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MULTIVARIABLE GENERALIZED POLYNOMIAL SET $A_n\{(x_m),y\}$ IN TERMS OF THE LAGUERRE POLYNOMIAL

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ABSTRACT :

The present paper provides operational representation with the operation (expansion) of the multivariable generalized polynomial set $A_n\{(x_m),y\}$ in terms of Laguerre polynomial where many new interesting results are given along with discussion and applications.

Key Words : Operation, generalized polynomial, expansion, multivariable, Lagurre polynomial, parameters.

1. Introduction

Special functions include certain set of functions as well as polynomials, such as Legendre functions, Bessel functions, Hypergeometric function, Hermite polynomials, Laguerre polynomials and Jacobi polynomials etc. [1-4].

These functions and polynomials have wide applications in mathematical physics, biophysics, transport phenomena, flow through proous media, automatic computers and various other branches of science and modern technologies. These funcitons occur in the solution of boundary value problems [5-7].

Here in this paper we have focussed towards lagurre polynomials which are not mainly useful in the quantum-mechanical study of the hydrogen atom and alkali atom but they also find applications in transmission line theory and seismological investigations. As such here this paper deals with the operation (expansion) of the multivariable generalized polynomial set $A_n\{(x_m), y\}$ interms of the Laguerre polynomials. Many interesting new results have been obtained on using the respective parameters along with discussion and application.

2. Expansion of Generalized Polynomial set $A_n\{(x_m), y\}$ in terms of the Laguerre Polynomials.

We have [8]

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$$\sum_{n=0}^{\infty} A_n\{(x_m),y\}t^n = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{k_1=0}^{\infty}\cdots\cdots\sum_{k_m=0}^{\infty}$$

$$\times\frac{\left(\mu t^{e}\right)^{k}(v_z)_k\left[(a_g)\right]_{k_1}\left(\lambda_1 y^{-\mu_1 e_1}t^{e_1}\right)^{k_1}\left[(A_r)\right]_{n+k_2+.....+k_m}}{\kappa!\quad \kappa_1!\quad \left[(b_h)\right]_{k_1}\quad \left[(B_S)\right]_{n+k_2+......+k_m}}$$

$$\times\frac{\left[(C_p)\right]_n\left[\left(\alpha_{u_m}\right)\right]_{k_m}\lambda^n\left(x_1^d t\right)\left(\lambda_2 x_2^{e_2}y^{-\mu_2 e_2}\right)^{k_2}.......\left(\lambda_m x_m^{e_m t^{e_m}}\right)^m}{\left[(D_q)\right]_n\quad \left[\left(\beta_{v_m}\right)\right]_{k_m}\quad n!\quad k_2!}$$

$$=\sum_{n=0}^{\infty}\sum_{k,k_1k_2.....k_m=0}^{\infty}\frac{\left[(A_r)\right]_{n+k_2+.....+k_m}\left[(C_p)\right]_n}{\left[(B_s)\right]_{n+k_2+.....+k_m}\left[(D_q)\right]_n}$$

$$\times\frac{\left[\left(\alpha_{u_m}\right)\right]_{k_m}\left[(a_g)\right]_{k_1}(v_z)_{k_1}\mu^k\lambda_1^{k_1}\left(\lambda_2 x_2^{e_2}\right)^{k_2}}{\left[\left(\beta_{v_m}\right)\right]_{k_m}\left[(b_h)\right]_n\ \kappa!\,\kappa_1!\,\kappa_2!\,y^{e_1\mu_1k_1+e_2\mu_2k_2}}$$

$$\times\frac{\left(\lambda_m r_m^{e_m}\right)^{k_m}\lambda^n\left(x_1^d\right)t^{n+ek+e_1k_1+e_2k_2+....+e_mk_m}}{\kappa_m!\qquad\qquad n!}\qquad (2.1)$$

Also, we have

$$\left(x_1^d\right)^n = \sum_{i=0}^{n}\frac{(-1)^i n!(1+\alpha)_n L_i^{(\alpha)}\left(x_1^d\right)}{(n-i)!\quad (1+\alpha)}$$

