International Journal of Engineering, Science and Mathematics
Vol. 13 Issue 8, Aug 2024,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Quantum Teleportation using W-BELL and Bell-GHZ channels

Abhinav Pandey

***Keywords*—** Entanglement, Teleportation, No Cloning Theorem, Quantum Mechanics, Probability.

[1] INTRODUCTION

Quantum Teleportation is a significant phenomenon which occurs in the realm of Quantum Mechanics. In Quantum theory a classical bit of information has two possible states either a 1 or a 0. This is the binary system used by all computing and computing based devices in digital or classical computation. Information can be related and stored with either a 0 or a 1 but not with both the states being simultaneously present in a superposition form. This superposition of states is the essence of quantum computation. In quantum we have a Qubit to store information and a quantum state has qubits in a superposition form. Below we have an equation which states how a quantum state can be made by superposition of qubits.

$$|\Phi>= \alpha|0> + \beta|1> \text{ where } \alpha\hat{}2 + \beta\hat{}2=1 \qquad (1)$$

Above equation shows how a quantum state exists with qubit 0 and qubit 1 being in a superposition state. The alpha and beta in above equation are complex numbers and alpha square and beta square are the probabilities of existence of qubit 0 and qubit 1 in the superposition state $|\Phi>$. In quantum computation we can store more information than classical computation because we deal with all the possible states a piece of information can be found.

a.Quantum Entanglement

Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles are generated or interact in ways such that the quantum state of each particle cannot be described independently — instead, a quantum state must be described for the system as a whole. Measurements of physical properties such as position, momentum, spin, polarization, etc., performed on entangled particles are found to be appropriately correlated. For example, if a pair of particles are generated in such a way that their total spin is known to be zero, and one particle is found to have clockwise spin on a certain axis, then the spin of the other particle, measured on the same axis, will be found to be counterclockwise, as to be expected due to their entanglement. However, this behavior gives rise to paradoxical effects: any measurement of a property of a particle can be seen as acting on that particle (e.g., by collapsing a number of superposed states) and will change the original quantum property by some unknown amount; and in the case of entangled particles, such a measurement will be on the entangled system as a whole. It thus appears that one particle of an entangled pair "knows" what measurement has been performed on the other, and with what outcome, even though there is no known means for such information to be communicated between the particles, which at the time of measurement may be separated by arbitrarily large distances.

An entangled system is defined to be one whose quantum state cannot be factored as a product of states of its local constituents, that is to say, they are not individual particles but are an inseparable whole. If entangled, one constituent cannot be fully described without considering the other(s). Note that the state of a composite system is always expressible as a *sum*, or superposition, of products of states of local constituents; it is entangled if this sum necessarily has more than one term. As an example of entanglement: a subatomic particle decays into an entangled pair of other particles. The decay events obey the various conservation laws, and as a result, the measurement outcomes of one daughter particle must be highly correlated with the measurement outcomes of the other daughter particle (so that the total momenta, angular momenta, energy, and so forth remains roughly the same before and after this process). For instance, a spin-zero particle could decay into a pair of spin-½ particles. Since the total spin before and after this decay must be zero (conservation of angular momentum), whenever the first particle is measured to be spin up on some axis, the other, when measured on the same axis, is always found to be spin down. (This is called the *spin anti-correlated* case; and if the prior probabilities for measuring each spin are equal, the pair is said to be in the singlet state) The special property of entanglement can be better observed if we separate the said two particles. Let's put one of them in the White House in Washington and the other in UC Berkeley (think about this as a thought experiment, not an actual one). Now, if we measure a particular characteristic of one of these particles (say, for example, spin), get a result, and then measure the other particle using the same criterion (spin along the same axis), we find that the result of the measurement of the second particle will match (in a complementary sense) the result of the measurement of the first particle, in that they will be opposite in their values.

BELL STATES- the Bell states are a concept in quantum information science and represent the simplest examples of entanglement. They are named after John S. Bell because they are the subject of his famous Bell inequality. An EPR pair is a pair of qubits (or quantum bits) which are in a Bell state together, that is, entangled with each other. Bell states are 4 types of states given below.

