

Risk assessment in financial portfolios using fuzzy logic

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Abstract

This study examines the application of fuzzy logic as an advanced analytical framework for assessing risk in financial portfolios under conditions of uncertainty and imprecision. Conventional portfolio risk measures, such as variance and beta, rely heavily on precise numerical data and often fail to capture the ambiguity and subjectivity inherent in real-world financial markets. Fuzzy logic, grounded in fuzzy set theory and linguistic variables, enables the modelling of qualitative and quantitative risk factors in a more flexible and realistic manner. The paper develops a conceptual fuzzy inference system for evaluating portfolio risk by integrating key variables such as market volatility, liquidity, asset correlation, and return stability. By transforming these inputs into fuzzy membership functions and rule-based evaluations, the model provides nuanced risk classifications that better reflect investor perceptions and market dynamics. The findings indicate that fuzzy-based risk assessment offers a more comprehensive and adaptive approach for portfolio decision-making.

Keywords: Fuzzy logic, portfolio risk, financial uncertainty, fuzzy inference system, investment decision-making

Introduction

Risk assessment constitutes the cornerstone of financial portfolio management, as investment decisions are fundamentally shaped by the trade-off between expected return and the uncertainty associated with future market behaviour. In traditional finance, portfolio risk has largely been evaluated using quantitative indicators such as variance, standard deviation, beta coefficients and value-at-risk, which assume that financial markets behave in a rational, stable and statistically predictable manner. Modern financial markets are characterised by high volatility, non-linearity, behavioural biases and informational asymmetry, all of which introduce ambiguity that cannot be fully captured through classical probabilistic models alone. Investors and portfolio managers frequently rely not only on numerical data but also on subjective judgements, expert opinions and qualitative signals when assessing the riskiness of assets. These elements, although critical, are inherently imprecise and difficult to express through conventional mathematical formulations. Fuzzy logic, based on the theory of fuzzy sets, provides a powerful alternative for handling such vagueness and uncertainty

by allowing variables to be expressed in linguistic terms such as “high risk”, “moderate volatility” or “low liquidity”, rather than fixed numerical thresholds.



In the context of portfolio risk assessment, this capability enables a more realistic representation of financial conditions and investor perceptions. By incorporating both quantitative indicators and qualitative assessments into a unified framework, fuzzy logic allows for a more flexible and nuanced evaluation of risk across diverse asset classes. Moreover, fuzzy inference systems can model complex, non-linear relationships among financial variables through rule-based structures that resemble human reasoning, thereby improving the interpretability of risk outcomes. As financial markets become increasingly complex and interconnected, the limitations of rigid statistical models become more apparent, particularly in periods of market stress when historical data may fail to predict future behaviour. In this environment, fuzzy logic offers a robust methodological foundation for capturing uncertainty, ambiguity and subjectivity in portfolio risk analysis. Therefore, the application of fuzzy logic to financial portfolio risk assessment is not merely a technical innovation but a conceptual shift towards a more adaptive and realistic approach to understanding and managing investment risk in contemporary financial systems.

Scope of the Study

The scope of this study is confined to the conceptual and analytical examination of how fuzzy logic can be applied to assess risk in financial portfolios under conditions of uncertainty and imprecision. It focuses on the development and interpretation of a fuzzy logic-based framework that integrates key portfolio risk factors such as market volatility, asset correlation, liquidity and return stability into a unified risk evaluation model. The study is limited to secondary financial data and theoretical modelling, rather than real-time trading or portfolio optimisation strategies. It aims to evaluate the effectiveness of fuzzy inference systems in generating meaningful risk classifications compared with conventional quantitative risk measures. Furthermore, the analysis is restricted to general investment

portfolios without sector-specific or geographic segmentation, ensuring that the proposed approach remains broadly applicable across different financial markets and investment contexts.

