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Super Contra Harmonic Mean Labeling of Graphs

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Abstract

The concept of mean labeling of graphs was introduced by S. Somasundaram and R. Ponraj [5] . We have introduced Contra Harmonic mean labeling of graphs [6] . Futher, the concept of Super mean labeling was introduced by R. Ponraj and D.Ramya [3]. In this paper we analyze the Super Contra Harmonic mean labeling for some path and cycle related graphs.

Keywords: Contra Harmonic mean labeling , Super Contra Harmonic mean labeling .

1. Introduction

Let G = (V,E) be a finite, simple , undirected graph with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notation we follow Harray [2].We will give a brief summary of definition and other information which are useful for the present investigation.

A graph G = (V,E) with p vertices and q edges is called a Contra Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0,1,…,q in such a way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$ with distinct edge labels . Then f is called a Contra Harmonic mean labeling of G.

Definition: 1.1

Let f: V(G)→{1,2,…,p+q} be an injective function. The induced edge labeling f*(e=uv) is defined by f*(e) = $\left\lceil \frac{(f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{(f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$, then f is called Super Contra Harmonic mean labeling if {f(V(G))}∪{f(e)/e∈E(G)} ={1,2,…,p+q}. A graph which admits Super Contra Harmonic mean labeling is called Super Contra Harmonic mean graph .

Definition 1.2 :

The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Theorem 1.3: Any Path is a Contra Harmonic mean graph.

Theorem 1.4: Any cycle is a Contra Harmonic mean graph.

Theorem 1.5: Any Comb is a Contra Harmonic mean graph.

Theorem 1.6: Any Crown is a Contra Harmonic mean graph.

2.Main Results:

Theorem:2.1

A path admits Super Contra Harmonic mean labeling.

Proof:

Let P_n be a path $v_1, v_2 \ldots, v_n$ with edge set $E=\{u_iu_{i+1}/1\leq i\leq n-1\}$

Define a function $f:V(P_n)\rightarrow\{1,2,\ldots,p+q\}$ by

$f(u_i)=2i-1,\ 1\leq i\leq n$

with edge labels

$f(u_iu_{i+1})=2i,\ 1\leq i\leq n-1$

Here the vertices and edges together get distinct labels $\{1,2,\ldots,p+q\}$

Hence a path P_n is a Super Contra harmonic mean labeling.

A Super Contra Harmonic mean labeling of P_6 is

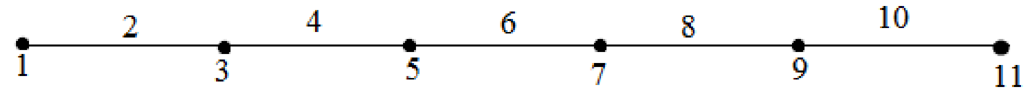

Figure 1

Theorem:2.2

$P_n\odot K_1$ admits Super Contra Harmonic mean labeling.

Proof:

Let $G = P_n\odot K_1$ be a comb obtained from a path P_n with vertices $u_1,\ldots,u_n$ and by joining the vertex u_i to v_i, $1\leq i\leq n$.

Define a function $f:V(G)\rightarrow\{1,2,\ldots,p+q\}$ by

$f(u_i) = 4i-3,\ 1\leq i\leq n$

$f(v_i)) = 4i-1,\ 1\leq i\leq n$

with edge labels

$f(u_iu_{i+1}))=4i,\ 1\leq i\leq n-1$

$f(u_iv_i)=4i-2,\ 1\leq i\leq n$

Here the vertices and the edges together get distinct labels $\{1,2,\ldots,p+q\}$.

Hence $P_n\odot K_1$ admits Super Contra harmonic mean labeling.

A Super Contra Harmonic mean labeling of $P_6\odot K_1$ is

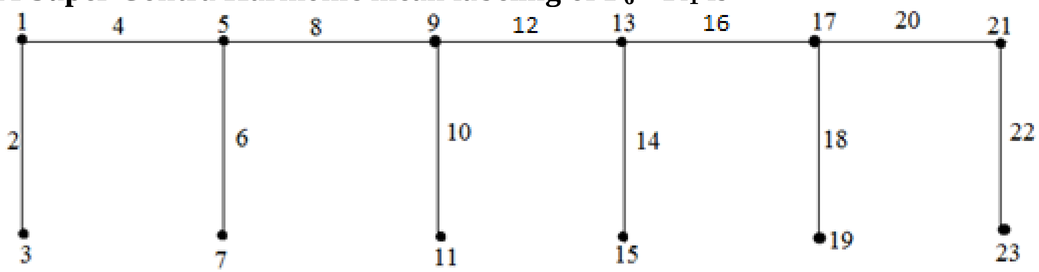

Figure 2

Theorem:2.3

$P_n \odot K_{1,2}$ admits a Super Contra Harmonic mean labeling.

