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Mathematical Modelling of System Parameters and Optimal Maintenance of Three Units System with Intermittent and Non-Intermittent Repair Using RPGT

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Abstract: This paper discusses mathematical modelling and optimal maintenance of a three-unit system with intermittent and non- intermittent repair policies using Regenerative Point Graphical Technique (RPGT). There are various repair policies, of this one is intermittent repair is feasible, with perfect repair for two of the subsystems, other is non-intermittent repairable considered for the subsystem C which consists of three online parallel working components which may work in degraded state upon the failure of one or two subcomponents of subsystem C and after the complete failure is imperfectly repaired. All the units are served by the same single repairperson who is 24x7 hours available. The system can operate in a reduced state when subsystem C partially fails. Various path probabilities, mean sojourn time and system behavior for increasing repair and failure rates of various units is discussed drawing tables, graphs, followed by analysis.

Keywords: - Intermittent and Non-intermittent Repair**,** Optimization, RPGT, Optimal Maintenance, Modeling

Introduction

There are several types of repair policies which may be categorized into three main types of repair types as under: -

(i) Intermittent repair is feasible.

(ii) No Intermittent repair is feasible.

(iii) Periodic and Scheduled Repair

In this paper an analysis of the first two types of repair policies is taken only, for optimum operation of the plant, for intermittent and non- intermittent repair cases with perfect and

partial repairs (post failure) is discussed. In the third type of systems, periodic and scheduled repair, there are very few systems available in practice due to the heavy costs involved. Whereas partial repairable subsystems work in reduced capacity after repair, perfectly repairable subsystems work as if a new one after repair. The Markov process has been used to draw the working transition diagram the system. A general system has been taken which may be observed in several operating industries in practical, such as nuclear plants, space satellites, etc. Subsystems replacement and repair policies can be established by optimizing system parameters by the managements. Various subsystems in an operational plant may fail while working online, repair of these units may be feasible or non-feasible of various types. This paper presents mathematical modelling and optimization of parameters of three-unit system considering various repair policies with post-repair degradation in one-unit, perfect repair in another subsystems, and a single permanent repairman who is 24x7 available using the RPGT.

The considered system is made up of three online working subsystems C having components in parallel which may have only non-intermittent repair and is in degraded state (post repair), and other two subsystems D and E upon failure have intermittent repair and perfectly repaired by a single repairman. The system operates in a reduced capacity when subsystem C operates in a reduced state. Whenever subsystems D and E fail are fixed by a repairman. A unit's failure or operational state is ascertained using a fuzzy concept. A state transition diagram of the system is shown in figure 1, considering various probabilities, repair rates and failure rates of units independent, to identify the Primary, Secondary, and Tertiary circuits, as well as the Base state.

To solve the problem and to model the system parameters, RPGT is used. Tables and graphs are used to assist in explaining system behaviour. The impact of varying repair and failure rates on system parameters is investigated through simulations. Additionally, profit optimization is also discussed. However, using the RPGT, solutions are obtained swiftly and simply without having to write any state equations, going through cumbersome computations. The values of the repair rates of different units are assessed by setting the system parameters to their ideal levels. The management will have the best possible profit and cost values, if it can maintain the recommended values of repair and failure rates of subsystems. Simulation, when repair and failure rates of all subsystems are equal is also presented and discussed.

(Devi and Garg 2022) discussed the three algorithms specifically HA, COGA and HGAPSO are applied to solve RAP. Present paper carriages a comprehensive literature review to classify, evaluate and intercept the standing studies related to the RAP (Devi et al. 2023), Kumar et al. 2018 studied behaviour of bread manufacturing plant was analysed by, also Kumar et al. 2019 presented sensitivity analysis of a cold standby framework consisting of two identical units with server failure and preventative maintenance, used RPGT, two halves make up the paper, one of which is in use and the other of which is in cold standby mode. (Shakuntla et al., 2011) discussed the behaviour analysis of a polytube plant applying supplementary technique. PSO was used by (Kumari et al. 2021) to analyze systems with limited situations. (Kumar et al. 2019) carried out mathematical formulation and behaviour study of a paper mill's washing unit. Rajbala et al. 2022, making use of a heuristic approach, analysed the redundancy allocation problem for the cylinder manufacturing plant. Shakuntla et al. 2011, carried the comparative analysis of the subsystem in which failures occur simultaneously.

