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MAX-MIN MATRIX OF INTUITIONISTIC FUZZY GRAPH STRUCTURE

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ABSTRACT

The Max- Min Matrix of an intuitionistic fuzzy graph structure (IFGS) $\tilde{G}$ is introduced and derived as M($\tilde{G}$) in this paper. Subsequently, the expressions for the coefficients of the characteristic polynomial of M($\tilde{G}$) are also explained explicitly.

KEYWORDS:

Max-Min Matrix, Max degree matrix, Min degree matrix.

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1. **INTRODUCTION** With the introduction of fuzzy sets by Prof. Zadeh [8] and intuitionistic fuzzy sets by Atanassov [5], the graph structure was discussed by Sampathkumar [2]. Dinesh and Ramakrishnan [17] contributed fuzzy graph structure. The notion of intuitionistic fuzzy graph structure (IFGS) $\tilde{G}$= ($A,B_1,B_2,....,B_k$) are defined and introduced by the author in [6] ,[7] and [13]. In this paper, Max- Min Matrix of intuitionistic fuzzy graph structure is discussed. Then the coefficients of the characteristic polynomial of M($\tilde{G}$) are also defined and explained.

2. **II PRELIMINARIES**

Some definitions and results that are necessary in this paper are reviewed,

Definition (2.1) [11]: A graph *G* is a pair of set (V, E), denoted by *G* = (V, E), where V is a set of vertices and E is a set of edges. Each edge in E is a pair of vertices in V. Each edge is associated with a set consisting of either one or two vertices called its endpoints.

Definition (2.2) [11]: An edge whose endpoints are the same is called a loop.

Definition (2.3) [11]: A graph without loops and parallel edges is called a simple graph.

Definition (2.4) [2]: $G = (V,R_1,R_2,...,R_k)$ is a graph structure if V is a non empty set and $R_1, R_2,...,R_k$ are relations on V which are mutually disjoint such that each R_i, i=1,2,3,...,k, is symmetric and irreflexive.

Definition (2.5) [6]: Let $G = (V, R_1, R_2,..., R_k)$ be a graph structure and let A be an intuitionistic fuzzy subset (IFS) on V and $B_1,B_2,..., B_k$ are intuitionistic fuzzy relations (IFR) on V which are mutually disjoint, symmetric and irreflexive such that

$\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v)$ $\forall\, u,v \in V$ and $i = 1,2, ..., k$.

Then $\tilde{G} = (A, B_1, B_2,...,B_k)$ is an intuitionistic fuzzy graph structure (IFGS) of G.

III. MAX - MIN MATRIX OF AN INTUITIONISTIC FUZZY GRAPH STRUCTURE

Definition (3.1): Let $\tilde{G} = (A,B_1,B_2,....,B_k)$ be an intuitionistic fuzzy graph structure (IFGS) of G for each vertex *i*. Define $\alpha_p(j)$ and $\beta_p(j)$ as follows

$$\alpha_p(j) = \max\{\ \mu_{B_p}(i\,j), \forall\ i\ \},\ p = 1, 2, ..., k.$$
$$\alpha_1(j) = \max\{\ \mu_{B_1}(1j),\ \mu_{B_1}(2j),\ \mu_{B_1}(3j), ..., \mu_{B_1}(k\,j)\ \},$$
$$\alpha_2(j) = \max\{\ \mu_{B_2}(1j),\ \mu_{B_2}(2j),\ \mu_{B_2}(3j), ..., \mu_{B_2}(k\,j)\ \},$$
$$\alpha_3(j) = \max\{\ \mu_{B_3}(1j),\ \mu_{B_3}(2j),\ \mu_{B_3}(3j), ..., \mu_{B_3}(k\,j)\ \},$$
$$..,$$
$$..,$$
$$\alpha_k(j) = \max\{\ \mu_{B_k}(1j),\ \mu_{B_k}(2j),\ \mu_{B_k}(3j), ..., \mu_{B_k}(k\,j)\ \}$$