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B).$$

Fig. 1

b.Quantum teleportation

This is a process by which quantum information (e.g. the exact state of an atom or photon) can be transmitted (exactly, in principle) from one location to another, with the help of classical communication and previously shared quantum entanglement between the sending and receiving location. Because it depends on classical communication, which can proceed no faster than the speed of light, it cannot be used for faster-than-light transport or communication of classical bits. It also cannot be used to make copies of a system, as this violates the no-cloning theorem. While it has proven possible to teleport one or more qubits of information between two (entangled) atoms, [this has not yet been achieved between molecules or anything larger.

By looking at the various terminologies of Quantum Computation we can go deep into the teleportation aspects. In the paper next we have related work followed by Quantum teleportation using W-BELL and BELL-GHZ channels. In the End of the paper we have conclusion, future work and references.

[2] RELATED WORK

Information Processing is a growing field of research in quantum mechanics. Information processing in classical physics is done using classical bits which exist in two definite states either 1 or 0. However things change in quantum physics where qubits represented by Ket |> are used and can exist in states |0>, |1> and also in the superposition of |0> and |1>. As explained in [2] quantum information processing is an emerging field of quantum computation which uses qubits for information processing. Use of qubits aid in processing complex information [5] in short span of time. Another striking feature of Quantum mechanics is entanglement. Entanglement not only provides possibilities for testing quantum mechanics against local hidden variable theories [5] but also has many practical application in information theory processing. Many work has been done using quantum information processing algorithm which help in understanding the quantum nature of particles. Quantum Entanglement is defined as the interaction of two quantum particles in which measurement on one particle gives the state of another particle even if two particles are separated by large distances. Quantum entanglement was brought up by famous 1935 paper presented by Albert Einstein, Boris Podolsky, and Nathan Rosen which describes the EPR paradox. The paradox were that first the Quantum mechanics system does not have a definite value before measurement and after the measurement is acquires a definite value. However in classical systems the physical systems have a definite value whether or not a measurement is taken or not. Second if two particles interacted and then separated then state of one particle can tell about the state of the other particle since they were entangled before separation [2]. It follows either the EPR does not follow the principal of locality or the measurement is done faster than the speed of light which is impossible [3]. There has to be a hidden variable which describes the nature of the particles. However Bell proved that theory of Einstein was false and non-local theories of quantum mechanics was accepted.
We can use the principal of entanglement explained in teleportation has we can predict the state and measurement of the other entangled pair without actually physically measuring it. Classical System uses two state systems to encode a single bit of information consequently one has to physically transfer the number of such systems. Thus the question boils down to whether you can use quantum systems to communicate the classical information more efficiently or is it possible to transfer the quantum systems as a whole.

In original proposal of quantum teleportation (Bennett el al. 1993) it was realized that the foremost requirement for the sender is to share an entangled pair of particle. The long range quantum correlation could be used to transport data was used in a protocol to transport a state of quantum system to a distant place. The original idea of long correlation was brought up by EPR paradox [1]. The problem to be solved was that Alice who wanted to transfer Bob who was at distant location with the full description of a particle n state. Now Alice could send the particle to Bob but let's assume that the particle could not survive the classical channel and that there was no measurement that Alice could use to obtain sufficient information for Bob to reconstruct the state of the particle. This kind of measurement will project the total state on only one of the superposition states of which a quantum state usually consists. However using the same projection postulate it was possible for Alice to let the particle interact with another particle with known state leaving it in state containing all information about the initial particle. By sending the particle to Bob he can reverse the actions of Alice to get the original particle [2]. During the interaction of two particles Alice destroys the quantum state to be teleported and thus obeying the no-cloning theorem of quantum mechanics [1]. In the given paper we show the comparison between the teleportation of W-Bell and Bell-GHZ channel and determine which channel has better teleportation capability using probabilistic teleportation results.

[3] Quantum Teleportation using BELL Channel

Anyone who has read science fiction or watched it on television is familiar with teleportation—a device or process by which people can jet around the galaxy. The basic idea is that you get scanned somehow, turned into energy, then beamed to where you want to go and rematerialized.

While such a procedure is likely to remain in the realm of science fiction, quantum mechanics does allow us to do something almost as magical. It allows us to send a quantum state from one place to another without that state traversing the space in between. Good thing Einstein wasn't around to hear about this. If entanglement itself made him feel "spooky," then teleportation is sure to be making him turn over in his grave.