Proof:

Let G = $P_n \odot K_{1,2}$ be a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$.

Define $f:V(G)\rightarrow\{1,2,\ldots,p+q\}$ by

$f(u_i)=6i-3,\ 1\le i\le n$

$f(v_i)=6i-5,\ 1\le i\le n$

$f(w_i)=6i-1,\ 1\le i\le n$

with edge labels

$f(u_iu_{i+1})=6i,\ 1\le i\le n-1$

$f(u_iv_i)=6i-4\ ,\ 1\le i\le n$

$f(u_iw_i)=6i-2\ ,\ 1\le i\le n$

Here the vertex and the edge labels are distinct

Hence $P_5 \odot K_{1,2}$ admits Super Contra Harmonic mean labeling.

A Super Contra Harmonic mean labeling of $P_4 \odot K_{1,2}$ is

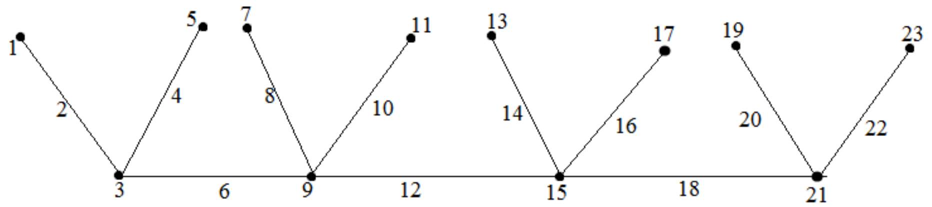

Figure 3

Theorem2.4

$P_n \odot K_{1,3}$ admits a Super Contra Harmonic mean labeling.

Proof:

Let G= $P_n \odot K_{1,3}$ be a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,3}$.

Define f: $V(G)\rightarrow\{1,2,...,p+q\}$ by

$f(u_1)=1,\ f(u_2)=8i-5,\ 2\le i\le n$

$f(v_1)=3,\ f(v_2)=8,\ f(v_i)=8i-7,\ 3\le i\le n$

$f(w_i)=8i-3,\ 1\le i\le n$

$f(x_i)=8i-1,\ 1\le i\le n$

with edge labels

$f(u_1u_2)=10,\ f(u_iu_{i+1})=8i,\ 2\le i\le n-1$

$f(u_1v_1)=2,\ f(u_2v_2)=9,\ f(u_iv_i)=8i-6,\ 3\le i\le n$

$f(u_iw_i)=8i-4,\ 1\le i\le n$

$f(u_ix_i)=8i-2,\ 1\le i\le n$

The vertices and the edges get distinct labels $\{1,2,\ldots,p+q\}$.

Hence $P_n \odot K_{1,3}$ admits a Super Contra Harmonic mean labeling .

A Super Contra Harmonic mean labeling of $P_5 \odot K_{1,3}$ is

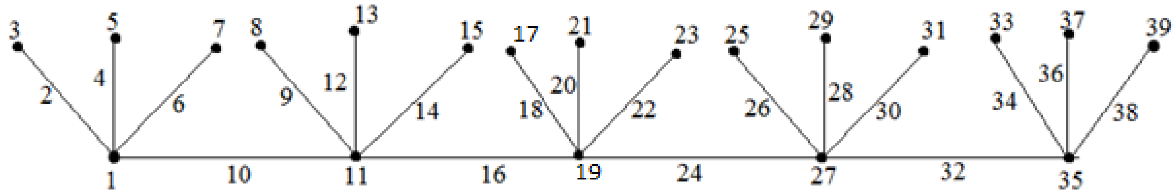

Figure 4

Theorem:2.5

$(P_n \odot K_1) \odot K_{1,2}$ admits Super Contra Harmonic mean labeling

Proof:

Let G = $(P_n \odot K_1) \odot K_{1,2}$ be a graph obtained by attaching the central vertex of $K_{1,2}$ at each pendent vertex of a comb.