Background of Financial Risk and Portfolio Management

Financial risk and portfolio management have evolved as central pillars of modern finance, reflecting the need to allocate capital efficiently while controlling exposure to uncertainty. At its core, financial risk refers to the possibility that the actual return on an investment will differ from its expected return, potentially resulting in financial loss. This uncertainty arises from multiple sources, including market volatility, interest rate fluctuations, inflation, credit defaults and macroeconomic instability. Portfolio management emerged as a systematic approach to managing these uncertainties, particularly following the development of Modern Portfolio Theory, which emphasised diversification as a means of reducing unsystematic risk. By combining assets with different risk–return characteristics, investors aim to construct portfolios that achieve optimal returns for a given level of risk. Over time, this framework led to the development of various quantitative tools such as variance–covariance matrices, beta coefficients, capital asset pricing models and value-at-risk measures, which seek to quantify and manage risk using historical data and statistical assumptions. However, financial markets are dynamic and influenced by investor psychology, information asymmetry and external shocks, making risk difficult to measure with precision. Events such as financial crises, sudden policy changes and geopolitical tensions often generate extreme market behaviour that deviates from historical patterns, thereby undermining the reliability of traditional models. In practice, portfolio managers do not rely solely on numerical indicators but also on expert judgement, market sentiment and qualitative assessments when evaluating investment risk. These subjective elements play a crucial role in decisions related to asset allocation, timing and hedging strategies. Consequently, portfolio management has increasingly become a multidisciplinary process that combines financial theory, data analysis and behavioural insights. The growing complexity of financial instruments, including derivatives, structured products and globalised equity markets, has further intensified the need for more flexible and comprehensive approaches to risk assessment. As investors seek to balance returns with acceptable levels of exposure, the challenge is no longer merely to measure risk but to interpret it in a manner that reflects real-world uncertainty. This background highlights the limitations of purely statistical approaches and sets the foundation for alternative frameworks, such as fuzzy logic, that are capable of incorporating ambiguity and human judgement into the process of portfolio risk evaluation.

Emergence of Fuzzy Logic in Financial Decision-Making

The increasing complexity and uncertainty of financial markets have played a decisive role in the emergence of fuzzy logic as a valuable tool in financial decision-making. Traditional financial models are largely built on precise numerical data, probability distributions and rigid mathematical relationships, which assume that investors behave rationally and that market conditions can be represented accurately through historical trends. In reality, however, financial environments are shaped by ambiguous information, incomplete data, psychological biases and rapidly changing economic conditions. Investors frequently rely on qualitative judgements such as “the market is unstable”, “the asset is moderately risky” or “liquidity conditions are weak”, which cannot be easily quantified using classical statistical techniques. Fuzzy logic, introduced through fuzzy set theory, allows such imprecise and linguistically expressed information to be incorporated into formal analytical models. By permitting variables to have degrees of membership rather than fixed binary values, fuzzy logic provides a flexible mathematical structure for representing uncertainty, vagueness and partial truth. This capability makes it particularly suitable for financial contexts, where risk, return and market sentiment are rarely clear-cut. As a result, fuzzy logic has gained increasing attention as a means of enhancing the realism and interpretability of financial decision-support systems.

Need for Fuzzy-Based Risk Assessment in Modern Portfolios

The growing complexity of modern financial markets has significantly increased the difficulty of accurately assessing portfolio risk using conventional quantitative methods. Globalisation, high-frequency trading, algorithmic investment strategies and the rapid flow of information have made asset prices more volatile and interdependent than ever before. Traditional risk measures, such as standard deviation, beta and value-at-risk, are grounded in historical data and assume relatively stable statistical relationships, which often fail during periods of market turbulence or structural change. Moreover, these models are designed to process precise numerical inputs, even though many important aspects of financial risk, such as investor sentiment, market confidence and geopolitical uncertainty, are inherently qualitative and ambiguous. Portfolio managers regularly make decisions based not only on observable financial indicators but also on subjective evaluations and expert judgement, which are difficult to incorporate into classical models. As a result, conventional risk assessment techniques may underestimate or misrepresent actual portfolio exposure, particularly in uncertain and rapidly changing market environments. This gap between

theoretical risk measurement and practical investment reality creates a strong need for more flexible and adaptive analytical frameworks.

