International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 4, April 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

A STUDY ONMARKOV'S INEQUALITY AND ZERO SET POLYNOMIAL FACTOR MAPPINGS APPLICATION

Vikas Kumar[1], Dr. Rajeev Jha[2]

Department of Mathematics

[1,2]Shri Venkateshwara University, Gajraula (Uttar Pradesh)

Abstract

This paperdiscus theMarkov's inequality and zero set polynomial factors mappings application. Markov's inequality is a sure indicator of the subordinate standard of a polynomial in terms of the degree and standard of this polynomial. It has numerous interesting applications in estimation theory, value theory theory and examination (for example, Sobolev's inequalities or Whitney-type expansion problems). One of the reasons for this research is to give an answer to an old problem, among which Baran and Plesniak, and the invariance of Markov inequality under polynomial mappings (polynomial images). We also address the issue of protecting Markov's inequality when we take polynomial pre-images. In conclusion, we give an appropriate condition for a subset of a Markov set to be a Markov set..

1. OVERVIEW

The previous inequality for quadratic polynomials was found by the praised scientist Mendeleev. Markov inequality and different speculations found numerous applications in the theory hypothesis, research theory of useful functions, but also in other branches of science (for example, materials science or science). Currently there is such a broad formulation of Markov's inequalities in writing that goes beyond the scope of this research to obtain a complete bibliography.

In fact, the previous inequality for polynomials of the second degree was discovered by the famous chemist Mendeleev. Markov inequality and its various generalizations have found many applications in the theory of approximation, analysis, theory of the constructive function, but also in other branches of science (for example, in physics or chemistry). Now there is such ample literature on Markov type inequalities that are beyond the scope of this article provides a comprehensive bibliography. We mention only some works that are more closely related to our work (with emphasis on those dealing with the generalizations of Markov inequality in sets that admit cusps), we must emphasize here that this document owes a great debt especially to the work of Pawłucki and Pleśniak, because they laid the foundations of the theory of polynomial inequalities in "domesticated" sets (for example, semihialgia) with cusps.

From the point of view of the applications, it is important that the constant $(\deg P)^2$ in the Markov inequality does not grow too fast (ie, polynomial) the degree of the polynomial P. For this reason, the concept of a set is widely Markov studied.

Markov Inequality: The inequality

$$\|p'\|_{[-1,1]} \leq n^2 \|p\|_{[-1,1]}$$

Holds for every $p \in \mathcal{P}_n$

Bernstein Inequality: The inequality

$$|p'(y)| \leq \frac{n}{\sqrt{1-y^2}} \|p\|_{[-1,1]}$$

Holds for every $p \in \mathcal{P}_n$ and $y \in (-1, 1)$.

In the two previous theorems and in the whole thesis $\|\cdot\|_A$ denotes the supreme norm on $A \subset \mathbb{R}$.

(Markov) If P is a polynomial of one variable, then

$$\|P'\|_{[-1,1]} \leq (\deg P)^2 \|P\|_{[-1,1]}.$$

Moreover, this inequality is optimal, because for the Chebyshevpolynomials $T_n\ (n \in \mathbb{N}_0)$ we have $T_n'(1) = n^2$ and $\|T_n\|_{[-1,1]} = 1$.

Recall that

$$T_n(u) = \frac{1}{2}\left[\left(u + \sqrt{u^2-1}\right)^n + \left(u - \sqrt{u^2-1}\right)^n\right].$$

In fact, the previous inequality for polynomials of the second degree was discovered by the famous chemist Mendeleev. Markov inequality and its various generalizations have found many applications in the theory of approximation, analysis, theory of the constructive function, but also in other branches of science (for example, in physics or chemistry). Now there is such ample literature on Markov type inequalities that are beyond the scope of this article provides a comprehensive bibliography. We mention only a few jobs that are more closely related to our work (with emphasis on those related to the generalizations of Markov inequality in the sets that allow the cusps), for example [1-12].