and similarly,

$$\beta_p(j) = \min\{ \nu_{B_p}(i\,j), \forall\ i \}, p = 1, 2, ..., k.$$

And

$$\beta_1(j) = \min\{ \nu_{B_1}(1j),\ \nu_{B_1}(2j),\ \nu_{B_1}(3j), ..., \nu_{B_1}(kj) \},$$
$$\beta_2(j) = \min\{ \nu_{B_2}(1j),\ \nu_{B_2}(2j),\ \nu_{B_2}(3j), ..., \nu_{B_2}(kj) \},$$
$$\beta_3(j) = \min\{ \nu_{B_3}(1j),\ \nu_{B_3}(2j),\ \nu_{B_3}(3j), ..., \nu_{B_3}(kj) \},$$
$$..,$$
$$..,$$
$$\beta_k(j) = \min\{ \nu_{B_k}(1j),\ \nu_{B_k}(2j),\ \nu_{B_k}(3j), ..., \nu_{B_k}(kj) \}$$

where $\mu_{B_p}(i\,j)$ represents the strength of relationship between u_i and u_j and $\nu_{B_p}(i\,j)$ represents the strength of non-relationship between u_i and u_j.

Remark (3.2): For convenience, here some notations are used.

$\mu_{B_p}(u_i, u_j)$ is denoted by $\mu_{B_p}(i\,j)$ and $\nu_{B_p}(u_i, u_j)$ by $\nu_{B_p}(i\,j)$, $\forall\ p = 1, 2, ..., k.$.

Definition (3.3): Let $\tilde{G}$ = (A,B_1,B_2,….,B_k) be given IFGS of G. Two vertices u_i and u_j are mutually adjacent if there is an edge between u_i and u_j i.e. there is an edge from u_i to u_j and there is an edge from u_j to u_i .

Definition (3.4): Let $\tilde{G}$ = (A,B_1,B_2,….,B_k) be an IFGS of G. Three vertices u_i , u_j and u_p are said to be cyclic if there is an edge from u_i to u_j , from u_j to u_p and from u_p to u_i .

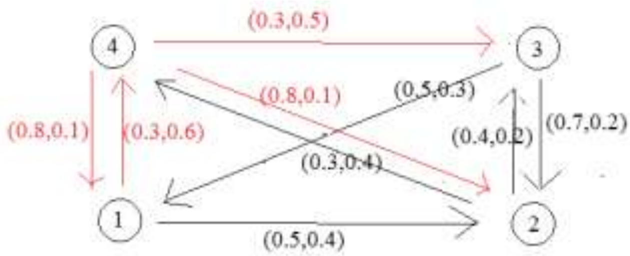
Definition (3.5): Let $\tilde{G}$ = (A,B_1,B_2,….,B_k) be an IFGS of G . The B_p-Max-Min Matrix Of IFGS $\tilde{G}$ is defined as $M_p(\tilde{G})$=[(t_{pij}, m_{pij})] , where t_{pij}= 0, m_{pij}= 0 ∀ i = j and for i ≠ j,

$$t_{pij} = \begin{cases} \max(\alpha_p(i), \alpha_p(j)) & \text{if } \mu_{B_p}(i\,j) \neq 0, \forall \text{ p} \\ 0 & \text{if } \mu_{B_p}(i\,j) = 0, \forall \text{ p} \end{cases} \text{ and } m_{pij} = \begin{cases} \min(\beta_p(i), \beta_p(j)) & \text{if } \nu_{B_p}(i\,j) \neq 0, \forall \text{ p} \\ 0 & \text{if } \nu_{B_p}(i\,j) = 0, \forall \text{ p} \end{cases}$$

Definition (3.6): Let $\tilde{G}$ = (A,B_1,B_2,….,B_k) be an IFGS of G . The Max-Min Matrix Of $\tilde{G}$ is defined as M($\tilde{G}$)=[(max(t_{pij}), min(m_{pij}))] =[(t_{ij}, m_{ij})].