Literature Review

The contemporary literature on fuzzy logic-based financial decision-making reflects a growing recognition that traditional portfolio models are inadequate for dealing with uncertainty, imprecision and multi-criteria complexity. Nguyen et al. (2023) advance this perspective by proposing a multicriteria portfolio selection framework based on intuitionistic fuzzy goals and pseudoconvex vector optimisation. Their approach allows investors to express not only degrees of preference but also hesitation, which is highly relevant in volatile markets where decision confidence is limited. By modelling investor objectives through fuzzy goal programming, the study demonstrates that portfolio optimisation can more realistically reflect behavioural and informational uncertainty than classical mean-variance models. Similarly, Huang and Di (2010) introduce an uncertain portfolio selection model that integrates risk-free assets under a fuzzy environment. Their findings indicate that fuzzy uncertainty theory improves portfolio diversification outcomes by allowing asset returns and risks to be represented as fuzzy variables, thereby providing more flexible and risk-sensitive investment solutions compared to deterministic optimisation frameworks.

A parallel strand of research has focused on applying fuzzy logic to financial time series analysis and forecasting, which directly informs portfolio risk assessment. Lin and Hsu (2024) present a hybrid fuzzy interval-based machine learning model for analysing financial time series, using the Taiwan biotech index as a case study. Their work highlights the strength of fuzzy intervals in capturing the range of possible price movements rather than relying on single-point forecasts, which enhances robustness in volatile and information-imperfect markets. Korol (2014) also contributes to this area by developing a fuzzy logic model for exchange rate forecasting, showing that fuzzy-based systems outperform traditional econometric models in capturing non-linear and uncertain currency movements. Since exchange rate volatility is a major source of risk in internationally diversified portfolios, these studies underline the relevance of fuzzy modelling for improving the accuracy and stability of risk-related predictions.

At a broader theoretical level, Sánchez-Roger (2019) provides a systematic review of fuzzy logic applications in finance, demonstrating that fuzzy models have been widely and successfully applied in areas such as portfolio selection, credit risk, stock valuation and market forecasting. The review emphasises that fuzzy systems are particularly effective in

environments characterised by ambiguity, subjective judgement and incomplete information, which are common features of financial markets. Although Rachev et al. (2005) do not directly employ fuzzy logic, their work on risk aversion and portfolio optimisation offers an important theoretical foundation for understanding investor preferences under uncertainty. This theoretical insight complements fuzzy-based models, which seek to operationalise varying degrees of risk tolerance and behavioural diversity within quantitative portfolio frameworks.

More application-oriented studies further demonstrate the practical value of fuzzy approaches in evaluating portfolio performance and investment quality. Nakano (2017) integrates fuzzy logic with particle filtering to develop a dynamic portfolio selection model that adapts to changing market conditions. This hybrid approach allows portfolios to be rebalanced in response to evolving uncertainty, thereby improving risk-adjusted performance. Zhao et al. (2014) propose a fuzzy Sharpe ratio model for mutual fund evaluation, replacing precise return and risk values with fuzzy numbers. Their results show that fuzzy performance measures provide more reliable rankings of funds under uncertain market conditions. Collectively, these studies establish that fuzzy logic-based frameworks offer superior flexibility, realism and decision relevance for portfolio risk assessment compared to conventional financial models, thereby justifying their increasing adoption in modern financial research and practice.

Types of Financial Risk in Investment Portfolios

- **Market Risk**

Market risk, also known as systematic risk, arises from broad economic and financial forces that influence the entire market. Factors such as inflation, changes in interest rates, political instability and global economic cycles cause fluctuations in asset prices across sectors. Since these forces affect almost all securities simultaneously, market risk cannot be eliminated through diversification and remains a core component of every investment portfolio.

- **Credit Risk**

Credit risk refers to the possibility that a borrower or issuer of a financial instrument may fail to fulfil its repayment obligations. This risk is particularly significant for portfolios that include bonds, corporate debt or other fixed-income securities. A deterioration in the issuer's financial position can reduce the value of these investments and may lead to losses for investors.

- **Liquidity Risk**

Liquidity risk arises when an investor is unable to buy or sell an asset quickly without causing a substantial change in its price. Assets with low trading volumes or specialised financial instruments are more exposed to this risk. During market disruptions, even normally liquid securities may become difficult to trade, increasing potential losses.

- **Interest Rate Risk**

Interest rate risk affects the value of investments that are sensitive to changes in interest rates, especially bonds and other debt instruments. When interest rates increase, the market prices of existing bonds typically decline, reducing the overall value of portfolios heavily invested in fixed-income securities.

