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Application of Normal distribution

By

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Application of Normal Distribution

1. **Abstruct :** The normal distribution was first discovered in 1733 by English mathematician DeMoivre who obtained this continuous distribution as a limiting case of the binomial distribution and applied it to problem arising in the game of chance. It was also known to Laplace, no later than 1774 but through a historical error, it was credited to Gauss, who made references to it in the beginning of 19th century, as the distribution of errors in Astronomy. Gauss used the normal curve to the theory of accidental measurement involved in the calculation of orbits of heavenly bodies. The main objective of this paper is to raise awareness of numerous applications of this wonderful distribution.

2. **Key Words :** Statistics, probability, distribution, normal distribution.

3. **Definition :** A random variable X is said to have a normal distribution. If its probability density function is

given by the law $f(x) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{(x-u)^2}{26^2}}$ $\quad -\infty < x < \infty$, $\quad -\infty < \mu < \infty$

here 6 is standard deviation and μ stands for mean and $z = \frac{x-\mu}{\sigma}$ is standard normal variate.

4. **Importance of Normal distribution :** Normal distribution is very useful in statistical theory because of the following reasons :

(i) Various sampling distribution : e.g. student 's 't', Snedecor'sF Chi-square distributions etc tend to normality for large samples. Many of the distributions occuring in practice. e.g. Binomial, Poisson, Hypergeometric distribution etc can be approximated by normal distribution.

(ii) A variable can be brought to normal form by simple transformation of the variable if it is not normally distributed.

(iii) If XN (μ, 6^2), then

$p(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$

Now $z = \frac{x - \mu}{6}$

When X = μ – 3σ

So, Z = = – 3

When X = μ + 3σ

Z == 3

So p (– 3 < Z < 3) = 0.9973

i.e. p (|Z| < 3) = 0.9973

= p (|Z| > 3) = 1 – 0.9973 = 0.0027

(iv) The distribution of sample mean, sample variance, etc tend to normality for large samples and so they can be best studied with the help of normal curves.

(v) The entire theory of small sample tests t, F, x^2 tests etc is based on the fundamental assumption that the parent population from which the samples have been drawn follow normal distribution.

(vi) Normal distribution finds large applications in statistical quality control in Industry for setting control of limites.

5. Application of Normal Distribution :

I. **In agriculture,** it is employed in agriculture to predict the yield. e.g. The mean yield of an acre plot is 662 kilos with s.d. 32 kilos. Assuming normal distribution, we have to calculate the number of plots in a batch of 1000 plots which are expected to have yield (i) over 700 kilos. (ii) below 650 kilos (iii) lowest yield of best 100 plots.

Let X denoted the yield in kilos for one acre. XN (μ, 6^2)

here μ = 662

6 = 32,

Normal variate Z is given by

$$Z = \frac{X - 662}{32}$$

The probability that a plot has a yield over 700 kilos is given by

p (X > 700) = p (Z > 1.19)

[because $z = \frac{700 - 662}{32} = 1.19$]

= p (0 < Z < ∞) – p (0 < Z < 1.19)

$= 0.5 - 0.3830$ (From table of normal distn)

$= 0.1170$

So, in a batch of 100 plots with yield over 700 kilos is given by $= 1000 \times 0.1170 = 117$

II. Probability that a plot will yield below 650 kilos is given by p (X < 650) = p (Z < – 0.38)

because $Z = \frac{650 - 662}{32} = -0.38$

So, $p\ (z < -0.38)$

$= p\ (-\infty < z < 0) - p\ (-0.38 < z < 0)$

$= 0.5 - p\ (0 < z < 0.38)$

$= 0.5 - 0.1480 = 0.352$

So, required number of plots $= 1000 \times 0.352 = 352$

The lowest yield say x of 100 plots is given by

$p\ (X < x) == 0.1$

$Z = \frac{X - \mu}{6} = \frac{x - 662}{32} = Z_1$ (ray)

So, $p\ (Z > Z_1) = 0.1$

$\Rightarrow p\ (Z > Z_1) = 0.1$

$\Rightarrow p\ (0 < z < Z_1) = 0.5 - 0.1 = 0.4$

$Z_1 = 1.28$ [From normal probability table]

So, $\frac{x - 662}{32} = 1.28$

$\Rightarrow x = 32 \times 1.28 + 662 = 702.96$

The lowest yield given by best 100 plots = 702.96 kilos

(ii) **In Examination :** In IGU, MEERPUR University, in a particular year, 1500 students appeared for examination. BAI (History). The mean marks obtained by students are 46 with standard deviation 6.

