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HYDRODYNAMICAL LUBRICATION OF MICROPOLAR FLUID BETWEEN TWO SYMMETRYCAL ROLLERS.

KAMINI YADAV, Dr. A. K. YADAV
1. Research Scholar, Govt. KRG PG Collage, Gwalior, MP.
2. Department of Mathematics, Govt. KRG PG collage, Gwalior, MP.

ABSTRACT

Many researchers are describing the flow of fluid. The flow in Newtonian and non-newtonian fluids is important. Flow in Newtonian and non-newtonian represent a systematic flow through the tube and channel. The flow through the tube is also describing by many authers [2,3,6]. We study the behaviour of hydrodynamical fluid in sliding form in this paper. We also study the pressure and load capacity in this condition. We observe that the pressure increases with the value of x^* as well as λ^*. . The load capacity increases with the increasing the value of x^* and also increasing with increasing the value λ^*.

KEYWORDS:- Micro-polar fluid, couple stress, micro-rotation field, hydrodynamical lubrication, non-newtonion fluid, fluid mechanics.

INTRODUCTION

Many researchers are describe the flow of fluid. The flow in Newtonian and non-newtonian fluids is important. Flow in Newtonian and non-newtonian represent a systematic flow through the tube and channel. The flow through the tube is also describing by many authers [1,2].

The theory of micropolar fluids introduced by Erigen [1]. It deals with fluids which has microscopic effects arising from micro-motions of fluid elements. Micropolar fluid are continuum fluids whose particles can translate and rotate. The model adds a microrotation (spin) field and couple – stresses to the usual velocity and stress field. Stokes [5] present the theory of couple stress fluid. This paper represent the inertia effect Its used to represent colloidal suspension, polymeric lubrication with microstructure, liquid crystals, and some nano- fluids. The flow behavioring of micro-polar fluid through the tube and channel is necessary for this study. [2,3,4] examins the stability problem of flow between two concentric rotating cylinder in their paper.

The solution for microrotation velocity fields and velocity are obtained. Shear stress difference, strain rate for basic flow and couple stress are also derived former. Micro-polar fluids also define the behaviour of animal blood, colloidal solution, liquid crystal etc. sometimes, the fluid is not pure but it contains many particles. The particles mixture not inadequate with scientific and industrial importance. With the presence of suspended particles , the effect of rotating on the micro – polar fluid is interesting for the physical point of view. It is happen because the large stabilizing effect of rotation and the destabilizing effect of suspended particles having competition between them. The application in meterphysics and oceanography with the rotating field is also find.

In fluid mechanics newtonian fluids have viscosity in constant, it is because of shear stress and this stress rate having a linear relationship. Fluids not having these properties are visco elastic or non- newtonian fluids. Non newtonian fluid attracted scientist for research. If non symmetrical stress tensor and micro structure are combined with non-newtonian fluids are called micropolar fluids.As a model, these fluids are the result from the classical Navier stokes equation. Micropolar fluid. Support couples stress and dispersed body couples due to micro rotational motions and spin inertia. This fluids contains solid particles. Erigen firstly formulated the theory of micropolar fluids. Micro polar fluids is investigated by many researchers in the last few years. We calculate the pressure and load carrying capacity with regarding two rotating field in micro- polar fluid.

HYDRODYNAMICAL LUBRICATION

Hydrodynamical lubrication occurs When (a) two surfaces in relative motion are separated by a continuously maintained film of lubricant. (b) the pressure in the fluid film developed by the motion and geometry of surfaces support the external load applied to the surfaces. Basically, when one surface moves relative to another with the lubricant between them, the geometry causes a pressure build-up in the fluid film.

The pressure derived from the Renolds. This Renolds equation comes from the Navier-Stokes equation under thin film assumption.

In slide bearing, or plane bearing supports a moving surface that slide over a stationary pad separated by a thin lubricant film. As the top plate drags lubricant into the narrowing gap, the velocity gradient and pressure distribution create a lift force that balances the external load. This is the main concept about the slide bearing.

