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On Zero Power Valued Generalized Homoderivation in Semi Prime Rings

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Abstract

The Purpose of this paper is to investigate commutativity of semi prime rings in case of generalized homoderivation of semi prime rings with Lie ideal.

Keywords:

Lie ideal,
generalized homoderivation,
commutator,
semi prime ring

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1. Introduction

Throughout this paper, R denotes an associative ring with centre $Z(R)$. For any x, $y \in R$, the notation $[x, y]$ denotes commutator $xy - yx$ and $x \circ y$ denotes an anti-commutator $xy + yx$. Recall that a ring R is prime if for any $x, y \in R$, $xRy = \{0\}$ implies that $x = 0$ or $y = 0$ and R is semi prime if $xRx = \{0\}$ implies that $x = 0$. An additive subgroup U of R is said to be a Lie ideal of R if $[u, r] \in U$, for all $u \in U$ and $r \in R$. An additive mapping $d : R \longrightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$, for all x, $y \in R$. In [4.], El-Soufi introduced the concept of homoderivation as follows: An additive mapping $h : R \longrightarrow R$ is called a homoderivation if $h(xy) = h(x)h(y) + h(x)y + xh(y)$, for all $x, y \in R$. An example of such mapping is to let $h(x) = F(x) + x$, for all $x \in R$. where F is an endomorphism on R. Thus, it is clear that a homoderivation h is also a derivation if $h(x)h(y) = 0$, for all $x, y \in R$.

Motivated by the definition of a homoderivation, the notion of generalized homoderivation was extended as follows : An additive mapping $F : R \longrightarrow R$ is called a right generalized homoderivation derivation if there exists a homoderivation $d : R \longrightarrow R$ such that $F(xy) = F(x)h(y) + F(x)y + xh(y)$, for all x, $y \in R$ and F is called a left generalized homoderivation if there exists a

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homoderivation $h : R \longrightarrow R$ such that $F(xy) = h(x)F(y) + h(x)y + xF(y)$, for all $x, y \in R$. F is said to be a generalized homoderivation associated with homoderivation h if it is both a left and a right generalized homoderivation associated with homoderivation h. If $S \subseteq R$, then a mapping $F{:}R \rightarrow R$ preserves S if $F(S) \subseteq S$. A mapping $F : R \rightarrow R$ is zero - power valued on S if F preserves S and for each $x \in S$, there exist a positive integer $n(x) > 1$ such that $F^{n(x)}(x) = 0$.

In [3.], Daif and Bell proved that if R is a semiprime ring U a nonzero ideal of R and d a derivation of R such that $d([x, y]) = [x, y]$, for all $x, y \in U$, then $U \in Z$. In 2007, Ashraf et al [2] prove that a prime ring R must be commutative if R satisfies any one of the following conditions

(i) $F(xy) = xy$, (ii) $F(x)F(y) = xy$, where F isa generalized derivation of R and I is a nonzero two sided ideal of R. Recently in 2023, Boua & Sogutcu [9] investigate the commutative of semiprime rings if R satisfies the following conditions : (i) $F[u,v] = \pm[u,v]$ (ii) $F[u,v] = uov$, for all $u, v \in I$. In this paper, we prove these results for generalized homoderivation with Lie ideals in semi prime rings.

1. Preliminaries

We shall use frequently the following basic commutator identities:

$$[a, bc] = b[a, c] + [a, b]c,$$
$$[ab, c] = [a, c]b + a[b, c],$$
$$a \, o \, (bc) = (a \, o \, b)c - b[a, c] = b(a \, o \, c) + [a, b]c,$$
$$(ab) \, o \, c = a(b \, o \, c) - [a, c]b = (a \, o \, c)b + a[b, c]$$

We began with the following lemma which is required to prove our results:

Lemma 2.1 [8, **Corollary** 2.1**]**. Let R be a 2-torsion free semi- prime ring, U a noncentral Lie ideal of R and $a, b \in U$.

(i) If $aUa = \{0\}$, then $a = 0$.

(ii) If $aU = \{0\}$, (or $Ua = \{0\}$), then $a = 0$.

2. Main Results

Theorem 3.1. Let R be a semi-prime ring with $CharR \neq 2$ and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F([u, v]) = (v \, o \, u)$, for all $u, v \in U$, then h is commuting map on U.