Factoring Polynomials

It is engineered division can be utilized to check if a given x-value is a zero of a polynomial function (by restoring a zero leftover portion) and it can likewise be utilized to partition out a linear factor from that polynomial (abandoning one with a little degree polynomial). On account of this cozy connection between zeroes (of polynomial functions) and arrangements (of polynomial equations), the techniques utilized for "fathoming" polynomials can be connected equally well to finding the entire

International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 4, April 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

factorization of a polynomial. The real contrast between the "understanding" activities on the last page and the "figuring" practices on this page are that, for considering, we need to monitor any numbers that we partition off.

2. ZOLOTAREV POLYNOMIALS

General properties. Here we describe some properties of Zolotarev polynomials which are solutions to the pointwise Markov problem and which bear a certain similarity with one-parameter families from the other Markov-type problems.

Definition: A polynomial$Z_n \in P_n$is called Zolotarev polynomial ifit has at least n equioscillations on [−1, 1], i.e. if there exist n points

$-1 \le \tau_1 < \tau_2 < \cdots < \tau_{n-1} < \tau_n \le 1$

such that

$$(-1)^{n-i} Z_n(\tau_i) = \|Z_n\| = 1.$$

(1.1)

There are many Zolotarev polynomials, for example the Chebyshev poly-nomialsT_n and T_{n-1} of degree n and n −1, with n + 1 and n equioscillation points, respectively. One needs one parameter more to get uniqueness. A convenient parametrization (due to Voronovskaya) is through the value of the leading coefficient:

$$\frac{1}{n!} Z^{(n)} \equiv \theta \quad \Leftrightarrow \quad Z_n(x) := Z_n(x,\theta) := \theta x^n + \sum_{i=0}^{n-1} a_i(\theta) x^i .$$

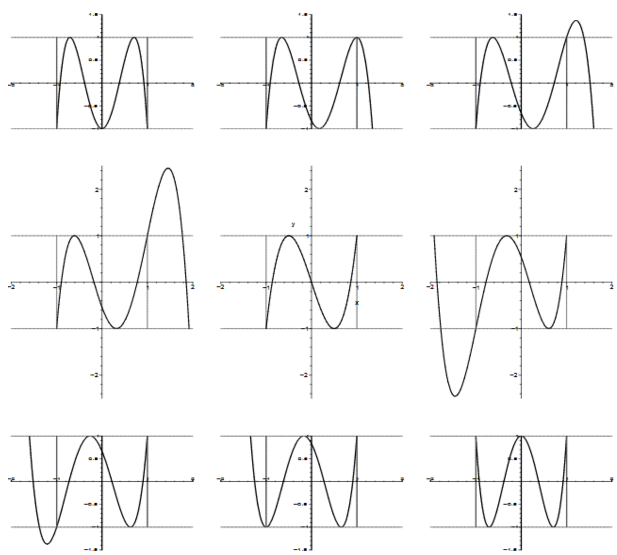
By Chebyshev's result, $\|p^{(n)}\| \le \|T_n^{(n)}\| \, \|p\|$, so the range of the parameter is

$-2^{n-1} \le \theta \le 2^{n-1}$.

As θ traverses the interval $[-2^{n-1}, 2^{n-1}]$, Zolotarev polynomials go through the following transformations:

$$-T_n(x) \to -T_n(ax+b) \to Z_n(x,\theta) \to T_{n-1}(x) \to Z_n(x,\theta) \to T_n(cx+d) \to T_n(x).$$

The next figure illustrates it for n = 4.



There are many other parametrizations in use. The classical one is based on the definition of Zolotarev polynomial as the polynomial that deviates least from zero among all polynomials of degree n with two leading coefficients fixed:

3. VARIATIONAL APPROACH

3.1 General considerations

Maximizing M_k over the one-parameter family. The following approach is perhaps the only one that can be applied to any problem of the Markov type in the sense that, initially, it does not rely on any particular properties of polynomials or splines or whatsoever. (It is another question whether it will work or not, sometimes it does, sometimes is does not.)