Define $f:V(G) \rightarrow \{1,2,\ldots,p+q\}$ by

$f(u_1)=1$, $f(u_2)=9$, $f(u_i)=8i-1$, $3 \le i \le n$

$f(v_1)=3$, $f(v_2)=11$, $f(v_i)=8i-6$, $3 \le i \le n$

$f(w_1)=5$, $f(w_2)=13$, $f(w_i)=8i-8$, $3 \le i \le n$

$f(x_1)=7$, $f(x_2)=15$, $f(x_i)=8i-2$, $3 \le i \le n$

with edges labels

$f(u_1u_2)=8$, $f(u_iu_{i+1})=8i+3$, $2 \le i \le n-1$

$f(u_1v_1)=2$, $f(u_2v_2)=10$, $f(u_iv_i)=8i-4$, $3 \le i \le n$

$f(w_1v_1)=4$, $f(w_2v_2)=12$, $f(w_iv_i)=8i-7$, $3 \le i \le n$

$f(v_1x_1)=6$, $f(v_2x_2)=14$, $f(v_ix_i)=8i-3$, $3 \le i \le n$

The vertices and edges get distinct labels $\{1,2,\ldots,p+q\}$.

Hence $(P_n \odot K_1) \odot K_{1,2}$ admits Super Contra Harmonic mean labeling.

A Super Contra Harmonic mean labeling of $(P_5 \odot K_1) \odot K_{1,2}$ is

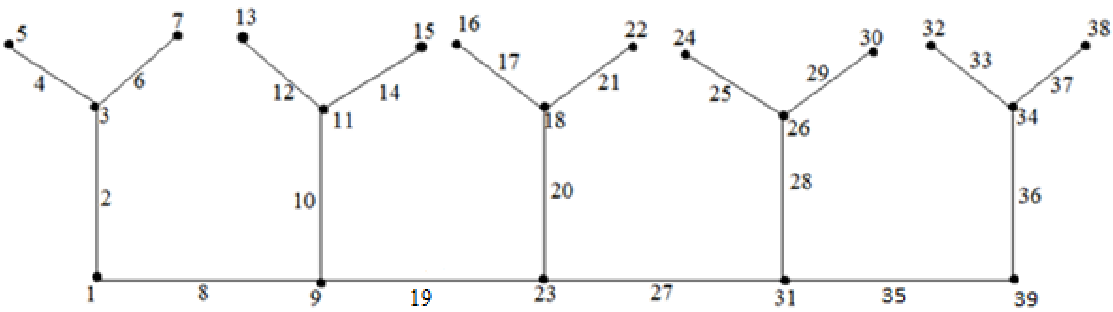

Figure 5

Theorem:2.6

$(P_n \odot K_1) \odot K_3$ admits Super Contra Harmonic mean labeling.

Proof

Let $G=(P_n \odot K_1) \odot K_3$ be a graph obtained by attaching a K_3 at each pendent vertex of a comb .

Define f: $V(G) \rightarrow \{1,2,3,\ldots,p+q\}$ by

$f(u_1)=1$, $f(u_2)=10$, $f(u_i)=9i-5$, $3 \le i \le n$

$f(v_1)=3$, $f(v_2)=12$, $f(v_i)=9i-1$, $3 \le i \le n$

$f(x_1)=5$, $f(v_2)=14$, $f(x_i)=9i-8$, $3 \le i \le n$

$f(y_1)=8$, $f(y_2)=17$, $f(y_i)=9i-6$, $3 \le i \le n$

with edge labels

$f(u_i u_{i+1})=9i$, $1 \le i \le n-1$

$f(u_1 v_1)=2$, $f(u_2 v_2)=11$, $f(u_i v_i)=9i-2$, $3 \le i \le n$

$f(v_1 x_1)=4$, $f(v_2 x_2)=13$, $f(v_i x_i)=9i-4$ $3 \le i \le n$

$f(v_1 y_1)=7$, $f(v_i y_i)=9i-3$, $2 \le i \le n$,

$f(x_i y_i)=10i-4$, $i=1,2$ and $f(x_i y_i)=9i-7$ $3 \le i \le n$

The vertices and edges get distinct labels $\{1,2,\ldots,p+q\}$.

Hence $(P_n \odot K_1) \odot K_3$ admits Super Contra Harmonic mean labeling .

