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PROOF OF GOLDBACH'S CONJECTURE

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ABSTRACT: Goldbach's conjecture that every even integer greater than 2 can be expressed as the sum of two primes have been proved.

Keywords: Goldbach's conjecture, Even integers, Prime numbers.

Mathematics Subject Classification (2010): 11N13 (primary); 11N80 (secondary).

1-INTRODUCTION:

Goldbach's conjecture (sometimes called the "ternary" Goldbach conjecture) is one of the oldest and well-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes [1]. The conjecture has been shown to hold for all integers less than 4×10^{18} [2], but remains unproven despite considerable effort of mathematicians.

The conjecture mentioned in a letter from Goldbach to Euler dated on 7. June 1742 (Latin-German) [3]. On 7 June 1742, the German mathematician Christian Goldbach wrote a letter to Leonhard Euler (letter XLIII) [4] in which he proposed the following conjecture:

Every integer which can be written as the sum of two primes can also be written as the sum of as many primes as one wishes, until all terms are units.

He then proposed a second conjecture in the margin of his letter:

Every integer greater than 2 can be written as the sum of three primes.

He considered 1 to be a prime number, a convention is no longer followed [1]. The two conjectures are now known to be equivalent, but this did not seem to be an issue at the time. A modern version of Goldbach's marginal conjecture is:

Every integer greater than 5 can be written as the sum of three primes.

Euler replied in a letter dated 30 June 1742, and reminded Goldbach of an earlier conversation they had ("…so Ew vormals mit mir communicirt haben…"), in which Goldbach remarked his original (and not marginal) conjecture followed from the following statement:

Every even integer greater than 2 can be written as the sum of two primes, which is, thus, also a conjecture of Goldbach. In the letter dated 30 June 1742, Euler stated:

"Dass … ein jeder numerus par eine summa duorum primorum sey, halte ich für ein ganz gewisses theorema, ungeachtet ich dasselbe nicht demonstriren kann." ("That … every even integer is a sum of two primes, I regard as a completely certain theorem, although I cannot prove it.") [5][6].

Goldbach's third version (equivalent to the two other versions) is the form in which the conjecture is usually expressed today. It is also known as the "strong", "even", or "binary" Goldbach conjecture, to distinguish it from a weaker conjecture, known today variously as the Goldbach's weak conjecture, the "odd" Goldbach conjecture, or the "ternary" Goldbach conjecture. This weak conjecture asserts that all odd numbers greater than 7 are the sum of three odd primes, and appears to have been proved in 2013 [7][8]. The weak conjecture is a corollary of the strong conjecture, as, if $(n – 3)$ is a sum of two primes, and then n is a sum of three primes. The converse implication and the strong Goldbach conjecture remain unproved.

Proof of Goldbach's conjecture:

The conjecture is true for $n = 4 = 2 + 2$. i. e. $p(1)$ is true

Let the conjecture $p(k)$ be true for even integer $n = k = a + b$ where and a, b are both prime.

Now for even integer k the next even integer is k+2.

Therefore $p(k+1) = k+2 = a+b+2 = a + (b+2)$

Now either b+2 prime integer or composite integer . If it is prime integer , then we have done. If it is a composite odd integer then we have,

$p(k+1) = k+2 = a+b+2 = a + (b+2) = a + c$, where c is odd number.

$\Rightarrow p(k+1) = 2h + 1$, which is odd integer either prime or not.

Now odd integer either prime or not prime may be expressed as a sum of two prime numbers in general. Therefore continuing this process we must reach to the reality that every even integer greater than 2 can be expressed as the sum of two primes.

Otherwise,

p(k+1) = k = e + f, say e prime and f not but mostly even.

⇒ p(k+1) = k = e + 2g, where e prime and g any integer.

⇒ p(k+1) = k = e + 2g +1 – 1 = e - 1 + 2g +1 = m + l, where both m and l are odd.

⇒ p(k+1) = k = n, where n even number. This is a contradiction of the hypothesis which leads us to the first conclusion to be true.

Hence by mathematical Induction the conjecture is true.

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