Hence

$$\sum_{n=0}^{\infty} A_n\{(x_m),y\}t^n = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{k_1=0}^{\infty}\cdots\cdots\sum_{k_m=0}^{\infty}\sum_{i=0}^{n}$$

$$\times\frac{\left[(A_r)\right]_{n+k_2+....+k_m}\left[(C_p)\right]_n\left[\left(\alpha_{u_m}\right)\right]_{k_m}\left[(a_g)\right]_{k_1}(v_z)_{k_1}}{\left[(B_S)\right]_{n+k_2+......k_m}\left[(D_q)\right]_n\left[\left(\beta_{v_m}\right)\right]_{k_m}\left[(b_h)\right]_{k_1}\kappa!\ \kappa_1!}$$

$$\times \frac{\mu^k \lambda_1^{k_1} \left(\lambda_2 x_2^{e_2}\right)^{k_2} \ldots\left(\lambda_m r_m^{e_m}\right)^{k_m} \lambda^n (-1)^i (1+\alpha)_h L_1^{(\infty)}\left(x_1^d\right)}{k_2! \quad y^{e_1\mu_1 k_1 + e_2\mu_2 k_2} \quad k_m! \quad (n-i)! \quad (1+\alpha)_i}$$

$$\times t^{n+ek+e_1k_1+e_2k_2+\ldots\ldots+e_mk_m}$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\ldots\ldots\sum_{k_m=0}^{\infty}\sum_{i=0}^{\infty} \frac{\left[(A_r)\right]_{n+i+k_2+\ldots+k_m}}{\left[(B_s)\right]_{n+i+k_2+\ldots+k_m}}$$

$$\times \frac{\left[(C_p)\right]_{n+i}\left[(\alpha_{u_m})\right]_{k_m}\left[(a_g)\right]_{k_1}(v_z)_{k_1}\mu^k v^{k_1}\left(\lambda_2 x_2^{e_2}\right)^{k_2}}{\left[(D_q)\right]_{n+i}\left[(\beta_{v_m})\right]_{k_m}\left[(b_h)\right]_n \quad \kappa! \quad \kappa_1! \quad \kappa_2!}$$

$$\times \frac{\left(\lambda_m x_m^{e_m}\right)^{k_m} \lambda^n (-1)^i (1+\alpha)_{n+i} L_i^{(\infty)} t^{n+ek+e_1k_1+e_2k+\ldots e_mk_m}}{y^{e_1\mu_1k_1+e_2\mu_2k_2} \quad k_m! \quad n! \quad (1+\alpha)_i}$$

$$= \sum_{n=0}^{\infty}\sum_{i=0}^{n}\sum_{k=0}^{\left[\frac{n}{e}\right]}\sum_{k=0}^{\left[\frac{n-ek}{e_1}\right]}\sum_{k_2=0}^{\left[\frac{n-ek-e_1k_1}{e_2}\right]}\ldots\ldots\ldots\sum_{k_m=0}^{\left[\frac{n-ek-e_1k_1-\ldots-e_{m-1}k_{m-1}}{e_m}\right]}$$

$$\times \frac{\left[(A_r)_{n-ek-e_1k_1-(e_2-1)k_2-\ldots\ldots-(e_m-1)k_m}\right]}{(B_s)_{n-ek-e_1k_1-(e_2-1)k_2-\ldots\ldots-(e_m-1)k_m}}$$

$$\times \frac{\left[(C_p)\right]_{n-ek-e_1k_1-e_2k_2-\ldots\ldots-e_mk_m}\left[(\alpha_{u_m})\right]_{k_m}}{\left[(D_q)\right]_{n-ek-e_1k_1-e_2k_2-\ldots\ldots-e_mk_m}\left[(\beta_{v_m})\right]_{k_m}}$$

$$\times \frac{\left[(a_g)\right]_{k_1}(v_z)_{k_1} u^k \lambda_1^{k_1}\left(\lambda_2 x_2^{e_2}\right)^{k_2}\ldots\ldots\left(\lambda_3 r_m^{e_m}\right)^{k_m}}{\left[(b_h)\right]_{k_1} \quad \kappa! \quad \kappa_1! \quad \kappa_2! \; y^{e_1\mu_1k_1+e_2\mu_2k_2} \quad \kappa_m!}$$