Proof. we have

$$F([u, v]) = (v \, o \, u), \; for \, all \, u, v \in U. \tag{3.1}$$

Replacing v by $2vu$ in equations (3.1), we obtain that

$$F([u, vu]) = (vu \text{ o } u), \ for\ all\ u, v \in U.$$
$$F([u, v]u) = (v \text{ o } u)u, \ for\ all\ u, v \in U.$$

i.e.,

$$F[u, v]h(u) + F[u, v]u + [u, v]h(u) = (v \text{ o } u)u, \quad for\ all\ u, v, \in U.$$
$$F[u, v](h(u) + u) + [u, v]h(u) = (v \text{ o } u)u, \quad for\ all\ u, v \in U.$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x) = 0$, for all $x \in U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F[u, v]u + [u, v]h(u) = (v \text{ o } u)u, \quad for\ all\ u, v \in U.$$

Using the given hypothesis, the above relation yields that

$$[u, v]h(u) = 0, \ for\ all\ u, v \in U. \tag{3.2}$$

Again, replacing v by $2vw$ in equation (3.2) and using the fact that Char$R \neq 2$, we get

$$[u, vw]h(u) = 0, \quad for\ all\ u, v, w \in U.$$

which gives that $(v[u, w] + [u, v]w)h(u) = 0$, for all $u, v, w \in U$, i.e., $v[u, v]h(u) + [u, v]wh(u) = 0$, for all $u, v, w \in U$. Using the equation (3.2), the above relation yields that $[u, v]wh(u) = 0$, for all $u, v, w \in U$.

Now replace v by $h(u)$, we get

$$[u, h(u)]wh(u) = 0, \ for\ all\ u, w \in U. \tag{3.3}$$

Right multiplication of equation (3.3) by u, we get

$$[u, h(u)]wh(u)u = 0, \ for\ all\ u, w \in U. \tag{3.4}$$

Replacing w by $2wu$ in equation (3.3) and using the fact that Char$R \neq 2$, we get

$$[u, h(u)]wu\,h(u) = 0, \ for\ all\ u, w \in U. \tag{3.5}$$

Now Subtracting equation (3.4) from equation (3.5), we arrived that

$$[u, h(u)]wuh(u) - [u, h(u)]wh(u)u = 0, for\ all\ u, w \in U.$$
$$[u, h(u)]w(uh(u)u - h(u)u) = 0, for\ all\ u, w \in U.$$
$$[u, h(u)]w[u, h(u)] = 0, for\ all\ u, w \in U.$$
$$[u, h(u)]\,U\,[u, h(u)] = 0, for\ all\ u, \in U.$$

Using Lemma 2.1, we obtain that $[u, h(u)] = 0$, for all $u \in U$. Hence h is commuting map on U.

Theorem 3.2. Let R be a semi-prime ring with $Char$ R $\neq 2$ and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F([u, v]) = -(v \text{ o } u)$, for all $u, v \in U$, then h is commuting map on U.

Proof. we have,

$$F([u, v]) = -(v \text{ o } u), for\ all\ u, v \in U. \tag{3.6}$$

Replacing v by $2vu$ in equations (3.6) and using the fact that CharR $\neq 2$, we obtain that

$$F([u, vu]) = -(vu \text{ o } u), for\ all\ u, v \in U.$$

$$F([u, v]u) = -(v \text{ o } u)u, for\ all\ u, v \in U.$$

i.e.

$$F\,[u,v]h(u) + F\,([u,v])u + [u,v]h(u) = -(v\ o\ u)u, for\ all\ u,v \in U.$$

$$F\,[u,v]\,(h(u) + u) + [u,v]h(u) = -(v\ o\ u)u, for\ all\ u,v \in U.$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x)=0$, for all $x \in U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F[u,v]u + [u,v]h(u) = -(v\ o\ u)u, for\ all\ u,v \in U.$$

Using the equation (3.6), the above relation yields that

$$[u,v]h(u) = 0, for\ all\ u,v \in U. \tag{3.7}$$

Proceeding in the same manner as in the proof of Theorem 3.1., we get the required result.

Theorem 3.3. Let R be a semi-prime ring with $Char\ R \neq 2$ and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F([u,v]) = [v,u]$, for all $u,v \in U$, then h is commuting map on U.