Assumptions: - The following assumptions are taken: -

1. There is a single repairperson for repair of failed units.
2. Failure and repair of subsystems follow general distributions and are independent.
3. Nothing fails further when the system is down,

Notations

s_r: rth simple path

S_{ff}: simple failure free path

$\left(i \overset{s_r}{\rightarrow} j\right)$: rth directed simple path from the state i to state j;

$\left(\xi \overset{sff}{\longrightarrow} i\right)$: failure free path from the $base$ state ξ to state i.

$V_{m,m}$: Probability factor of state m of the cycle.

$V_{\overline{m,m}}$: Probability factor of the state m reachable from the terminal state m of the $m - \overline{cycle}$.

$R_i(t)$: Reliability at time t of the system, provided the system is in good state 'i' at t = 0.

$A_i(t)$: Probability of the system in good at time 't',

$B_i(t)$: transition probability that the server is busy at time 't;'

$V_i(t)$: The expected number of server visits for doing a job in interval (o, t]

'' : Dash denotes derivative.

μ_i : Mean sojourn time spent in state i.

$\mu_i = \int_0^\infty R_i(t)dt$

μ_i^1 : The total unconditional time spent before transiting to any other regenerative states, given that the system entered regenerative state 'i'.

η_j : Expected waiting time in regenerative state 'j' .

ξ : Base state.

f_j : Fuzzy measure of availability in the jth-state.

w_p = $(1+\beta)\, w_{p-1}$ where β is degradation factor for repair of subsystem C post repair.

C, D, E: Units are good in working state.

m_i : failure rates of units.

n_i: repair rates of units

c, d, e: respective units in failed states

$\overline{C}$: unit C working in reduced state

g(t): pdf for the repair of failed unit C (post failure)

$\overline{g(t)}$: complement of g(t)

C^1: Unit C in reduced state (post repair)

* : Laplace Transformation

State	Symbol
Up	
Failed	
Reduced	
Post Repair Reduced	

1. **Transition Diagram of the System with Intermittent Repair with Perfect and Partial Repairable cases**

Considering the above assumptions and notations the state Transition Diagram of the system is drawn below in figure 1.