While teleportation seems to work almost by magic Einstein can breathe a sigh of relief because special relativity seems to step in to prevent faster than light communication. To see how this works, let's go through the basic formalism. The task at hand is that Alice wants to transmit an unknown quantum state to Bob. Let's denote the state that Alice wants to send Bob by $|\chi>$. The state is a qubit:

$$|\chi> = \alpha|0> + \beta|1>$$

By saying the state is unknown, we are saying we don't necessarily know what α and β are. All we assume is that the state is normalized, so $|\alpha|^2+|\beta|^2 = 1$. Teleportation takes place in a series of steps. We begin by creating an entangled EPR pair.

Teleportation Step 1: Alice and Bob Share an Entangled Pair of Particles

Alice and Bob create the entangled state

$$|\beta_{00}\rangle = \frac{|0_A\rangle|0_B\rangle + |1_A\rangle|1_B\rangle}{\sqrt{2}} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Fig. 2

Here we’ve decided that the first member of the pair belongs to Alice and the second member of the pair belongs to Bob. Now Alice and Bob physically separate. Alice decides that she wants to send the state (10.1) to Bob. She can do it by letting it interact with her member of the EPR pair.

Teleportation Step 2: Alice Applies a CNOT Gate

Let’s begin by writing down the state of the entire system. It’s a product state of the unknown state and the EPR pair.

Fig. 3

$$|\psi\rangle = |\chi\rangle \otimes |\beta_{00}\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)$$

$$= \frac{\alpha(|000\rangle + |011\rangle) + \beta(|100\rangle + |111\rangle)}{\sqrt{2}}$$

The first two qubits in this state belong to Alice, while the third rightmost qubit belongs to Bob. So |011 indicates that Alice has a 01 in her possession while Bob has a 1.

Alice begins interacting her member of the EPR pair, which is the second qubit in with the unknown state—the first qubit in —by applying a CNOT gate. She uses the unknown state |χ as the control qubit and her member of the EPR pair as the target qubit. Remember, if the control qubit is 0, nothing happens; if the control qubit is 1, the target qubit is flipped.

Teleportation Step 3: Alice Applies a Hadamard Gate

Teleportation Step 4: Alice Measures Her Pair

|00 → |00, |01 → |01, |10 → |11, |11 → |10 So, when Alice applies the CNOT gate to the state becomes

At this stage of the game special relativity surprisingly enters the game. Alice has to somehow tell Bob her measurement result, and she has to do it using a classical communications channel—a telephone, email message, radio wave, or something—some mechanism governed by the speed of light limit. It’s this nec- essary step that prevents Alice and Bob from faster than light communication. But security is maintained—Alice just calls Bob and for instances, says she got 01, then Bob applies his X gate to obtain the state Alice wanted to send to Bob. Nothing about that state is communicated over the classical channel—Bob can obtain it because they shared an entangled EPR pair of particles.

The lesson here is that quantum information based communication can be characterized by two key aspects—local operations and classical communications (LOCC). That is, each party has two tasks:

1) Performs local quantum mechanical (local unitaries) operations on their respective states.

2) Uses classical communication to communicate measurement results.

If classical communications is not used, then the state will appear totally random to Bob.

[4] Teleportation Using W-BELL Channel

A W channel can be written as

$$|W>= (|001>+|100>+|010>)/\sqrt{3}$$

and we can write the BEL channel as

$$|B>= (|00> + |11>)/\sqrt{2}$$

Combining these two channels we get the W-BELL channel as

$$|\Phi>= (|00100>+|00111>+|10000>+|10011>+|01000>+|01011>)/\sqrt{6}.$$

Above channel will be used to teleport a single qubit state $\alpha|0> + \beta|1>$ where $\alpha^2 + \beta^2=1$

The final 6 qubit state can be written as

$$\alpha/\sqrt{6}[|001000>+|001110>+|100000>+|100110>+|010000>+|010110>]+\beta/\sqrt{6}[|001001>+|001111>+|100001>+|100111>+|010001>+|010111>]$$

Now we assign the first 5 qubits to Alice and last qubit to Bob. First we apply a control not gate with control bit 1 and target bit 5.Then another control not gate with control bit 1 and target bit 5.Finally a hadamard gate is applied on 1, 2, and 5th bit. The final result obtained is shown in table 1.In the final result measurements are done in computational basis in which we successfully teleport single qubit in 24 measurements out of 32.This accounts to be 75% success. Bob can finally get the desired state by applying some suitable gates after the measurements by Alice.