Proof. we have,

$$F\,([u,v]) = [v,u], for\ all\ u,v, \in U. \tag{3.8}$$

Replacing v by $2vu$ in equations (3.8) and using the fact that $Char$ R $\neq 2$, we obtain that

$$F\,([u,vu]) = [vu,u], for\ all\ u,v \in U.$$

$$F\,([u,v]u) = [v,u]u, for\ all\ u,v \in U.$$

i.e.,

$$F\,[u,v]h(u) + F\,[u,v]u + [u,v]h(u) = [v,u]u, for\ all\ u,v \in U.$$

$$F\,[u,v](h(u) + u) + [u,v]h(u) = [v,u]u, for\ all\ u,v \in U.$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x) = 0$, for all $x \in U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F\,[u,v]u + [u,v]h(u) = [v,u]u, for\ all\ u,v \in U.$$

$$F\,[u,v]u + [u,v]h(u) = [v,u]u, for\ all\ u,v \in U.$$

Using the given hypothesis, the above relation yields that

$$[u,v]h(u) = 0, for\ all\ u,v \in U. \tag{3.9}$$

Proceeding in the same manner as in the proof of Theorem 3.1., we obtain the required result.

Theorem 3.4. Let R be a semi-prime ring with $Char\ R \neq 2$ and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F\,([u,v]) = -[v,u]$, for all $u,v \in U$, then h is commuting map on U.

Proof. we have,

$$F\,([u,v]) = -[v,u], for\ all\ u,v \in U. \tag{3.10}$$

Replacing v by $2vu$ in equations (3.10) and using the fact that Char R $\neq 2$, we obtain that

$$F\,([u,vu]) = -[vu,u], for\ all\ u,v \in U.$$

$$F\,([u,v]u) = -[v,u]u, for\ all\ u,v \in U.$$

i.e.,

$$F\,[u,v]h(u)\ +\ F\,[u,v]u\ +\ [u,v]h(u)\ =\ -\,[\,v,u]u, for\ all\ u,v\ \epsilon\ U.$$

$$F\,[u,v](h(u)\ +\ u)\ +\ [u,v]h(u)\ =\ -[v,u]u, for\ all\ u,v\ \epsilon\ U.$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x) = 0$, for all $x \in U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F\,[u,v]u\ +\ [u,v]h(u)\ =\ -[v,u]u, for\ all\ \ u,v\ \epsilon\ U.$$

Using the given hypothesis, the above relation yields that

$$[u,v]h(u)\ =\ 0, for\ all\ \ u,v\ \epsilon\ U. \qquad (3.11)$$

Proceeding in the same manner as in the proof of Theorem 3.1, we obtain the srequired result.

Theorem 3.5. Let R be a semi-prime ring with *Char* R $\neq 2$ and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F(u \text{ o } v) = [v, u]$, for all $u, v \in U$, then h is commuting map on U.

Proof. we have,

$$F\,(u\ o\ v)\ =\ [v,u], for\ all\ u,v,\epsilon\ U. \qquad (3.12)$$

Replacing v by $2vu$ in equations (3.12) and using the fact that Char$R \neq 2$, we obtain that

$$F\,(u\ o\ vu)\ =\ [vu,u],\ \ for\ all\ u,v\ \epsilon\ U.$$

$$F\,((u\ o\ v)u)\ =\ [v,u]u,\ \ for\ all\ u,v\ \epsilon\ U.$$

i.e.,

$$F\,(u\ o\ v)h(u) +\ F\,(u\ o\ v)u\ +\ (u\ o\ v)h(u) =\ [v,u]u,\ \ for\ all\ u,v\ \epsilon\ U.$$

$$F\,(u\ o\ v)(h(u) +\ u) +\ (u\ o\ v)h(u) =\ [v,u]u,\ \ \ for\ all\ u,v\ \epsilon\ U.$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x)=0$, for all $x \in U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F\,((u\ o\ v)u)\ +\ (u\ o\ v)h(u)\ =\ [v,u]u,\qquad for\ all\ u,v\ \epsilon\ U.$$

Using the equation (3.12), the above relation yields that

$$(u\ o\ v)h(u)\ =\ 0,\ \ for\ all\ u,v\ \epsilon\ U. \qquad (3.13)$$

Again, replacing v by $2wv$ in equation (3.13) and using the fact that *Char* $R \neq 2$, we get $(u\ o\ wv)h(u) = 0$, which gives that $(w(u\ o\ v) + [u,w]v)h(u) = 0$, for all $u,v,w\ \epsilon\ U$, i.e., $w(u\ o\ v)h(u) + [u,w]vh(u) = 0$, for all $u,v,w\ \epsilon\ U$. Using the equation (3.13), the above relation yields that $[u, w]vh(u) = 0$, for all $u, v, w \in U$.

Proceeding in the same manner as in the proof of Theorem 3.1., we obtain the required result.