FORMULATION OF PROBLEM

The governing equation of micro-polar fluids for two-dimensional flow between two symmetric rollers are as follows-

$$(\mu_v+k_v)\frac{\partial^2\mu_x}{\partial x^2}+k_v\frac{\partial v_x}{\partial y}-\frac{\partial p}{\partial x}=0 \qquad \text{----(1)}$$

$$\gamma_v\frac{\partial^2 v_x}{\partial y^2}-k_v\frac{\partial \mu_x}{\partial y}-2k_vV_x=0$$

The equation (1) and (2) are solved under the following boundary conditions-

$\mu_x=0$ at y = h

$\frac{\partial\mu_x}{\partial y}=0$ at y = 0 ----

(2)

$v_x=0$ at $y=\pm h$

We Introduce the following non-dimensional quantities-

$\mu_x=U_0\mu$ when y = h_0y^*

$v_x=\frac{U_0v}{h_0}$ when x = $\sqrt{2Rh_0x^0}$

$h^*=\frac{h}{h_0}$ when $n_1=1+x^{*^2}$ ----

(3)

$n_2=\frac{\mu_vh_0^2}{\gamma_v}$ $n_3=\frac{k_vh_0}{\gamma_v}$

$p_x=\frac{U_0(\mu_v+k_v)p}{L}$ $Q^*=\frac{Q}{2h_0U_0}$ --------(4)

The governing equation takes the flow from after-

$$\frac{\partial^2U}{\partial^{*2}}+\frac{n_3}{n_2+n_3}\frac{\partial v}{\partial y^*}-\frac{\partial p^*}{\partial x}=0 \qquad \text{-------(5)}$$

$$\frac{\partial^2 v}{\partial y^{*2}}-n_3\frac{\partial\mu}{\partial y^*}-2n_3V=0 \qquad -----(6)$$

It is clear that the relationship between h , R and x can be obtained in the form –

h = $h_\circ + R - \sqrt{R^2 - r^2}$ – – – – – – (7)

an usual approximation to the equation (7) may be obtained by explaning the square root term in the binomial series and dropping all but first two terms

h = $h_\circ\left(1 + \frac{x^2}{2Rh_\circ}\right)$ $\quad or \quad$ $h^* = (1 + x^*)$ – – – – (8)

the boundary condition (3) reduce to

μ = 0 at $y^* = h^*$ ------------(9)

$$\frac{\partial\mu}{\partial y^*} = 0 \quad at\ y^* = 0 \qquad ----(10)$$

V = 0 at $y^* = \pm h^*$ -----------(11)

SOLUTION OF THE PROBLEM

Solving the equation (5) and (6) under the boundary condition (10) we get

$$\mu = 1 + \frac{\partial p^*}{\partial x^*}[\frac{(n_2+n_3)}{(2n_2+n_3)}\left(y^{*^2} - h^{*^2}\right) - \frac{h^*(n_2+n_3)}{(2n_2+n_3)^2}\alpha\left(\frac{\cosh\alpha y^* - \cosh\alpha h^*}{\sinh\alpha h^*}\right) \qquad ---(12)$$

$$V = \frac{n_3}{\alpha^2}\frac{\partial p^*}{\partial x^*}\left[\frac{h\sinh\alpha y^*}{\sinh\alpha h^*} - y^*\right] \qquad ----(13)$$

Where

$$\alpha^2 = \frac{n_2(2n_2+n_3)}{(n_2+n_{3)}} \qquad ----(14)$$

The volume flow rate is given by

$$Q = 2\int_0^{h^*}\mu\, dy \qquad ----(15)$$

$$Q = 2h^* - 2h^*\frac{(n_2+n_3)}{(2n_2+n_3)}\frac{\partial p^*}{\partial x^*}\left[\frac{2}{3}h^{*^2} + \frac{1}{(2n_2+n_3)}\left\{1 - \alpha h\frac{\cosh\alpha h^*}{\sinh\alpha h^*}\right\}\right] \qquad ----(16)$$

But, the rollers are large and the angle between them is very small i.e. m is very small as well as h^* can not exceed to writing the expression of cosh $\alpha h^*$and sinh $\alpha h^*$is ascending power of x^*