$$\times \frac{\lambda^{n-ek-e_1k_1-e_2k_2\ldots\ldots e_mk_m}(-1)^i(1+\alpha)_{n-ek-e_1k_1-e_2k_2\ldots\ldots e_mk_m} L_i^{(\infty)}\left(x_1^d\right)t^n}{(1+\alpha)_i \quad (n-i-ek-e_1\kappa_1-e_2\kappa_2\ldots\ldots e_m\kappa_m)!}$$

On comparing the co-efficient of t^n from both sides and we finally achieve after little

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simplification for $e_2, e_3 \ldots\ldots e_m > 1$

$$A_n\{(x_m),y\} = M_1 \sum_{i=0}^{n} \sum_{k,k_1,k_2\ldots k_m=0}^{\infty}$$

$$\times \frac{[1-(B_s)-n]_{ek+e_1k_1-(e_2-1)k_2+\ldots+(e_m-1)k_m}}{[1-(A_r)-n]_{ek+e_1k_1-(e_2-1)k_2+\ldots+(e_m-1)k_m}}$$

$$\times \frac{[1-(D_q)-n]_{ek+e_1k_1+e_2k_2+\ldots+e_mk_m}\,[(\alpha_{u_m})]_{k_m}\,[(a_g)]_{k_1}}{[1-(C_p)-n]_{ek+e_1k_1+e_2k_2+\ldots+e_mk_m}\,[(\beta_{v_m})]_{k_m}\,[(b_h)]_{k_1}}$$

$$\times \frac{(V_z)_{k_1}\,\mu^k(-1)^{e(r+s+p+q+1)k}\,\lambda^{k_1}(-1)^{e_1(r+s+p+q+1)+g+h}\left(\lambda_2 x_2^{e_2}\right)^{k_2}}{\kappa!\qquad \kappa_1!\qquad y^{e_1\mu_1k+e_2\mu_2k_2}\qquad \kappa_2!}$$

$$\times \frac{(-1)^{e_1(r+s+p+q+1)+r+s)k_2}\left(\lambda_m r_m^{e_m}\right)^{k_m}(-1)^{e_m(r+s+p+q+1)+r+s)k_m}}{\kappa_2!\qquad (1+\alpha)_i\qquad \kappa_m!\ \lambda^{ek+e_1k_1+e_2k_2+\ldots+e_mk_m}}$$

$$\times \frac{(-n+i)_{ek+e_1k_1+e_2k_2+\ldots+e_mk_m}\,L_i^{(\alpha)}\left(x_1^e\right)(1+\alpha)}{(-\alpha+n)_{ek+e_1k_1+e_2k_2+\ldots+e_mk_m}\qquad (n-i)!} \tag{2.2}$$

where a is non negative integer.

The single terminating factor $(-n+i)_{ek+e_1k_1+e_2k_2+\ldots+e_mk_m}$ makes all summations in (2.2) run up to ∞.

where $M_1 = \dfrac{[(A_r)][(C_p)]\,\lambda^n}{[(B_s)][(D_q)]}$

3. Some Corollaries

Corollary I : For $e_2 > 1, \ldots\ldots e_m > 1$, we have

$$A_n\{(x_m),y\} M_1 \sum_{i=1}^{n} \frac{(-1)^i(1+\alpha)_n\,L_i^{(\alpha)}\left(x_1^e\right)}{(1+\alpha)_i\ \ (n-i)!}$$