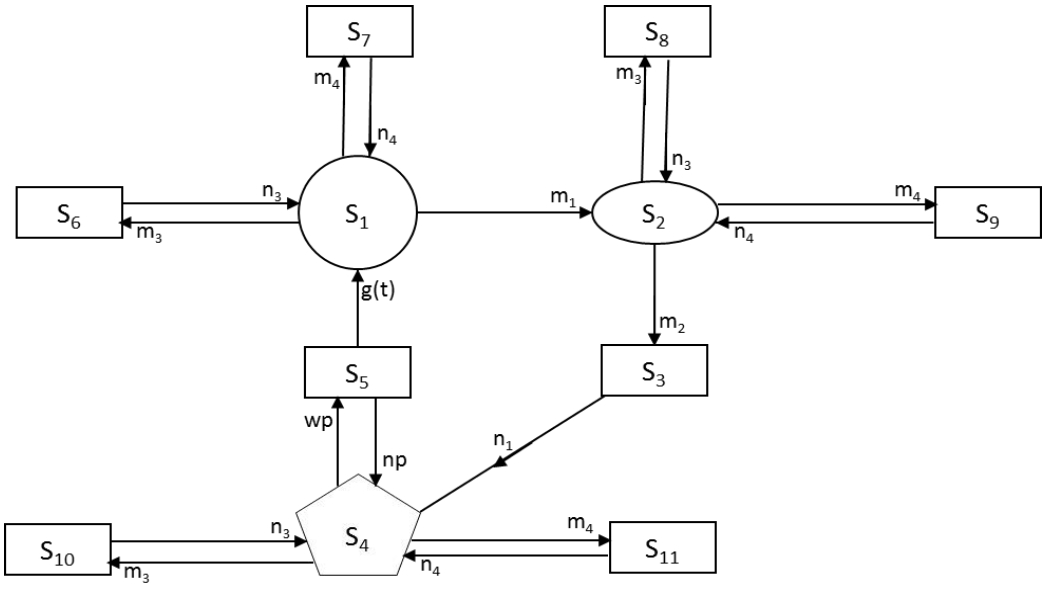


Figure 1: Transition Diagram of the System

$S_1 = CDE$, $S_2 = \overline{C}DE$, $S_3 = cDE$, $S_4 = C^1DE$

$S_5 = c^1DE$, $S_6 = CdE$, $S_7 = CDe$, $S_8 = \overline{C}dE$

$S_9 = \overline{C}De$, $S_{10} = C^1dE$, $S_{11} = C^1De$

3.1 Model Description: -

Initially the system is initially in full capacity working state S_1, as all the units C, D and E are in good state. There is a single repairperson available in the system. The failure rates of units D to d, $\overline{C}$ to c, and E to e, are m_1, m_2, m_3, m_4 and m_p repair rates n_1, n_2, n_3, n_4 and n_p respectively. As the Unit 'C' have subcomponents in parallel so it is works in reduced state $\overline{C}$ for which its partial failure rate is m_1 and the system enters the state $S_2 = \overline{C}DE$, if in this state unit 'D' fails further then the system enter the state $S_8 = \overline{C}dE$, from which unit is repaired by the repairman at the repair rate n_3, so the system again enters the state $S_2 = \overline{C}DE$ and again if in this state unit 'E' fails further then the system enter the state $S_9 = \overline{C}De$, from which unit is repaired by the repairman at the repair rate n_4, so the system again enters the state $S_2 = \overline{C}DE$. If in the state S_1, unit D or E fail for which the failure rates are m_3 and m_4 respectively, then the system enters the states $S_6 = CdE$, $S_7 = CDe$, again upon their repair with rates are n_3 and n_4, the system again reaches the state S_1. In state S_2, if unit C fails the system enters the failed state $S_3 = cDE$, and upon it repair enters the state $S_4 = C^1DE$, reduced state post failure of unit C, as it cannot be repaired perfectly. Similarly, while in state S4, if unit D and E fail, the system enters the states $S_{10} = C^1dE$, and $S_{11} =$

C^1De, and upon repair of respective units the system again enters the state S_4. Similarly, while in state S4, if unit C fails the system enters the states $S_5 = c^1DE$ and after its replacement, the system again reaches the state S_1. If the unit C upon failure is beyond repair, then it is replaced with new one with probability density function g(t).

Transition Probabilities and Mean Sojourn Time

The transition probabilities in moving from state i→ j, (1≤ i, j≤ 11) are given below in table 1.

Table 1: State Transition Probabilities

$q_{i,j}^{(t)}$	$p_{ij} = q^*_{i,j}(t)$
$q_{1,2} = m_1 e^{-k_1 t}$ $q_{1,6} = m_3 e^{-k_1 t}$ $q_{1,7} = m_4 e^{-k_1 t}$ Where $k_1 = m_3+m_4+m_1$	$p_{1,2} = m_1/k_1$ $p_{1,6} = m_3/k_1$ $p_{1,7} = m_4/k_1$
$q_{2,3} = m_2 e^{-k2t}$ $q_{2,8} = m_3 e^{-k2t}$ $q_{2,9} = m_4 e^{-k2t}$ Where $k_2 = m_3+m_4+m_2$	$p_{2,3} = m_2/k_2$ $p_{2,8} = m_3/k_2$ $p_{2,9} = m_4/k_2$
$q_{3,4} = n_1 e^{-n_1 t}$	$p_{3,4} = 1$
$q_{4,5} = m_p e^{-k3t}$ $q_{4,10} = m_3 e^{-k3t}$ $q_{4,11} = m_4 e^{-k3t}$ Where $k_3 = m_4 + m_3 + m_p$	$p_{4,5} = m_p/k_3$ $p_{4,10} = m_3/k_3$ $p_{4,11} = m_4/k_3$
$q_{5,1} = g(t) e^{-n_p t}$ $q_{5,4} = n_p e^{-n_p t} \overline{g(t)}$	$p_{5,1} = g^* n_p$ $p_{5,4} = 1-g^* n_p$
$q_{6,1} = n_3 e^{-n_3 t} = q_{8,2} = q_{10,4}$	$p_{6,1} = 1$
$q_{k,1} = n_4 e^{-n_4 t}$, k=7, 9, 11	$p_{7,1} = 1 = p_{9,2} = 1 = p_{11,4}$

Mean Sojourn Time: The mean sojourn times are given in table 2.