Definition (3.7): Max-Min Matrix Of $\tilde{G}$ consists of two different matrices one containing the entries as membership values and the other containing the entries as non-membership values as M($\tilde{G}$)=[(max(t_{pij}), min(m_{pij}))] =[(t_{ij}, m_{ij})] where the matrix containing the entries as membership values are termed as **Max degree matrix of intuitionistic fuzzy graph structure** and is denoted by T= [t_{ij}] and the matrix containing the entries as non-membership values are termed as **Min degree matrix of intuitionistic fuzzy graph structure** and is denoted by N= [m_{ij}].

Example (3.8): Take example of link structure of a website and represent it by directed IFGS. The links are considered as vertices and the path between the links are considered as edges. The strength of each edge is taken as the membership value and the non- strength as non-membership value. Suppose an IFGS $\tilde{G} = (A, B_1, B_2)$ as displayed in the following diagram.



$M(\tilde{G})=[(t_{ij}, m_{ij})]$,

$$\therefore \; M(\tilde{G}) = \begin{bmatrix} (0,0) & (t_{12},m_{12}) & (t_{13},m_{13}) & (t_{14},m_{14}) \\ (t_{21},m_{21}) & (0,0) & (t_{23},m_{23}) & (t_{24},m_{24}) \\ (t_{31},m_{31}) & (t_{32},m_{32}) & (0,0) & (t_{34},m_{34}) \\ (t_{41},m_{41}) & (t_{42},m_{42}) & (t_{43},m_{43}) & (0,0) \end{bmatrix} \quad \text{where}$$

$$T = \begin{bmatrix} 0 & t_{12} & t_{13} & t_{14} \\ t_{21} & 0 & t_{23} & t_{24} \\ t_{31} & t_{32} & 0 & t_{34} \\ t_{41} & t_{42} & t_{43} & 0 \end{bmatrix} \quad \textit{and} \quad N = \begin{bmatrix} 0 & m_{12} & m_{13} & m_{14} \\ m_{21} & 0 & m_{23} & m_{24} \\ m_{31} & m_{32} & 0 & m_{34} \\ m_{41} & m_{42} & m_{43} & 0 \end{bmatrix}$$

$$\text{Here in this example,} \quad M_1(\tilde{G}) = \begin{bmatrix} (0,0) & (0.7,0.1) & (0.5,0.1) & (0.5,0.4) \\ (0.7,0.1) & (0,0) & (0.7,0.1) & (0.7,0.1) \\ (0.5,0.2) & (0.7,0.2) & (0,0) & (0.4,0.2) \\ (0.5,0.4) & (0.7,0.1) & (0.4,0.2) & (0,0) \end{bmatrix}$$

$$\text{and } M_2(\tilde{G}) = \begin{bmatrix} (0,0) & (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.1) & (0,0) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.1) & (0.8,0.1) & (0,0) & (0.3,0.5) \\ (0.8,0.1) & (0.8,0.1) & (0.3,0.5) & (0,0) \end{bmatrix}$$

$$\therefore \ M(\tilde{G}) = \begin{bmatrix} (0,0) & (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.1) & (0,0) & (0.8,0.1) & (0.8,0.1) \\ (0.8,0.1) & (0.8,0.1) & (0,0) & (0.4,0.2) \\ (0.8,0.1) & (0.8,0.1) & (0.4,0.2) & (0,0) \end{bmatrix} \quad \text{where}$$

$$T = \begin{bmatrix} 0 & 0.8 & 0.8 & 0.8 \\ 0.8 & 0 & 0.8 & 0.8 \\ 0.8 & 0.8 & 0 & 0.4 \\ 0.8 & 0.8 & 0.4 & 0 \end{bmatrix} \quad and \quad N = \begin{bmatrix} 0 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0 \end{bmatrix}$$

Result (3.9): Let $M(\tilde{G})=[(t_{ij}, m_{ij})]$, where $T=[t_{ij}]$, $N=[m_{ij}]$. The characteristic polynomial of T and N is an equation of order n if G has n vertices.

Note (3.10): In this paper, only the polynomial of order 4 will be used.