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$$\times F^{1+q+s+h:u_1;u_2;u_m}_{p+r+g;v_1;v_2;v_m}\left[\begin{matrix}\left[(-n+i):e,e_1,e_2,\ldots\ldots e_m\right]\\ \left[(-n-\alpha);e,e_1,e_2,\ldots\ldots e_m\right]\end{matrix}\right.$$

$$\left[\left(1-(B_s-n\right):e,e_1,e_2-1,\ldots..e_m-1\right],\left[\left(1-(D_q)-n\right):e,e_1,e_2,\ldots..e_m\right],$$

$$\left[\left(1-(A_r)-n\right):e,e_1,e_2-1,\ldots..e_m-1\right],\left[\left(1-(C_p)-n\right):e,e_1,e_2,\ldots..e_m\right],$$

$$\left[\left(a_g\right):1\right]\left[\left(\alpha_{u_1}\right):1\right],\left[\left(\alpha_{u_2}\right):1\right],\ldots\ldots\left[\left(\alpha_{u_m}\right):1\right],\left[\left(v_z\right):\right]$$

$$\left[\left(b_h\right):1\right]\left[\left(\beta_{v_1}\right):1\right],\left[\left(\beta_{v_2}\right):1\right],\ldots\ldots\left[\left(\beta_{v_m}\right):1\right],----$$

$$\frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^{e}},\frac{v(-1)^{e_r(r+s+p+q+h+g+1)}}{\lambda^{e_1}y^{e_1u_1}},$$

$$\frac{\lambda_2 x_2^{e_2}(-1)^{e_2(r+s+p+q+1)+r+s}}{\lambda^{e_2}y^{e_2u_2}},\ldots\ldots\frac{\lambda_m x_m^{e_m}(-1)^{e_m(r+s+p+q+1)+r+s}}{\lambda^{e_m}} \qquad (3.1)$$

Corollary II : For $e_1 = 1$ and $e_2 > 1$ ……..$e_m > 1$, we have

$$A_n\left\{\left(x_m\right),y\right\}=M_1\sum_{i=1}^{n}\frac{(-1)^n(1+\alpha)_n L_i^{(u)}\left(x_1^d\right)}{(1+\alpha)_i\ \ (-n-i)!}$$

$$\times F^{1+q+s+h:u_1;u_2;u_m}_{r+p+g:v_1;v_2;v_m}\left[\begin{matrix}\left[(-n+i):e,e_1,e_2,\ldots\ldots e_m\right]\\ \left[(-n-\alpha);e,e_1,e_2,\ldots\ldots e_m\right]\end{matrix}\right.$$

$$\left[\left(1-(B_s-n\right):1,e,e_1,e_2-1,\ldots..e_m-1\right],\left[\left(1-(D_q)-n\right):1,e_1,e_2-1,\ldots..e_m-1\right],$$

$$\left[\left(1-(A_r)-n\right):1,e_1,e_2-1,\ldots..e_m-1\right],\left[\left(1-(C_p)-n\right):1,e_1,e_2-1,\ldots..e_m-1\right],$$

$$\left[\left(a_g\right):1\right]\left[\left(\alpha_{u_1}\right):1\right],\left[\left(\alpha_{u_2}\right):1\right],\ldots\ldots\left[\left(\alpha_{u_m}\right):1\right],\left[\left(v_z\right):\right]$$

$$\left[\left(b_h\right):1\right]\left[\left(\beta_{v_1}\right):1\right],\left[\left(\beta_{v_2}\right):1\right],\ldots\ldots\left[\left(\beta_{v_m}\right):1\right],----$$

$$\frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^{e}},\frac{v(-1)^{e_1(r+s+p+q+g+h+1)}}{\lambda y^{\mu_1}},$$

$$\frac{\lambda_2 x_2^{e_2}(-1)^{e_2(r+s+p+q+1)+r+s}}{\lambda^{e_2} y^{e_2 u_2}},\ldots\ldots\frac{\lambda_m x_m^{e_m}(-1)^{e_m(r+s+p+q+1)+r+s}}{\lambda^{e_m}} \tag{3.2}$$