Table 2: Mean Sojourn Times

$R_i(t)$	$\mu_i=R_i^*(0)$
$R_1^{(t)}= e^{-k1t}$	$\mu_1 = 1/ k_1$
$R_2^{(t)}= e^{-k2t}$	$\mu_2 = 1/ k_2$
$R_3^{(t)}= e^{-(n_1)t}$	$\mu_3 = 1/(n_1)$
$R_4^{(t)}= e^{-k3t}$	$\mu_4 = 1/ k_3$
$R_5^{(t)}= e^{-n_4 t}\overline{g(t)}$	$\mu_5 = 1-g^*(n_4)/n_1$
$R_6^{(t)}= e^{-n_3 t} = R_8^{(t)} = R_{10}^{(t)}$	$\mu_6 = 1/n_3 = \mu_8 = \mu_{10}$
$R_7^{(t)}= e^{-(n_4)t} = R_9^{(t)} = R_{11}^{(t)}$	$\mu_7 = 1/n_4 = \mu_9 = \mu_{11}$

2. Path Probabilities: -

As at vertex '4', there are maximum number of primary cycles, so taking '4' as the base-state, using RPGT, the path probabilities are as below: -

$V_{4,1} = (4, 5, 1)/ [1- (1, 6, 1)] [1- (1, 7, 1)]$

$= p_{4,5}\, p_{5,1}/ (1-p_{1,6}\, p_{6,1}) (1-p_{1,7}\, p_{7,1})$

$V_{4,2} = (4, 5, 1, 2)/ [1- (1, 6, 1)] [1-(1, 7, 1)] [1 -(2, 8, 2)] [1-(2, 9, 2)]$

$= p_{4,5}\, p_{5,1}\, p_{1,2}/ (1-p_{1,6}\, p_{6,1}) (1-p_{1,7}\, p_{7,1}) (1-p_{2,8}\, p_{8,2}) (1-p_{2,9}\, p_{9,2})$

$V_{4,3}$ = ……. to continue.

$V_{1,1}$ =1 (verified)

$V_{1,2} = (1, 2)/ [1- (2, 8, 2)] [1- (2, 9, 2)]$

$= p_{1,2}/ (1- p_{2,8}\, p_{8,2}) (1 -p_{2,9}\, p_{9,2})$

$V_{1,3}$ = ……. to continue.

3. Evaluation of system parameters:

Mean Time to System Failure **T_0:** With initial state '1', the system before entering any failed state are: 'i' = 1, 2

$$\text{MTSF } (T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} i\right)\right\}\mu i}{\Pi_{m_2\neq\xi}\left\{2-V_{\overline{m_2 m_2}}\right\}}\right\}\right] \div \left[1-\sum_{sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} \xi\right)\right\}}{\Pi_{m_3\neq\xi}\left\{2-V_{\overline{m_3 m_3}}\right\}}\right\}\right]$$

$= (V_{1,1}\, \mu_1 + V_{1,2}\, \mu_2) / (V_{1,1}\, \mu_1 + V_{1,2}\, \mu_2 + V_{1,3}\, \mu_3 + V_{1,4}\, \mu_4 + V_{1,5}\, \mu_5 + V_{1,6}\, \mu_6 + V_{1,7}\, \mu_7$

$+ V_{1,8}\, \mu_8 + V_{1,9}\, \mu_9 + V_{1,10}\, \mu_{10} + V_{1,11}\, \mu_{11})$

Availability of the System (A_0): With base state 'ξ' = '4' the system is available in states 'j' = 1, 2, 4 total fractions of time for which the system is available using RPGT is as under,

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\}fj\mu j}{\Pi_{m_2\neq\xi}\{1-V_{\overline{m_2m_2}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{m_3\neq\xi}\{1-V_{\overline{m_3m_3}}\}}\right\}\right]$$

$$= \left[\sum_j V_{\xi,j}\, f_j \mu_j\right] \div \left[\sum_i V_{\xi,i}\, f_j \mu_i^1\right]$$

$$= (V_{4,1}\,\mu_1 + V_{4,2}\,\mu_2 + V_{4,4}\,\mu_4)/\,(V_{4,1}\,\mu_1 + V_{4,2}\,\mu_2 + V_{4,3}\,\mu_3 + V_{4,4}\,\mu_4 + V_{4,5}\,\mu_5 + V_{4,6}\,\mu_6 + V_{4,7}\,\mu_7 + V_{4,8}\,\mu_8 + V_{4,9}\,\mu_9 + V_{4,10}\,\mu_{10} + V_{4,11}\,\mu_{11})$$

Busy Period of the Server (B_0): With base state ξ = '4', the total fraction of time for which the server remains busy in states 'j' = 3 to 11, using RPGT is