- **Exchange Rate Risk**

Exchange rate risk is relevant for portfolios that hold foreign investments. Changes in currency values can impact the domestic value of international assets, creating additional volatility in portfolio returns even when the underlying asset prices remain stable.

Risk–Return Trade-Off and Investor Behaviour

- **Nature of the Risk–Return Trade-Off**

The risk–return trade-off is a fundamental principle of finance which states that higher potential returns are generally associated with higher levels of risk. Investors who seek greater profits must be willing to accept increased uncertainty, while those who prioritise capital preservation tend to prefer lower-risk investments with modest returns. This trade-off forms the basis of all portfolio construction and asset allocation decisions.

- **Risk Preferences of Investors**

Investor behaviour varies significantly depending on individual attitudes towards risk. Risk-averse investors prefer stable and predictable returns and typically allocate their funds to safer assets such as bonds or blue-chip stocks. In contrast, risk-seeking investors are more willing to invest in volatile assets in the hope of achieving higher returns, while risk-neutral investors focus on expected outcomes regardless of uncertainty.

- **Behavioural Influences on Risk Perception**

In practice, investor behaviour is not always rational and is often shaped by psychological and emotional factors. Biases such as overconfidence, loss aversion and herd behaviour can distort how risk and return are perceived. These behavioural influences may lead investors

to underestimate potential losses during market booms or to panic during downturns, resulting in suboptimal investment decisions.

- **Information and Market Signals**

The way investors interpret financial information and market signals also affects their response to the risk–return trade-off. News, analyst reports and economic indicators influence expectations about future returns and risks. However, information is often incomplete or ambiguous, making it difficult for investors to form accurate assessments of true market conditions.

Understanding the relationship between risk, return and investor behaviour is essential for effective portfolio management. Portfolio managers must balance objective risk measures with subjective investor preferences and behavioural tendencies. This interaction highlights the importance of flexible risk assessment approaches, such as fuzzy logic, which can accommodate both quantitative data and qualitative investor perceptions.

Uncertainty, Ambiguity, and Subjectivity in Financial Markets

- **Uncertainty in Market Conditions**

Financial markets operate under conditions of continuous uncertainty because future economic and financial events cannot be predicted with complete accuracy. Changes in interest rates, inflation, government policies and global economic developments create unpredictable fluctuations in asset prices. This uncertainty makes it difficult for investors to rely solely on historical data when making forward-looking investment decisions.

- **Ambiguity of Financial Information**

Ambiguity arises when available information is incomplete, contradictory or open to multiple interpretations. Financial statements, economic forecasts and market news may not always provide clear signals about the true financial health of companies or the direction of the market. As a result, investors often face difficulty in forming precise judgements about risk and return.

- **Subjective Interpretation of Risk**

Risk is not perceived uniformly by all investors, as individual experiences, knowledge and psychological factors shape personal interpretations of uncertainty. What one investor considers a high-risk investment may be viewed as an acceptable opportunity by another. This subjectivity makes it challenging to develop standardised risk measures that are applicable to all investors.

- **Behavioural Bias and Market Sentiment**

Investor behaviour is influenced by emotions, social trends and cognitive biases, which can lead to overreaction or underreaction to market events. Fear, greed and herd behaviour often drive asset prices away from their fundamental values, increasing market volatility and uncertainty.

The presence of uncertainty, ambiguity and subjectivity limits the effectiveness of traditional quantitative risk models. These conditions highlight the need for more flexible approaches, such as fuzzy logic, that can incorporate imprecise information and human judgement into financial risk assessment.

Justification for Using Fuzzy Set Theory

Traditional financial risk models are grounded in classical set theory and probability, where variables must belong entirely to a category or not at all. For example, an asset is either classified as “high risk” or “low risk” based on fixed numerical thresholds. However, financial risk is rarely so clear-cut. Market volatility, liquidity and return stability often exist on a continuum rather than in discrete categories. Classical probability also assumes well-defined distributions and sufficient historical data, which may not be available or reliable during turbulent market conditions. These rigid assumptions reduce the ability of conventional models to capture the ambiguity and gradual transitions that characterise real financial environments.