TABLE I

OUTCOME OF MEASUREMENT	STATE OBTAINED
|00000>	(α|0> + β|1>)
|00000>	(α|0> + β|1>)
|00001>	(α|1> + β|0>)
|00001>	(-α|1> - β|0>)
|00010>	(α|0> + β|1>)
|00010>	(α|0> + β|1>)
|00011>	(α|1> + β|0>)
|00011>	(-α|1> - β|0>)
|00100>	(α|0> + β|1>)
|00100>	(α|0> + β|1>)
|00101>	(α|1> + β|0>)
|00101>	(-α|1> - β|0>)
|01000>	(-α|0> - β|1>)
|01000>	(α|0> + β|1>)
|01001>	(-α|1> - β|0>)
|01001>	(α|1> + β|0>)
|01010>	(α|1> + β|0>)
|01010>	(α|0> + β|1>)
|01011>	(α|1> + β|0>)
|01011>	(-α|1> - β|0>)
|01100>	(α|0> + β|1>)
|01100>	(α|0> + β|1>)
|01101>	(α|1> + β|0>)
|01101>	(-α|1> - β|0>)

[5] Teleportation Using BELL- GHZ Channel

A W channel can be written as

$|GHZ>= (||000>+|111>)/\sqrt{2}$

and we can write the BEL channel as

$|B>= (|00> + |11>)/\sqrt{2}$

Combining these two channels we get the W-BELL channel as

$|\Phi> = (|0000> +|0001>+|1110>+|1111>)/\sqrt{4}$.

Above channel will be used to teleport a single qubit state $\alpha|0> + \beta|1>$ where $\alpha^2 + \beta^2=1$.

The final 5 qubit state can be written as

α/√4[|00000>+|00010>+|11100>+|11110>]+ β/√4[||00001> +|00011>+|11101>+|11111>]

Now we assign the first 4 qubits to Alice and last qubit to Bob. Here we assign first 4 qubits to Alice and last qubit to Bob. First we apply a control not gate with control bit 1 and target bit 5.Then another control not gate with control bit 1 and target bit 4.Finally a hadamard gate is applied on 1, 2, and 5th bit. The final result obtained is shown in table 2.In the final result measurements are done in computational basis in which we successfully teleport single qubit in 12 measurements out of 16.This accounts to be 75% success. Bob can finally get the desired state by applying some suitable gates after the measurements by Alice.

TABLE II

OUTCOME OF MEASUREMENT	STATE OBTAINED
|0000>	(α|0> + β|1>)
|0000>	(α|0> + β|1>)
|0000>	(α|1> + β|0>)
|0000>	(-α|1> - β|0>)
|0001>	(α|0> + β|1>)
|0001>	(α|0> + β|1>)
|0001>	(α|1> + β|0>)
|0001>	(-α|1> - β|0>)
|0010>	(α|0> + β|1>)
|0010>	(α|0> + β|1>)
|0010>	(α|1> + β|0>)
|0010>	(-α|1> - β|0>)

[6] Conclusion

In this paper we presented the proof of principle of quantum teleportation. We demonstrated the possibility of transferring the polarization state from one photon onto another. The techniques developed here also allow one to perform the transfer of any arbitrary quantum state, which in the process of entanglement swapping enables one to create non-classical correlations between particles that never interacted with each other (Zukowski et al. 1993; Bose et al. 1998). This was recently confirmed experimentally (Pan et al. 1998). Moreover, with these experimental techniques, generating entanglement between three out of four particles finally comes within reach (Zeilinger et al. 1997). In principle, any two quantum systems could be entangled with each other, for example atoms or ions in a trap. Then one could also consider schemes where the long coherence time of atomic states allows the storage of quantum states for longer times than would be possible for light; the transfer of quantum information into such a quantum memory would be achieved typically by quantum teleportation.