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\}\eta_j}{\Pi_{m_2\neq\xi}\{1-V_{\overline{m_2m_2}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{m_3\neq\xi}\{1-V_{\overline{m_3m_3}}\}}\right\}\right]$$

$$= \left[\sum_j V_{\xi,j}\,\eta_j\right] \div \left[\sum_i V_{\xi,i}\,\mu_i^1\right]$$

$$= (V_{4,3}\,\mu_3 + V_{4,5}\,\mu_5 + V_{4,6}\,\mu_6 + V_{4,7}\,\mu_7 + V_{4,8}\,\mu_8 + V_{4,9}\,\mu_9 + V_{4,10}\,\mu_{10} + V_{4,11}\,\mu_{11})/(V_{4,1}\,\mu_1 + V_{4,2}\,\mu_2 + V_{4,3}\,\mu_3 + V_{4,4}\,\mu_4 + V_{4,5}\,\mu_5 + V_{4,6}\,\mu_6 + V_{4,7}\,\mu_7 + V_{4,8}\,\mu_8 + V_{4,9}\,\mu_9 + V_{4,10}\,\mu_{10} + V_{4,11}\,\mu_{11})$$

Expected Fractional Number of Inspections (V_0) by the repairperson in unit time: the states server visits the system afresh are 3 to 11, with Base state 'ξ' = '4', the fractional number of visits per unit time by the repairman using RPGT is given by

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr}\rightarrow j)\}}{\Pi_{k_2\neq\xi}\{2-V_{\overline{k_2k_2}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr}\rightarrow i)\}\mu_i^1}{\Pi_{k_3\neq\xi}\{2-V_{\overline{k_3k_3}}\}}\right\}\right]$$

$$= \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}\,,\mu_i^1\right]$$

$$= (V_{1,3}\,\mu_3 + V_{1,5}\,\mu_5 + V_{1,6}\,\mu_6 + V_{1,7}\,\mu_7 + V_{1,8}\,\mu_8 + V_{1,9}\,\mu_9 + V_{1,10}\,\mu_{10} + V_{1,11}\,\mu_{11})/\,(V_{1,1}\,\mu_1 + V_{1,2}\,\mu_2 + V_{1,3}\,\mu_3 + V_{1,4}\,\mu_4 + V_{1,5}\,\mu_5 + V_{1,6}\,\mu_6 + V_{1,7}\,\mu_7 + V_{1,8}\,\mu_8 + V_{1,9}\,\mu_9 + V_{1,10}\,\mu_{10} + V_{1,11}\,\mu_{11})$$

4. Case:

When repair and failure rates of all subsystems are equal,

Taking $m_i = m$ and $n_i = n$, then we get

MTSF (T_0)

Table 3: - T_0

m↓\n→	0.50	0.60	0.70
0.10	**1.44**	**1.44**	**1.44**
0.20	1.42	1.42	1.42
0.30	1.40	1.40	1.40

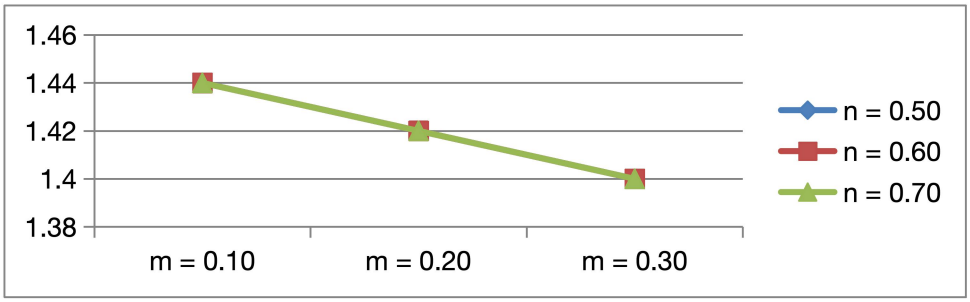


Figure 2: Graph T_0

According to Fig. 2 and table 3, MTSF decreases even more quickly when unit failure rates increase. A failure rate is comparable to MTSF, which is not designed with development in repair rate in mind. It is also seen that repair rates of units do not affect the value of MTSF, and its value is optimum as bolded in the above table for minimum values of failure rates of units.