4. LIMITATIONS OF VARIATIONAL AND "SMALL-O" METHODS

All three authors – Bernstein, Tikhomirov and Bojanov – while using the small-oarguments, arrived actually, at exactly the same conclusion which was providedby V.Markov.

Theorem 1 The local extreme values of $M_k(\cdot)$ attained by a polynomial other than T_n are local minima, or, equivalently, all local maximal values of $M_k(\cdot)$ are attained by the Chebyshev polynomial $\pm T_n$.

The only difference is that V. Markov proved that $M_k(\cdot)$ indeed have local maxima and minima.

What is important in such a conclusion is that it shows that we cannot apply the variational or a "small-o" method to the Markov-type problem, unless we are sure that the local behaviour of $M_{k,F}(\cdot)$ follows the pattern given by the theorem above.

Example 1. Consider the Landau–Kolmogorov problem

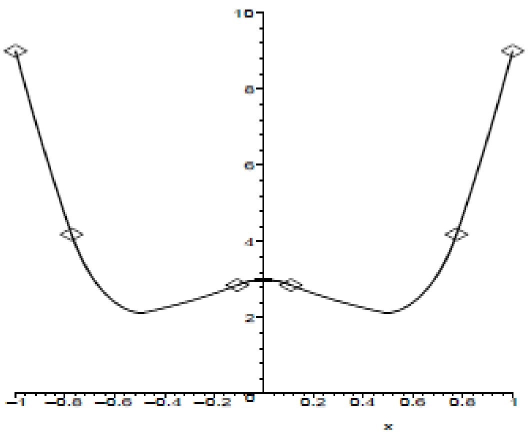
$$M_{k,\sigma}(z) = \sup_{f \in W_\infty^{n+1}(\sigma)} |f^{(k)}(z)|, \quad z \in [-1,1],$$

where

$$W_\infty^{n+1}(\sigma) = \{f : \|f\| \le 1, \|f^{(n+1)}\| \le \sigma\}.$$

For σ = 0 it reduces to the Markov problem for polynomials, hence for small σ, the pointwise bound $M_{k,\sigma}(z)$ should be close to the Markov pointwise bounds $M_k(z)$.

The function $M_k(z)$ has (n−k) local minima and (n−k −1) local maxima as illustrated on the graph below (for n = 3 and k = 1). Now, according to Pinkus' results [29], the Chebyshev-like function $T_* \in W_\infty^{n+1}(\sigma)$ that attains the value $M_{k,\sigma}(z)$ at z = 1 takes other values of $M_{k,\sigma}(z)$ only at a finite set of (n −k) points, and similarly for $\widehat{T}_*$ which is extremal for z = −1.As □ → 0, these points will tend to the ends of Zolotarev intervals (ξ_i) and (η_i), respectively, and we see that, for small σ, there are local maxima of $M_{k,\sigma}(\cdot)$ that are achieved by functions of Zolotarev type (the maximum at z = 0 on the figure).



Hence, for small σ, we cannot prove that T_* is the global solution using variational or "small-o" methods. It does not mean that this is not true, most likely it is, but we certainly need other methods to prove it. In fact, the same picture is true for any $\sigma > 0$ when the extremal functions for $z = 1$ and for $z = -1$ are two proper Zolotarev splines the function $M_{k,\sigma}(\cdot)$ is monotone on [0, 1] is not true, although it may still be true for $\sigma = \|T^{(n+1)}_{n+1,r}\|)$.