Assuming the marks to be normally distributed, we have to calculate the number of students who got more than 32 marks, i.e. passed, number of students who got more than 48 marks i.e. got First division and number of students who got 64 marks so got distinction. Here maximum marks are 80.

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Let X denotes the variable representing marks obtained by a student. i.e. XN (46, 36)

The Z, normal variate is given by

$$Z = \frac{X - 46}{6}$$

When, X = 32

$$Z = \frac{32 - 46}{6} = -2.33$$

$p\,[X > 32] = p\,[X > -2.33]$

$= p\,[-2.33 < z < 0] + p\,[0 < z < \infty]$

$= p\,[0 < z < 0.233] + 0.5$

$= 0.4901 + 0.5 = 0.9901$

So, number of students who passed

$= 1500 \times 0.9901 = 1485.151485$

When $x = 48$

$$Z = \frac{48 - 46}{6} = 0.33$$

$p\,[X > 48] = p\,[z > 0.33]$

$= p\,[0 < z < \infty] - p\,[0 < z < 0.33]$

$= 0.5 - 0.1293 = 0.3707$

So, Number of students who got first division

$= 0.3707 \times 1500 = 556.05 \cong 556$

When, X = 64

$$Z = \frac{64 - 46}{6} = 3$$

So, $p\,[X > 64] = p\,[z > 3]$

$= p\,[0 \le z \le \infty] - p\,[0 \le z \le 3]$

$= 0.5 - 0.4987 = 0.0013$

So, number of students who got more than 64 marks be distinction

$= 0.0013 \times 1500 = 1.95 \cong 2$

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In Army : It is found that mean height of Jawans in BSF is 66 inch with a standard deviation 4 inches. In a regiment of 20,000. We can estimate the number of Jawans over 78 inch, assuming the height's are normally distributed.

Let, x denote the height of a soldier. Then x will follow normal distribution with mean 66 inch and standard

deviation 4 inches. Let z be the standard normal variate.

Then, $z = \frac{x-66}{4}$

When, $x = 78$, then

$$z = \frac{78-66}{4} = \frac{12}{4} = 3$$

So, $p\ (x > 78) = p\ (z > 3)$

$= p\ (z \geq 0) - p\ (0 < z \leq 3)$

$= 0.5 - 0.4987$

$= 0.0013$

So, in a regiment of 20000, the number of Jawan with height over 78 inches

$= 0.0013 \times 20{,}000$

$= 26.0000 = 26$

Applications

Ex. (IV) **In Industry :**

(i) In a factory, there are 10,00000 workers whose monthly income are distributed normally with mean 12000 and standard deviation Rs. 1200. We can find the number of workers whose income exceed Rs. 12800.

Sol. Let x denotes monthly income of workers

$$Z = \frac{x-\mu}{6} = \frac{X-12000}{1200}$$

When, X = 11200

$$Z = \frac{12800-12000}{1200} = \frac{800}{1200} = \frac{2}{3} = 0.66$$

$p\ [z > 0.66] = p\ [0 < z < \infty] - p\ [0 < z \leq 0.66]$

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= 0.5 – 0.2454

= 0.2546

Number of workers whose income exceed 12800

= 0.2546 × 1000000

= 254600

(ii) A CFL manufacturers produces CFL with average span of 900 hours with standard deviation 120 hours. The manufacturers promises the customer that he return his money if CFL lasts for less than 540 hours. If he produces 10,00000 CFL in an year. What is number of CFL for which he has to return the money.

Let x denotes life span of CFL, assuming X to he normally distributed, we have z, the standard normal variate, given by $z = \frac{x-900}{120}$

When $x = 540$

$$Z = \frac{540-900}{120} = -3$$

$p\,[z < -3] = p\,[0 < z < \infty] - p\,[-3 < z < 0]$

$= p\,[0 < z < \infty] - p\,[0 < z < 3]$

because $p\,[-3 < z < 0] = p\,[0 < z < 3]$

= 0.5 – 0.4987 [from table of normal distribution]

= 0.0013

Number of CFL, for which he has to pay money back = 0.0013 × 10,00000 = 1300

5. **Conclusion :** It is obvious that normal distribution plays an important role in sampling theory. It is one of the most important distribution in probability and statistics. It usually works well with large data base. In the present paper, I have tried to apply normal distribution in various problems, arising in examination, in industry, in army, in agriculture. It is of immense use in our daily life problems.

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