Analysis for A_0

Table 4: A_0

m↓\n→	0.50	0.60	0.70
0.10	0.399	0.437	**0.472**
0.20	0.363	0.399	0.431
0.30	0.333	0.365	0.399

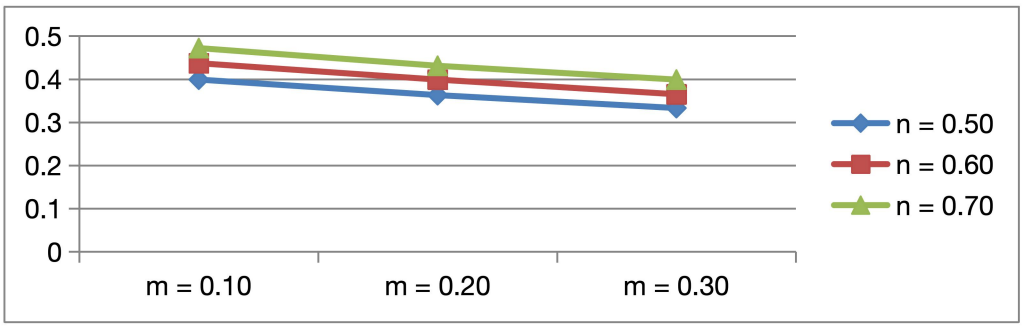


Figure 3: Graph A_0

The obtained values of A_0 are shown in Figure 3 and Table 4 to be increasing with an increase in the unit's maintenance rates and decreasing with an increase in the unit's failure rates. Its value is optimum i.e., **0.472** when failure rates of units are lowest and repair rates of units are highest.

Analysis for B_0

Table 5: B_0

m↓\n→	0.50	0.60	0.70

0.10	0.815	0.784	**0.749**
0.20	0.858	0.815	0.781
0.30	0.886	0.853	0.815

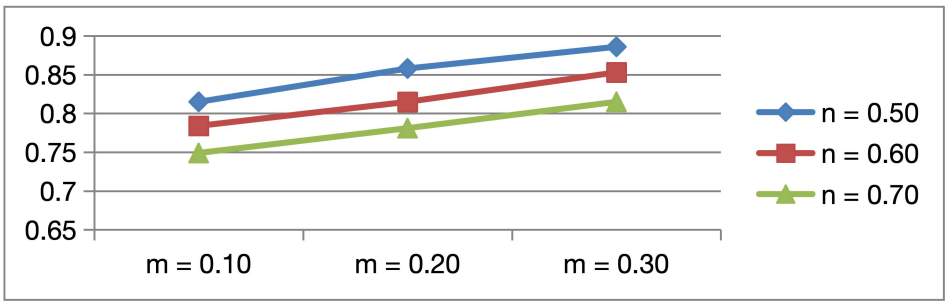


Figure 4: Graph B_0

Based on the data presented in Table 5 and fig. 4, the Busy Period of Server decreases with the increase in units maintenance rates. It is also evident that the units with the lowest value are those in the first row's last column i.e., **0.749**. The value in the last row is the highest, and the first column is the lowest.

Analysis for V_0

Table 6: V_0

m↓\n→	0.50	0.60	0.70
0.10	**0.626**	0.643	0.669
0.20	0.637	0.662	0.685
0.30	0.645	0.678	0.696

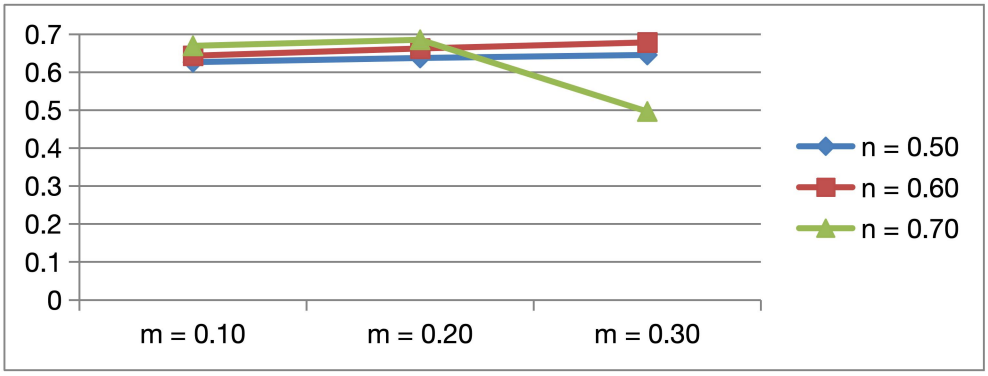


Figure 5: Graph V_0

Based on the data presented in Table 6 and Figure 5, It inferred that value of $\mathbf{V_0}$ by repairman is straightforwardly relative to repair and failure rates of subsystems. The optimum value of the parameter is **0.696** when failure rates of units are lowest.

5. Conclusion:

The results of the behavior analysis can be used to validate and challenge existing models with various assumptions about the system. For example, the analysis could show that a certain parameter has a much greater impact on system performance than previously thought upon. It can help optimize maintenance strategies, improve system design, and reduce downtime and maintenance costs.

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