5. APPLICATIONS

The expression "spline" is used to allude to a wide class of functions that are used in applications requiring data interpolation and additionally smoothing. The data might be it is possible that one-dimensional or multi-dimensional. Spline functions for interpolation are ordinarily decided as the minimizers of reasonable measures of harshness (for example integral squared curvature) subject to the interpolation imperatives. Smoothing splines might be seen as generalizations of interpolation splines where the functions are resolved to limit a weighted combination of the normal squared approximation error over watched data and the harshness measure. For various significant meanings of the unpleasantness measure, the spline functions are observed to be finite dimensional in nature, which is the essential explanation behind their utility in computations and portrayal. For whatever is left of this section, we center totally around one-dimensional, polynomial splines and utilize the expression "spline" in this confined sense.

Polynomials are as often as possible used to encode information about some other question. The trademark polynomial of a matrix or linear administrator contains information about the administrator's eigenvalues. The negligible polynomial of an algebraic component records the easiest algebraic connection fulfilled by that component. The chromatic polynomial of a graph checks the quantity of appropriate colourings of that graph.

From the perspective of applications, it is essential that the constant $(\deg P)^2$ in Markov's inequality develops not very quick (that is, polynomially) concerning the degree of the polynomial P. This is the motivation behind why the concept of a Markov set is generally investigated.

6. CONCLUSION

This kind of set was introduced by Federer in the study of a set of Steiner Polynomial (very smooth and regular), the polynomial that calculates at r> 0 provides the d-dimensional measure of the tubular r of a given set. The main interest in a class of sets of this type is that under this hypothesis (instead of a high degree of smoothness) it is possible to recover the Steiner Polynomial coefficients as Radon measurements, the Curvature Measures.

The resulting mesh is given by coating points on some appropriately selected flat surfaces of d {Ω}. The result is shown using the regularity property of the function d {Ω in a small tubular neighborhood of X and the tangential Markov inequality for the sphere.

Further motivations. Finally, it is worth to recall that the (weighted) Bernstein Markov property it is a key tool in a series of probabilistic results regarding zeros of random polynomials and eigenvalues of random matrices, vector energy problems in the complex plane and large deviations of random arrays generated by a determinantal point process.

REFERENCES

[1]. Baran, M.: Markov inequality on sets with polynomial parametrization. Ann. Polon. Math. 60, 69–79 (1994)

[2]. Białas-Cież, L., Eggink, R.: Equivalence of the local Markov inequality and a Kolmogorov type inequality in the complex plane. Potential Anal. 38, 299–317 (2013)

[3]. Bos, L.P., Brudnyi, A., Levenberg, N.:(2010) On polynomial inequalities on exponential curves in Cn . Constr. Approx. 31, 139–147.

[4]. Brudnyi, A.: (2008) On local behavior of holomorphic functions along complex submanifolds of CN . Invent. Math. 173, 315–363.

[5]. Burns, D., Levenberg, N., Ma'u, S., Révész, S.:(2010) Monge–Ampère measures for convex bodies and Bernstein–Markov type inequalities. Trans. Am. Math. Soc. 362, 6325–6340.

[6]. Erdélyi, T., Kroó, A.: (2004). Markov-type inequalities on certain irrational arcs and domains. J. Approx. Theory 130, 111–122.

[7]. Fefferman, C., Narasimhan, R.:(1993) Bernstein's inequality on algebraic curves. Ann. Inst. Fourier (Grenoble) 43, 1319–1348.

[8]. Frerick, L., Jordá, E., Wengenroth, J.: (2011) Tame linear extension operators for smooth Whitney functions. J. Funct. Anal. 261, 591–603.

[9]. Goncharov, A.: (1996) A compact set without Markov's property but with an extension operator for C∞-functions. Studia Math. 119, 27–35.

[10]. Pawłucki, W., Pleśniak, W.: (1986)Markov's inequality and C∞functions on sets with polynomial cusps. Math. Ann. 275, 467–480.

[11]. Pleśniak, W.:(2006) Inégalité de Markov en plusieurs variables, Int. J. Math. Math. Sci. 14, 1–12.

[12]. Totik, V.: (2001) Polynomial inverse images and polynomial inequalities. Acta Math. 187, 139–160.