Theorem 3.6. Let R be a semi-prime ring with *Char* $R \neq 2$ and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F(u \text{ o } v) = -[v, u]$, for all $u, v \in U$, then h is commuting map on U.

Proof. we have,

$$F\,(u\;o\;v) \;=\; -[v,u], for\,all\,u,v,\epsilon\,U. \qquad (3.14)$$

Replacing v by $2vu$ in equations (3.14), we obtain that

$$F\,(u\;o\;vu) = -[vu,u], \qquad for\,all\,u,v\,\epsilon\,U.$$

i.e.,

$$F\,((u\;o\;v)u) \;=\; -[v,u]u, \qquad for\,all\,u,v\,\epsilon\,U.$$

Or,

$$F\,(u\;o\;v)h(u) + F\,(u\;o\;v)u \;+\; (u\;o\;v)h(u) = -[v,u]u, \qquad for\,all\,u,v\,\epsilon\,U.$$

$$F\,(u\;o\;v)(h(u) \;+\; u) \;+\; (u\;o\;v)h(u) \;=\; -([v,u]u), \qquad for\,all\,u,v\,\epsilon\,U.$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x)=0$, for all $x \in U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F\,((u\;o\;v)u) \;+\; (u\;o\;v)h(u) \;=\; -[v,u]u, \;\; for\,all\,u,v\,\epsilon\,U.$$

Using the equation (3.14), the above relation yields that

$$(u\;o\;v)h(u) \;=\; 0, \;\; for\,all\,u,v\,\epsilon\;U. \qquad (3.15)$$

Again, replacing v by $2wv$ in equation (3.15) and using the fact that *Char* R $\neq 2$, we get $(u \text{ o } wv)h(u) = 0$, which gives that $(w(u \text{ o } v)+[u,w]v)h(u) = 0$, for all $u, v, w \in U$, i.e., $w(u \text{ o } v)h(u) + [u, w]vh(u) = 0$, for all $u, v, w \in U$. Using the equation (3.15), the above relation yields that $[u, w]vh(u) = 0$, for all $u, v, w \in U$. Now the proof runs as Theorem 3.1.

Theorem 3.7. Let R be a semi-prime ring with $CharR \neq 2$ and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F([u, v]) + h([u, v]) + [u, v] = 0$, for all $u, v \in U$, then h is commuting map on U.

Proof. we have,

$$F\,([u,v]) + \;h([u,v]) \;+\; [u,v] \;=\; 0, for\,all\,u,v\,\epsilon\;U. \qquad (3.16)$$

Replacing v by $2vu$ in equations (3.16) and using the fact that $CharR \neq 2$, we obtain that

$$F\,([u,vu]) + \;h([u,vu]) \;+\; [u,vu] \;=\; 0, for\,all\,u,v\,\epsilon\;\;U.$$

i.e.,

$$F\,([u,v]u) \;+\; h([u,v]u) \;+\; [u,v]u \;=\; 0, for\,all\,u,v\,\epsilon\;\;U.$$

$$F\,([u,v\,])h(u) + F\,([u,v])u + [u,v]h(u) + h[u,v]h(u) + h([u,v])u + [u,v]h(u) + [u,v]u = \;0,$$

$$F\,([u,v])(h(u) + u) + [u,v]h(u) + h[u,v](h(u) + u) + [u,v]h(u) + [u,v]u \;=\; 0,$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x)=0$, for all $x \in U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F(\,[u,v])u \;+\; [u,v]h(u) \;+\; h([u,v])u \;+\; [u,v]h(u) \;+\; [u,v]u \;=\; 0, for\,all\,u,v\,\epsilon\;U.$$

$$F\,([u,v])u \;+\; h([u,v])u \;+\; 2[u,v]h(u) \;+\; [u,v]u \;=\; 0, for\,all\,u,v\,\epsilon\;\;U.$$

Using the given hypothesis, the above relation yields that $2[u, v]h(u) = 0$, for all $u, v \epsilon U$. Since R is of $CharR \neq 2$, we have $[u, v]h(u) = 0, for\ all\ u, v \epsilon U.$

Now the proof runs as the proof of Theorem 3.1, we get the required result.

Theorem 3.8. Let R be a semi-prime ring with *Char* R≠2 and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F([u, v]) + h([u, v]) + (u o v) = 0$, for all $u, v \epsilon U$, then h is commuting map on U.