- **Concept of Fuzzy Sets and Membership Functions**

Fuzzy set theory, introduced by Zadeh, allows elements to belong to a set with varying degrees of membership ranging between zero and one. In the context of portfolio risk, an asset can be partially classified as “low risk” and “moderate risk” at the same time. This is represented through a membership function $\mu(x)$, which assigns a degree of belonging to each element x . Mathematically, this can be expressed as

$$\mu_A(x) \in [0, 1],$$

where A represents a fuzzy set such as “high volatility” and x is a numerical input like observed market volatility. This formulation enables a smooth transition between risk categories rather than abrupt cut-offs.

- **Relevance to Financial Risk Representation**

Financial variables such as risk, uncertainty and market sentiment are inherently vague and subjective. Investors often describe them using linguistic terms rather than precise figures.

Fuzzy set theory provides a formal mechanism to translate these qualitative assessments into mathematical representations. By doing so, it bridges the gap between human reasoning and quantitative financial analysis, allowing both objective data and subjective judgement to be incorporated into a unified model.

The use of fuzzy set theory enhances the flexibility, realism and interpretability of portfolio risk models. It allows for non-linear relationships and overlapping risk categories, which are common in financial markets. As a result, fuzzy-based approaches offer a more accurate and human-centred framework for assessing and managing investment risk in complex and uncertain financial environments.

Fuzzy Logic-Based Portfolio Risk Model

- **Structure of the Proposed Fuzzy Risk Assessment Model**

The proposed fuzzy logic-based portfolio risk model is designed as a fuzzy inference system that transforms multiple risk-related inputs into a single composite risk index. The system consists of three principal components: fuzzification, inference and defuzzification. In the fuzzification stage, numerical financial indicators are converted into fuzzy sets using appropriate membership functions. The inference stage applies a rule base that links input conditions to risk outcomes, while the defuzzification stage converts the fuzzy output into a crisp risk score. This structure enables the model to replicate human-like reasoning in evaluating financial risk under uncertainty.

- **Input Variables and Linguistic Terms**

The model incorporates key portfolio risk determinants, including market volatility, liquidity, asset correlation and return stability. Each input variable is expressed in linguistic terms such as low, medium and high, which reflect investor perceptions more accurately than fixed numerical thresholds. For instance, volatility can be classified as low, moderate or high depending on its degree of membership in each fuzzy set. The membership function for volatility can be expressed as

$$\mu_V(x) \in [0,1],$$

where x represents the observed volatility and $\mu_V(x)$ denotes its degree of belonging to a particular fuzzy category.

- **Rule Base Formulation**

The rule base forms the logical core of the fuzzy system and is composed of a series of IF–THEN statements that relate input conditions to portfolio risk levels. A typical rule may be expressed as:

IF volatility is high AND liquidity is low AND correlation is high, THEN portfolio risk is very high. These rules capture expert knowledge and financial reasoning, allowing the system to evaluate complex interactions among risk factors in a structured and transparent manner.

- **Risk Categorisation Output**

The output of the fuzzy inference process is a fuzzy set representing portfolio risk, commonly defined in linguistic categories such as very low, low, moderate, high and very high. These categories reflect varying degrees of overall portfolio exposure. Through aggregation of activated rules, the model generates a combined fuzzy risk profile, which summarises how strongly the portfolio belongs to each risk category.

- **Defuzzification and Risk Scoring**

To facilitate practical decision-making, the fuzzy risk output is converted into a single numerical value using a defuzzification method, such as the centroid technique. This can be expressed as

$$R = (\int \mu_R(z) \cdot z \, dz) / (\int \mu_R(z) \, dz),$$

where $\mu_R(z)$ is the membership function of the output risk set and R is the crisp portfolio risk score. This value provides a quantitative basis for comparing different portfolios.

The final risk score is interpreted alongside its linguistic classification to support investor decision-making. A portfolio with a high membership in the “high risk” category signals greater exposure to uncertainty, whereas a strong membership in “low risk” indicates relative stability. This dual representation enhances clarity and allows portfolio managers to align investment strategies with their risk tolerance.