Corollary III : For $e_2 = 1, \ldots\ldots e_m > 1$ we achieve

$$A_n\{(x_m),y\} = M_1\sum_{i=0}^{n}\frac{(-1)^n(1+\alpha)_n L_i^{(\alpha)}\left(x_1^d\right)}{(1+\alpha)_i\ (-n-i)!}$$

$$\times F^{1+q+s+h:u_1;u_2;u_m}_{p+r+g:v_1;v_2;v_m}\left[\begin{array}{l}\left[(-n+i):e,e_1,0,\ldots\ldots e_m-1\right],\\ \left[(-n-\alpha);e,e_1,0,\ldots\ldots e_m-1\right],\end{array}\right.$$

$$\left[(1-(B_s-n):e,e_1,0,\ldots..e_m-1\right],\left[(1-(D_q)-n):e,e_1-1,\ldots..e_m\right],$$

$$\left[(1-(A_r)-n):e,e_1,0,\ldots..e_m\right],\left[(1-(C_p)-n):e,e_1,1,\ldots..e_m\right],$$

$$\left[(a_g):1\right]\left[(\alpha_{u_1}):1\right],\left[(\alpha_{u_2}):1\right],\ldots\ldots\left[(\alpha_{u_m}):1\right],\left[(v_z):\right]$$

$$\left[(b_h):1\right]\left[(\beta_{v_1}):1\right],\left[(\beta_{v_2}):1\right],\ldots\ldots\left[(\beta_{v_m}):1\right],----$$

$$\frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^e},\frac{v(-1)^{e_1(r+s+p+q+g+h+1)}}{\lambda_1^{e_1}y^{e_1\mu_1}},$$

$$\frac{\lambda_2 x_2(-1)^{2(r+s)+p+q+1}}{\lambda y^{u_2}},\ldots\ldots\frac{\lambda_m x_m^{e_m}(-1)^{e_m(r+s+p+q+1)+r+s}}{\lambda^{e_m}} \tag{3.3}$$

Corollary IV : For $e_2 = 1, \ldots\ldots e_m = 1$ we get

$$A_n\{(x_m),y\} = M_1\sum_{i=0}^{n}\frac{(-1)^n(1+\alpha)_n L_i^{(\alpha)}\left(x_1^d\right)}{(1+\alpha)_i\ (-n-i)!}$$

$$\times F^{1+g+s+h:u_1;u_2;u_m}_{p+g+r+:v_1;v_2;v_m}\left[\begin{array}{l}\left[(-n+i):e,e_1,e_2,\ldots\ldots 1\right],\\ \left[(-n-\alpha);e,e_1,e_2,\ldots\ldots 1\right],\end{array}\right.$$

$$\left[(1-(B_s-n):e,e_1,e_2-1,\ldots..0\right],\left[(1-(D_q)-n):e,e_1e_2,\ldots..1\right],$$

$$\left[(1-(A_r)-n):e,e_1,e_2-1,\ldots..0\right],\left[(1-(C_p)-n):e,e_1,e_2,\ldots..1\right],$$

$$\left[\left(a_g\right):1\right]\left[\left(\alpha_{u_1}\right):1\right],\left[\left(\alpha_{u_2}\right):1\right],\ldots\ldots\left[\left(\alpha_{u_m}\right):1\right],\left[\left(v_z\right):\right]$$
$$\left[\left(b_h\right):1\right]\left[\left(\beta_{v_1}\right):1\right],\left[\left(\beta_{v_2}\right):1\right],\ldots\ldots\left[\left(\beta_{v_m}\right):1\right],----$$
$$\frac{\mu(-1)^{e(r+s+p+q+1)}}{\lambda^e},\frac{v(-1)^{e_2(r+s+p+q+g+h+1)}}{\lambda_1^{e_1}y^{e_1\mu_1}},$$
$$\frac{\lambda_2 x_2^{e_2}(-1)^{e_2(r+s+p+q+1)+r+s}}{\lambda_2^{e_2}y^{e_2u_2}},\ldots\ldots\frac{\lambda_m x_m(-1)^{2(r+s)+p+q+1)}}{\lambda_m} \qquad (3.4)$$

Corollary V : For e_2 1 = e_3 = ………=e_m we can find the result in a similar way

4. Discussion and Application

This paper presents expansion of the multivariable generalized polynomial set $A_n\{x_m),y\}$ in terms of Laguerre polynomial by using different equations and expressions along with various new results.

Such type of study is significant and useful in the study of quantum mechanics, transmission line theory and seismological investigations.

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