Proof. we have

$$F([u, v]) + h([u, v]) + (u\ o\ v) = 0, for\ all\ u, v, \epsilon U. \tag{3.17}$$

Replacing v by $2vu$ in equations (3.18) and using the fact that $CharR \neq 2$, we obtain that

$$F([u, vu]) + h([u, vu]) + (u\ o\ vu) = 0, for\ all\ u, v \epsilon\ U.$$

$$F([u, v])u + h([u, v])u + (u\ o\ v)u = 0, for\ all\ u, v \epsilon\ U.$$

$$F([u, v])h(u) + F([u, v])u + [u, v]h(u) + h([u, v])h(u) + h([u, v])u + [u, v]h(u) + (u\ o\ v)u = 0,$$

$$F[u, v](h(u) + u) + [u, v]h(u) + h([u, v])(h(u) + u) + [u, v]h(u) + (u\ o\ v)u = 0, for\ all\ u, v \epsilon\ U.$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x)=0$, for all $x \epsilon U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F([u, v]u) + [u, v]h(u) + h([u, v]u) + [u, v]h(u) + (u\ o\ v)u = 0, for\ all\ u, v \epsilon U.$$

$$F([u, v]u) + h([u, v]u) + 2[u, v]h(u) + (u\ o\ v)u = 0, for\ all\ u, v \epsilon U.$$

Using the given hypothesis, the above relation yields that $2[u, v]h(u) = 0$, for all $u, v \epsilon\ U$. Since R is semi prime ring with $CharR \neq 2$,

$$[u, v]h(u) = 0, for\ all\ u, v \epsilon U. \tag{3.18}$$

Proceeding in the same manner as in the proof of Theorem 3.1, we obtain the required result.

Theorem 3.9. Let R be a free semi-prime ring with *Char*R $\neq 2$ and U a nonzero Lie ideal of R. Suppose that R admits a right generalized homoderivation F associated with a homoderivation h of R such that $h(U) \subseteq U$. If $F(u\ o\ v) + h(u\ o v) + (u\ o v) = 0$, for all $u, v \epsilon U$, then h is commuting map on U.

Proof. we have,

$$F(u\ o\ v) + h(u\ o\ v) + (u\ o\ v) = 0, for\ all\ u, v \epsilon U. \tag{3.19}$$

Replacing v by $2vu$ in equations (3.20) and using the fact that $CharR \neq 2$, we obtain that

$$F(u\ o\ vu) + h(u\ o\ vu) + (u\ o\ vu) = 0, for\ all\ u, v \epsilon U.$$

$$F((u\ o\ v)u - v[u, u]) + h((u\ o\ v)u - v[u, u]) + ((u\ o\ v)u - v[u, u]) = 0, for\ all\ u, v \epsilon U.$$

i.e.

$$F\left((u\,o\,v)u\right) + h\left((u\,o\,v)u\right) + (u\,o\,v)u = 0, for\ all\ u, v \epsilon U.$$

$$F(uov)h(u) + F(uov)u + (uov)h(u) + h(uov)h(u) + h(uov)u + (uov)h(u) + (uov)u = 0,$$

$$F(uov)(h(u) + u) + (uov)h(u) + h(uov)(h(u) + u) + (uov)h(u) + (uov)u = 0,$$

Since h is zero-power valued on U, there exists an integer $n(x) > 1$ such that $h^{n(x)}(x)=0$, for all $x \epsilon U$. Replacing u by $u—h(u) + h^2(u) + ... + (—1)^{n(u-1)}h^{n(u)-1}(u)$ in the above equation, we get

$$F(u\,o\,v)u + (u\,o\,v)h(u) + h(u\,o\,v)u + (u\,o\,v)h(u) + (u\,o\,v)u = 0, for\ all\ u, v \epsilon U.$$

Using the given hypothesis, the above relation yields that $2[u \text{ o } v]h(u) = 0$, for all $u, v \epsilon U$. Since R is a semi prime ring with $CharR \neq 2$,

$$(u\,o\,v)h(u) = 0, for\ all\ u, v \epsilon U. \tag{3.20}$$

Again, replacing v by $2wv$ in equation (3.20) and using the fact that $CharR \neq 2$,we get $(uov)h(u) = 0$, for all $u, v \epsilon U$ which gives that $(w(u \text{ o } v)+[u, w]v)h(u) = 0$, for all $u, v, w \epsilon U$, i.e., $w(u \text{ o } v)h(u) + [u, w]vh(u) = 0$, for all $u, v, w \epsilon U$. Using equation (3.20), the above relation yields that $[u, w]vh(u) = 0$, for all $u, v, w \epsilon U$. Now follow the proof of Theorem 3.1, we get the required result.

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