A Super Contra Harmonic mean labeling of $(P_5 \odot K_1) \odot K_3$ is

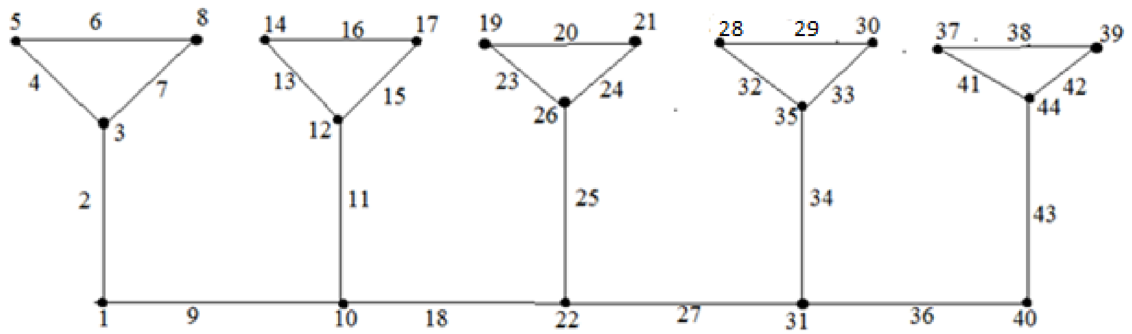

Figure 6

Theorem2.7

A Cycle is a Super Contra Harmonic mean labeling , for $n>3$.

Proof:

Let C_n be a cycle $u_1, u_2, \ldots, u_n$.

Define a function $f: V(C_n) \rightarrow \{1,2,3,\ldots,p+q\}$ by

$f(u_i)=2i-1$, $1 \le i \le n-1$, $f(u_n)=2n$

with edge labels

$f(u_i u_{i+1})=2i$, $1 \le i \le n-1$, $f(u_n u_1)=2n-1$,

Clearly, a Cycle admits a Super Contra Harmonic mean labeling.

A Super contra Harmonic mean labeling of C_4 and C_6 is

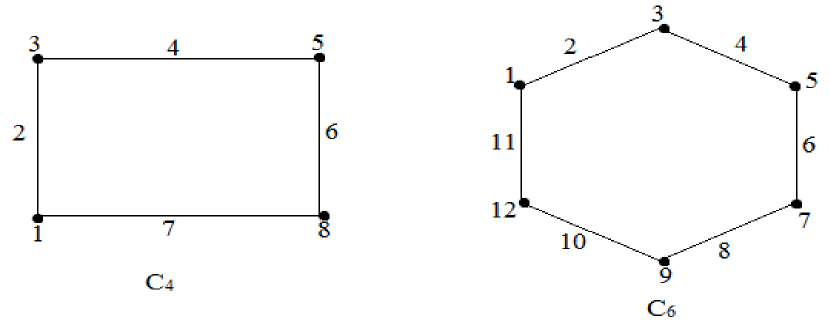

Figure 7

Theorem2.8

A Crown admits Super Contra Harmonic mean labeling.

Proof:

Let G = $C_n \odot K_1$ be a crown obtained from any cycle C_n with a pendant edge attached to each vertex of C_n, $1 \leq i \leq n$.

Define a function $f:V(G) \to \{1,2,\ldots,p+q\}$ by

$f(u_i)=4i-3$, $1\leq i \leq n-1$, $f(u_n)=4n-2$

$f(v_i)=4i-1$, $1\leq i\leq n-1$, $f(v_n)=4n$

with edge labels

$f(u_iu_{i+1})=4i$, $1\leq i\leq n-1$, $f(u_nu_1)=4n-3$

$f(u_iv_i) = 4i-2$, $1\leq i\leq n-1$,$f(u_nv_n)= 4n-1$

The vertices and the edges together get distinct labels $\{1,2,\ldots,p+q\}$.

Hence $C_n \odot K_1$ admits Super Contra Harmonic Mean labeling.

A Super Contra Harmonic mean labeling of $C_6 \odot K_1$ is

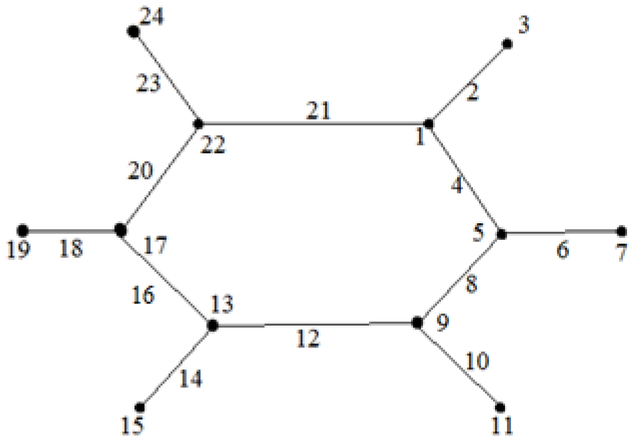

Figure 8

Theorem 2.9

$C_n \odot K_{1,2,}$ admits Super Contra Harmonic mean labeling.