We get,

$$\frac{\partial p^*}{\partial x^*} = \frac{3}{h^{*^2}} + \frac{\alpha^2}{2} - \frac{3Q^*}{2h^{*^3}} - \frac{Q^*\alpha^2}{4h^*} \qquad ----(17)$$

Using equation (4) we get,

$$\frac{\partial p^*}{\partial x^*} = \frac{3}{\left(1+x^{*^2}\right)^2} + \frac{\alpha^2}{2} - \frac{3Q^*}{2(1+x^{*^2})^3} - \frac{Q^*\alpha^2}{4\left(1+x^{*^2}\right)} \qquad ----(18)$$

The quantity Q^* is yet to be determined in a suitable form. This can be done by using the condition at $x^* = \lambda^*$ where the fluid flow looses contact with rollers i.e. where $\frac{\partial p^*}{\partial x^*} = 0$

Which gives the form

$$\frac{3}{\left(1+\lambda^{*^2}\right)^2} + \frac{\alpha^2}{2} - Q^*\left[\frac{3}{2\left(1+\lambda^{*^2}\right)^3} + \frac{\alpha^3}{4\left(1+\lambda^{*^2}\right)}\right] \qquad ----(19)$$

$$Q^* = \frac{\frac{3}{(1+\lambda^{*2})^2} + \frac{\alpha^2}{2}}{\frac{3}{2(1+\lambda^{*^2})^3} + \frac{\alpha^2}{4(1+\lambda^{*^2})}} \qquad ------(20)$$

Integrating equation (17) using condition $p^*(\lambda) = 0$ we get

$$p^* = \int (\frac{3}{\left(1+\lambda^{*^2}\right)^2} + \frac{\alpha^2}{2} - Q^*\left[\frac{3}{2\left(1+\lambda^{*^2}\right)^3} + \frac{\alpha^3}{4\left(1+\lambda^{*^2}\right)}\right])\, dx^* \qquad ------(21)$$

$$p^* = \frac{3x^*}{(1+x^*)} - \frac{3\lambda^*}{(1+\lambda^*)} - \frac{\alpha^2}{2}\left(x^{*^2} - \lambda^{*^2}\right) + \frac{3Q^*}{8}\left[\frac{1}{(1+x^*)^2} - \frac{1}{(1+\lambda^*)^2}\right] - \frac{Q^*\alpha^2}{4}\left[\log(1+x^*) - \log(1+\lambda^*)\right]$$

THE LOAD CARRING CAPACITY

$$W = \int_0^{\lambda^*} p^*\, d\lambda^* \qquad ------(22)$$

$$= \int_0^{\lambda^*} \left[\frac{3x^*}{(1+x^*)} - \frac{3\lambda^*}{(1+\lambda^*)} - \frac{\alpha^2}{2}\left(x^{*^2} - \lambda^{*^2}\right) + \frac{3Q^*}{8}\left[\frac{1}{(1+x^*)^2} - \frac{1}{(1+\lambda^*)^2}\right] - \frac{Q^*\alpha^2}{4}\left[\log(1+x^*) - \log(1+\lambda^*)\right]\right] d\lambda^*$$

$$W = \frac{3x^*\lambda^*}{(1+x^*)} + \frac{3Q^*}{8}\left(\frac{1}{(1+\lambda^*)} - \frac{1}{(1+x^*)^2}\right) + \frac{\alpha^2\lambda^*}{4}\left(2x^{*^2} - \lambda^{*^2}\right) - \frac{Q^*\alpha^2}{4}\left[\log(1+x^*)\,\lambda^* + \lambda^*\log(1+\lambda^*) - \lambda^* + \log(1+\lambda^*)\right]$$