Quantum memories find use in extensions of the standard quantum cryptography schemes and of course in the new field of quantum computation. By showing the various Quantum Channels like W and GHZ also have property to teleport a Quantum state we can use it further in many quantum related applications like Super Dense Coding. By showing that W-BELL and BELL-GHZ channel have a probabilistic success of 75% to teleport a Quantum State we have actually opened the door for many other such work and utilization of these channels for teleportation in Quantum Information and not just rely on the BELL channel for such operations.

[7] FUTURE WORK

In quantum information theory, superdense coding is a technique used to send two bits of classical information using only one qubit [1] [2]. It is the inverse of quantum teleportation, which sends one qubit with two classical bits. [3] Both superdense coding and quantum teleportation require, and use up, entanglement between the sender and receiver in the form of Bell pairs. Not only the Superdense coding can now be tested with the channels like W-BELL and BELL-GHZ but also other Quantum Related experiments like Quantum Cryptography can be carried out with these channels and we can find out the efficiency of these channels in the given applications.

Acknowledgment

It was a great opportunity for me to write on a topic like “Quantum Teleportation”. While writing this paper I have gone through a lot of websites, Books and research papers written by other authors which helped me to read and learn about new topics. I have been focussed on such topics which are relevant to my study and help me and guide me to complete my research. I acknowledge my professor who has been sincere and helpful in making me understand the different system of Quantum Information. Apart from me this paper will be very useful to any student, professor or any scholar who wants to understand the topic of Quantum Teleportation. I have tried hard and soul to gather all the relevant information and documents regarding this subject. Furthermore I do not claim that all information in this paper is my original and is included perfectly. There must be shortcomings errors mistakes which are all mine and I am responsible for those. I am very thankful to my professor for giving me this opportunity, to my fellow students who helped me in writing this paper and gathering the information.

References

5 [1] Nielsen, M.A., Chuang, I.L.: QuantumComputationandQuantumInformation. Cambridge University Press, Cambridge (2002).

6 [2] Bennett, C.H., Brassard, G.Crepeau, C., Joss, R., Peres, A., Wootters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein–Podolsky–Rosen channels. Phys. Rev.

7 [3] Agrawal, P., Pati, A.: Perfect teleportation and superdense coding with W states. Phys. Rev, 062320 (2006).

8 [4] Prasath, S., Muralidharan, S., Panigrahi P.K., Mitra, C. Multipartite entangled magnon states as quantum communication channels, eprintquant-ph/0905.1233v2.

9 [5] Raussendorf, R., Briegel, H.J.: Persistent entanglement in arrays of interacting particles. Phys. Rev.910 (2001).

10 [6] Ola M. Hegazy, Ayman M. Bahaa Eldin and Yasser H. Dakroury (2011) Quantum Secure Communication using Entanglement and Super Dense Coding.

11 [7] P Mladen Entaglement and Superdense Coding with linear optics (2011) International Journal for Quantum Information.

12 [8] H. Pawel and H. Ryszard Distillation and Bound Entaglement (2001) Quantum Computation and Information.

13 [9]A. Pandey, A. Verma, P.K.Panigrahi Quantum Teleportation and Superdense Coding using W-GHZ Channel (2012).

14 [10] A. Verma, A. Pandey, P.K.Panigrahi Entanglement Distillation using various Quantum channels (2013). [6] S. Kulik, Y. Kim Quantum Teleportation using Complete Bell States.

15 C. H. Bennett et al., Phys. Rev. Lett. 70, 1895 (1993).

16 J. S. Bell, Speakable and Unspeakable in Quantum Mechanics (Cambridge Univ. Press, New York, 1988), p. 196.

17 L. Davidovich et al., Phys. Rev. A 50, R895 (1994).

18 J. I. Cirac and A. S. Parkins, ibid, p. R4441.

19 T. Sleator and H. Weinfurter, Ann. N.Y. Acad. Sci. 755, 715 (1995).

20 S. L. Braunstein and A. Mann, Phys. Rev. A 51, R1727 (1995); ibid. 53, 630 (1996).

21 D. Boschi et al., Phys. Rev. Lett. 80, 1121 (1998).

22 D. Bouwmeester et al., Nature 390, 575 (1997).