Remark (3.11): The characteristic polynomial of T of order 4 is $a_0\lambda^4-a_1\lambda^3+a_2\lambda^2-a_3\lambda+a_4=0$, where $a_0=1$, $a_1=tr(T)$, $a_2 = \frac{1}{2}\left[(tr(T))^2 - (tr(T^2)\right]$, $a_3 = \frac{1}{6}\left[(tr(T))^3 - 3(tr(T^2)tr(T) + 2tr(T^3)\right]$, $a_4=det(T)$.

Remark (3.12): The characteristic polynomial of matix N as $b_0\delta^4- b_1\delta^3 +b_2\delta^2- b_3\delta+ b_4= 0$, where $b_0=1$, $b_1= tr(N)$, $b_2 = \frac{1}{2}\left[(tr(N))^2 - (tr(N^2)\right]$, $b_3 = \frac{1}{6}\left[(tr(N))^3 - 3(tr(N^2)tr(N) + 2tr(N^3)\right]$, $b_4=det(N)$.

Now to see the explicit expressions for the coefficients of a_2, a_3, b_2 and b_3.

Lemma (3.13): In the characteristic polynomial of T , $a_2 = -\sum_{1\le i<j\le n} (t_{ij})^2$ such that the vertices u_i and u_j are mutually adjacent.

Proof: Generally $a_2 = \frac{1}{2}\left[(tr(T))^2 - (tr(T^2)\right]$, (from Remark (3.11),)

If the vertices u_i and u_j are mutually adjacent, then $t_{ij} = t_{ji}$ otherwise any one of t_{ij} or t_{ji} will be zero.

$$\therefore \ a_2 = \sum_{1\le i<j\le n} \begin{vmatrix} 0 & t_{ij} \\ t_{ij} & 0 \end{vmatrix} = \begin{vmatrix} 0 & t_{12} \\ t_{21} & 0 \end{vmatrix} + \begin{vmatrix} 0 & t_{13} \\ t_{31} & 0 \end{vmatrix} + \begin{vmatrix} 0 & t_{14} \\ t_{41} & 0 \end{vmatrix} + \begin{vmatrix} 0 & t_{23} \\ t_{32} & 0 \end{vmatrix} + \begin{vmatrix} 0 & t_{24} \\ t_{42} & 0 \end{vmatrix} + \begin{vmatrix} 0 & t_{34} \\ t_{43} & 0 \end{vmatrix}$$

$$= -[t_{12} t_{21}+ t_{13} t_{31}+ t_{14} t_{41}+ t_{23} t_{32}+ t_{24} t_{42}+ t_{34} t_{43}] \quad \text{---- (1)}$$

$\therefore a_2 = -\sum_{1\le i<j\le n} (t_{ij})^2$ if the vertices u_i and u_j are mutually adjacent.

Cor (3.14): In the characteristic polynomial of N , $b_2 = \sum_{1 \le i \prec j \le n} (m_{ij})^2$ such that the vertices u_i and u_j are mutually adjacent.

Proof: Generally $b_2 = \frac{1}{2}\left[(tr(N))^2 - (tr(N^2)\right]$, (from Remark (3.12),)

If the vertices u_i and u_j are mutually adjacent, then $m_{ij} = m_{ji}$ otherwise any one of m_{ij} or m_{ji} will be zero.

$$\therefore\ b_2 = \sum_{1 \le i < j \le n} \begin{vmatrix} 0 & m_{ij} \\ m_{ij} & 0 \end{vmatrix} = \begin{vmatrix} 0 & m_{12} \\ m_{21} & 0 \end{vmatrix} + \begin{vmatrix} 0 & m_{13} \\ m_{31} & 0 \end{vmatrix} + \begin{vmatrix} 0 & m_{14} \\ m_{41} & 0 \end{vmatrix} + \begin{vmatrix} 0 & m_{23} \\ m_{32} & 0 \end{vmatrix} + \begin{vmatrix} 0 & m_{24} \\ m_{42} & 0 \end{vmatrix} + \begin{vmatrix} 0 & m_{34} \\ m_{43} & 0 \end{vmatrix}$$

$= - [m_{12} m_{21} + m_{13} m_{31} + m_{14} m_{41} + m_{23} m_{32} + m_{24} m_{42} + m_{34} m_{43}]$ ---- (1)

$\therefore b_2 = \sum_{1 \le i \prec j \le n} (m_{ij})^2$ if the vertices u_i and u_j are mutually adjacent.