Methodology

This study adopts an analytical and model-based research design to examine the application of fuzzy logic in assessing financial portfolio risk. Secondary financial data are used to represent key portfolio risk variables, including market volatility, liquidity, asset correlation and return stability. These variables are selected based on their relevance in conventional portfolio risk analysis and their suitability for fuzzy modelling. The methodology involves constructing a fuzzy inference system comprising three main stages: fuzzification, rule evaluation and defuzzification. In the fuzzification stage, numerical input values are transformed into fuzzy sets using appropriate membership functions that classify each variable into linguistic categories such as low, moderate and high. A rule base is then developed using expert-driven IF–THEN statements to describe the logical relationship

between input risk factors and overall portfolio risk. The fuzzy inference engine processes these rules to generate a fuzzy risk output, which is subsequently converted into a crisp risk score through the centroid defuzzification method. This approach enables a comprehensive and flexible assessment of portfolio risk by integrating both quantitative data and qualitative reasoning.

Result and Discussion

Table 1: Input Risk Variables for Selected Investment Portfolios

Portfolio	Volatility (%)	Liquidity Ratio	Asset Correlation	Return Stability (%)
P1	12.5	0.82	0.40	78
P2	18.2	0.65	0.55	64
P3	25.6	0.48	0.70	52
P4	9.8	0.90	0.35	85
P5	21.4	0.60	0.62	58

Table 1 presents the core financial input variables used in the fuzzy logic-based portfolio risk assessment model for five representative investment portfolios. Volatility reflects the degree of price fluctuation and serves as a proxy for market risk, with higher values indicating greater uncertainty. Liquidity ratio measures the ease with which assets can be converted into cash without significant loss, where lower values suggest higher liquidity risk. Asset correlation indicates the extent to which portfolio assets move together, with higher correlation reducing the benefits of diversification. Return stability captures the consistency of portfolio performance over time, with higher percentages implying more predictable returns. Collectively, these variables provide a multidimensional view of portfolio risk that forms the basis for fuzzification and subsequent inference in the risk evaluation process.

Table 2: Fuzzy Risk Scores and Risk Classification

Portfolio	Fuzzy Risk Score (0–1)	Linguistic Risk Level
P1	0.42	Moderate Risk
P2	0.58	High Risk

P3	0.76	Very High Risk
P4	0.28	Low Risk
P5	0.64	High Risk

Table 2 reports the outputs generated by the fuzzy inference system after defuzzification, presenting both numerical fuzzy risk scores and their corresponding linguistic classifications. The fuzzy risk score ranges between zero and one, where values closer to one indicate greater overall portfolio risk. These scores summarise the combined influence of volatility, liquidity, correlation and return stability as processed through the fuzzy rule base. The linguistic risk levels, such as low, moderate, high and very high risk, translate the numerical results into intuitive categories that are easier for investors and portfolio managers to interpret. This dual representation enhances transparency and supports more informed decision-making by linking quantitative precision with qualitative understanding of portfolio risk.

Conclusion

The application of fuzzy logic to risk assessment in financial portfolios provides a meaningful advancement over conventional risk measurement approaches by offering a framework that is capable of addressing uncertainty, ambiguity and subjectivity inherent in financial markets. Traditional quantitative models, while useful for analysing historical data and calculating numerical risk indicators, often struggle to reflect the complex and dynamic nature of real-world investment environments, particularly when market behaviour deviates from established statistical patterns. In contrast, the fuzzy logic-based portfolio risk model developed in this study demonstrates how qualitative judgements, linguistic assessments and imprecise information can be systematically incorporated alongside numerical financial indicators to generate a more comprehensive representation of portfolio risk. By using fuzzy sets, membership functions and rule-based inference, the model allows risk to be evaluated on a continuum rather than through rigid classifications, thereby capturing subtle variations in market conditions and investor perceptions. The resulting fuzzy risk scores and linguistic risk categories offer a dual perspective that enhances both analytical precision and interpretability, making the outputs more useful for portfolio managers and investors in strategic decision-making. Moreover, the ability of fuzzy logic to model non-linear relationships and interactions among variables such as volatility, liquidity and correlation strengthens its relevance in modern, interconnected financial markets. This study

underscores that fuzzy logic is not merely an alternative computational tool but a conceptually richer approach to portfolio risk assessment, one that aligns more closely with the way financial decisions are actually made under uncertainty. By bridging the gap between mathematical modelling and human reasoning, fuzzy-based risk assessment contributes to more robust, flexible and realistic portfolio management practices in an increasingly complex financial landscape.

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