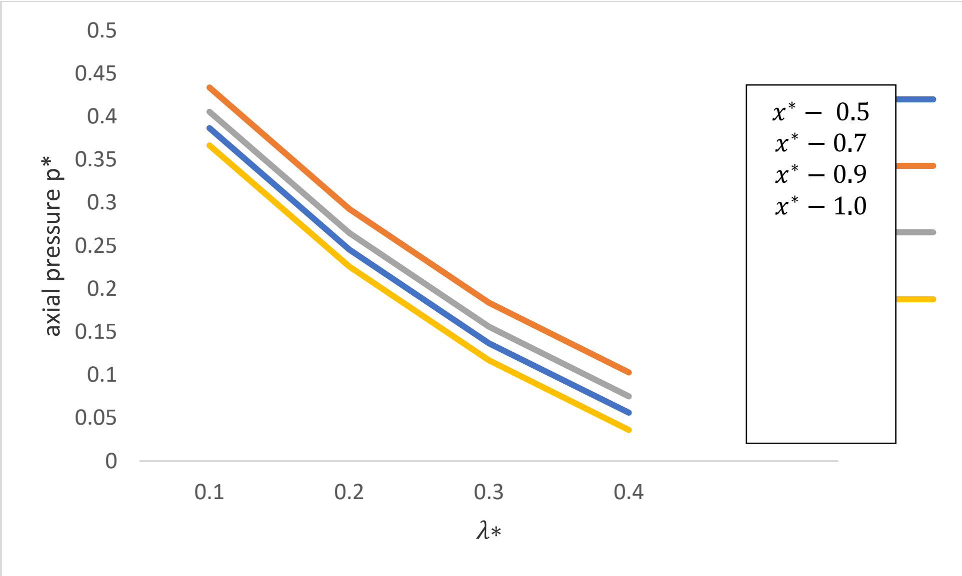

Figure 1: Graph of p* for different values of Q*=1, α = 1

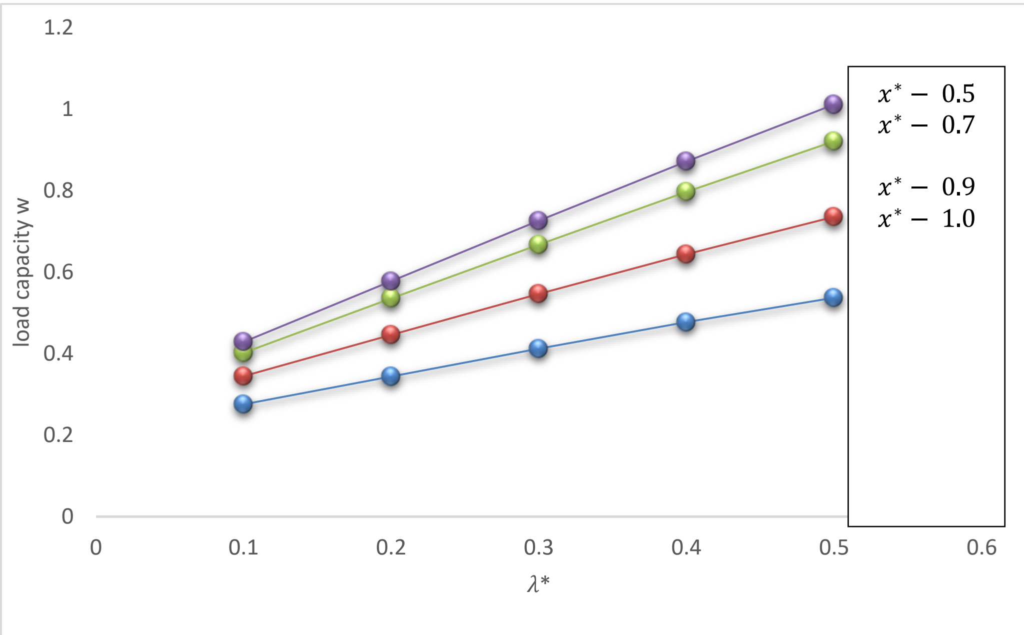

Figure 2:- Graph of w for different values of Q*=1, α = 1

RESULT AND DISCUSSION

The result for pressure and load capacity has been examined for different values of λ^* and x^*. The numerical values of different parameters are chosen. We observe that the pressure increases with the value of x^* as well as λ^*. . The load capacity increases with the increasing the value of x^* and also increasing with increasing the value λ^*.

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