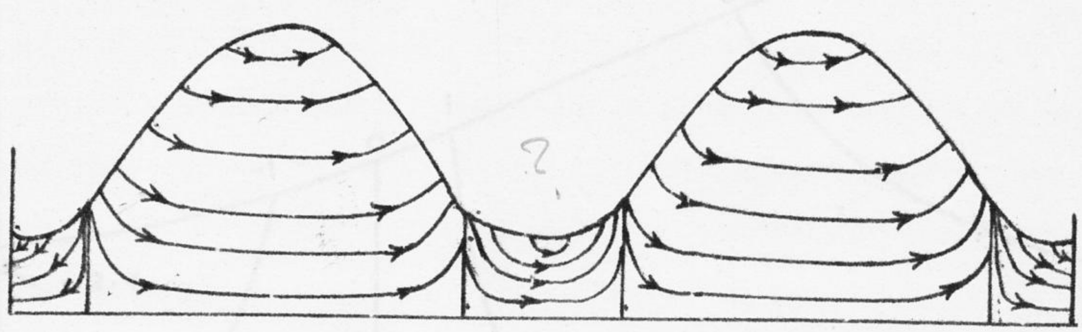

Fig: 7 Streamline pattern of a falling liquid film for C > 1 showing a flow reversal

When $C > 1$, we obtain reversal flow stream lines, a general view of which is shown in Fig. 7.

Fig. 8 gives a detailed stream line when ψ = -2.4, -1.0, 0.0, 1.0, 5.0 and 8.4 for C = 1.8. Thus when $C > 1$, we obtain reversal stream lines corresponding to positive and negative values of the stream function, suggesting a swirl in the liquid.

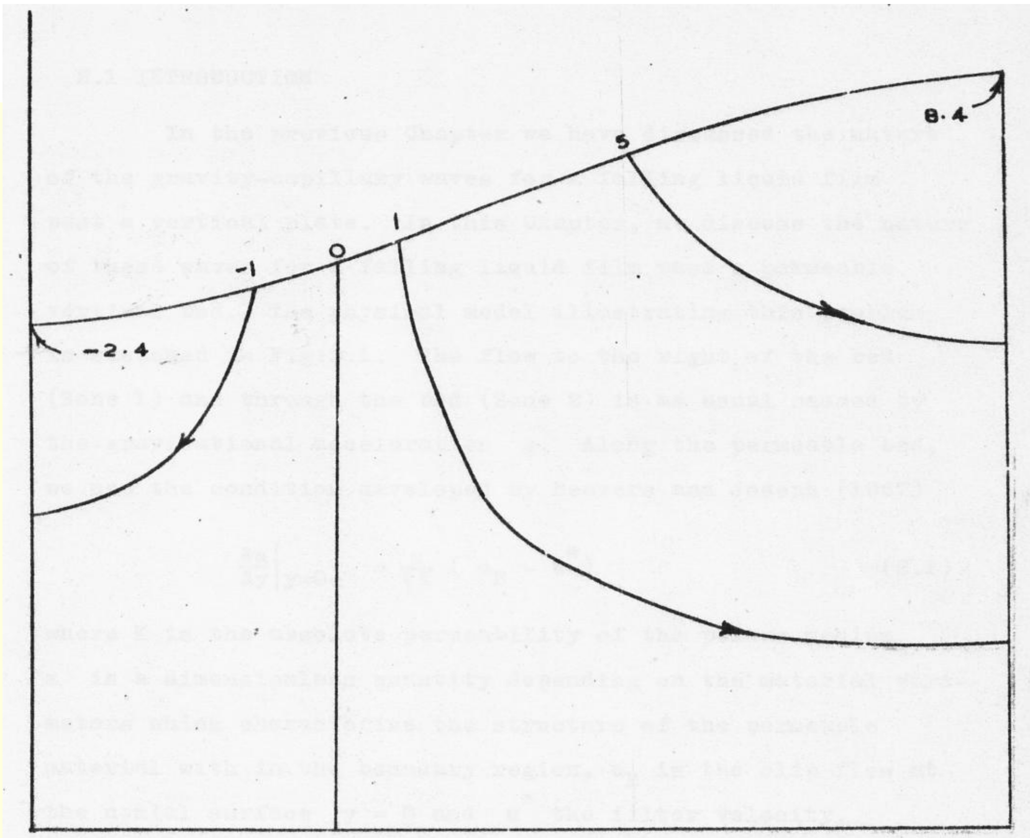

Fig. 8 Streamlines for C=1.8 > 1

Using equations (18), (19) and (23), we obtain

$$\bar{u} = \frac{c\psi + u_o}{1+\psi}. \qquad (44)$$

The transition between the above two types of flow is determined by setting $\bar{u} = 0$. We obtain, on using equation (43)

$$C\sin\frac{2\pi x}{\lambda} = -1. \qquad (45)$$

These two types of flow may be expected to reveal different mechanisms of surface renewal.

5. CONCLUSIONS

We have observed that for thin films (i.e., at low Weber number), we have the wave celerity $\eta = 3$ which is the value predicted by the theories of Kapitza, Yih, Benjamin, Hanratty and Hershman and hence these theories are asymptotic to the present solution at zero Weber number. Also, it has been found that when $\eta \to 1.689$, $We \to \infty$ which is the limiting value of Ishihara et al. [16] for thick films (large Weber number). Their analysis is therefore asymptotic to the present solution at high Weber number. At medium Weber number, where the other theories fail, there is a qualitative agreement between experimental results and the present predictions.

It is remarkable to note, from Fig. 2, that all experimental observations of η lie between 3 and 1.689 and this is the predicted range. It is noticed from Table 1 that the values of Weber number and wave number vary from zero to infinity as the wave celerity varies from 3 to 1.689. From Fig. 3, it is observed that the wavelength passes by a minimum when $\eta = 1.9$ and becomes very large when $We \to 0$ and $We \to \infty$. Such a behavior has been observed experimentally by Kapitza whose experimental results seem consistently higher than the theoretical predictions. It has been found from Fig. 4 that the path of the system for increasing Weber number goes through three regions to different known types of waves namely: long waves, gravity-capillary waves and deepwater waves. It is found that from Figs.5 and 6, when $C < 1$, we obtain the unidirectional stream lines. When $C > 1$, we obtain reversal flow stream lines, a general view of which is shown in Fig. 7. Also from Fig. 8, it has been observed that when $C > 1$, we obtain reversal stream lines corresponding to positive and negative values of the stream function, suggesting a swirl in the liquid.

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