Lemma (3.15): In the characteristic polynomial of T , $a_3 = \sum_{1 \le i \prec j \le n} t_{ij} t_{jp} t_{pi}$ where the summation is taken over all i, j and p if the vertices u_i , u_j and u_p are cyclic in $\tilde{G}$

Proof: From Remark (3.11), $a_3 = \frac{1}{6}\left[(tr(T))^3 - 3(tr(T^2)tr(T) + 2tr(T^3)\right]$,

$$\therefore\ a_2 = \sum_{1 \le i < j < p \le n} \begin{vmatrix} t_{ii} & t_{ij} & t_{ip} \\ t_{ji} & t_{jj} & t_{jp} \\ t_{pi} & t_{pj} & t_{pp} \end{vmatrix}$$

$= - [t_{12} t_{23} t_{31} + t_{12} t_{24} t_{41} + t_{13} t_{34} t_{41} + t_{13} t_{32} t_{21} + t_{14} t_{43} t_{31} + t_{14} t_{42} t_{21} + t_{23} t_{34} t_{42} + t_{24} t_{43} t_{32}]$ --- (2)

If the vertices u_i , u_j and u_p are cyclic in $\tilde{G}$, then $a_3 = \sum_{1 \le i < j < p \le n} t_{ij} t_{jp} t_{pi}$ otherwise any one of t_{ij} or t_{jp} or t_{pi} will be zero.

$\therefore$ (2) becomes $a_3 = \sum_{1 \le i < j < p \le n} t_{ij} t_{jp} t_{pi}$ where the summation is taken over all i, j and p if the vertices u_i , u_j and u_p are cyclic in $\tilde{G}$.

Cor (3.16): In the characteristic polynomial of N , $b_3 = \sum_{1 \le i \prec j \le n} m_{ij} m_{jp} m_{pi}$ where the summation is taken over all i, j and p if the vertices u_i , u_j and u_p are cyclic in $\tilde{G}$

Proof: From Remark (3.12), $b_3 = \frac{1}{6}\left[(tr(N))^3 - 3(tr(N^2)tr(N) + 2tr(N^3)\right]$,

$$\therefore\ b_2 = \sum_{1\le i<j<p\le n} \begin{vmatrix} m_{ii} & m_{ij} & m_{ip} \\ m_{ji} & m_{jj} & m_{jp} \\ m_{pi} & m_{pj} & m_{pp} \end{vmatrix}$$

$= -[m_{12}m_{23}m_{31} + m_{12}m_{24}m_{41} + m_{13}m_{34}m_{41} + m_{13}m_{32}m_{21} + m_{14}m_{43}m_{31} + m_{14}m_{42}m_{21} + m_{23}m_{34}m_{42} + m_{24}m_{43}m_{32}]$ --- (2)

If the vertices u_i , u_j and u_p are cyclic in $\tilde{G}$, then $b_3 = \sum_{1\le i<j\le n} m_{ij}m_{jp}m_{pi}$ otherwise any one of m_{ij} or m_{jp} or m_{pi} will be zero.

$\therefore$ (2) becomes $b_3 = \sum_{1\le i<j\le n} m_{ij}m_{jp}m_{pi}$ where the summation is taken over all i, j and p if the vertices u_i , u_j and u_p are cyclic in $\tilde{G}$.

IV. CONCLUSION

In the present paper, the max-min matrix of intuitionistic fuzzy graph structure is introduced. It can be further used in the development of new concepts related to Max- min matrix of intuitionistic fuzzy graph structure.

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