Proof: Let $u_1,u_2,,\ldots,u_n$ be the cycle C_n and and vi,wi be the vertices which are joined to the vertex u_i , $1\leq i\leq n-1$of the cycle .

Let $G = C_n \odot K_{1,2}$. Define f: V(G) →{1,2,…,p+q} by

$f(u_1)=1$, $f(u_i)=6i-3$, $2 \le i \le n-1$, $f(u_n)=6n$

$f(v_1)=3$ $f(v_i)=6i-6$, $2 \le i \le n$

$f(w_i)=6i-1$, $1 \le i \le n-1$, $f(w_n)=6n-5$,

with edge labels

$f(u_1u_2)=8$, $f(u_iu_{i+1})=6i+1$, $2 \le i \le n-2$

$f(u_{n-1}u_n)= 6(n-1)+2$, $f(u_nu_1)=6n-1$

$f(u_1v_1)=2$, $f(u_2v_2)=7$, $f(u_iv_i)=6i-4$, $3 \le i \le n-1$, $f(u_nv_n)=6n-3$

$f(u_iw_i)=6i-2$, $1 \le i \le n$

Here the vertices and the edges together get distinct labels {1,2,…,p+q} . Hence $C_n \odot K_{1,2}$ admits Super Contra Harmonic mean labeling.

A Super Contra Harmonic Mean Labeling of $C_6 \odot K_{1,2}$ is

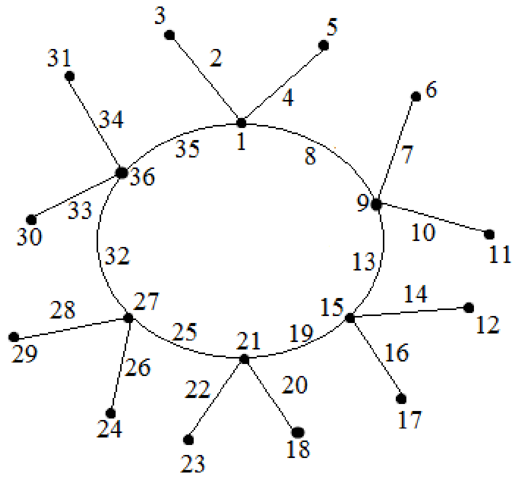

Figure 9

Theorem : 2.10

$(C_n \odot K_1) \odot K_3$ is a Super Contra Harmonic mean graph.

Proof:

Let $u_1, u_2, \ldots, u_n$ be a cycle C_n and let v_i, be the vertex adjacent to u_i , $1 \le i \le n$. The resultant graph is $C_n \odot K_1$. Let x_i, y_i be the vertices of K_3 which are attached to each of the vertex v_i.

The resultant graph is $(C_n \odot K_1) \odot K_3$.

Define a function f: V(G) →{1,2,….,p+q} by

$f(u_1)=1$, $f(u_2)=10$, $f(u_i)=9i-5$, $3 \le i \le n$

$f(v_1)=3$, $f(v_2)=12$, $f(v_i)=9i-1$, $3 \le i \le n-1$, $f(v_n)=9i-8$,

$f(x_1)=5$, $f(x_2)=14$, $f(x_i)=9i-8$ $3 \le i \le n-1$, $f(x_n)=9n -2$,

$f(y_1)=8$, $f(y_2)=17$, $f(y_i)=9i-6$, $3 \le i \le n-1$ and $f(y_n)=9n$

with the edges labels

$f(u_iu_{i+1})=9i$, $1 \le i \le n-1$, $f(u_nu_1) = 9n-6$

$f(u_1v_1)=2$, $f(u_2v_2)=11$, $f(u_iv_i)=9i-2$, $3 \le i \le n-1$, $f(u_nv_n)=9n-7$

$f(v_1x_1) =4$, $f(v_2x_2)=13$, $f(v_ix_i)=9i-4$, $3 \le i \le n$

$f(v_1y_1)=7$, $f(v_iy_i)=9i-3$, $2 \le i \le n$

$f(x_iy_i)=10i-4,\ 1\le i\le 2$, $f(x_iy_i)=9i-7,\ 3\le i\le n-1$, $f(x_ny_n)=9n-1$,
Clearly, f is a Super Contra Harmonic mean labeling of G .

A Super Contra Harmonic mean labeling of $(C_6\odot K_1)\odot K_3$ **is**

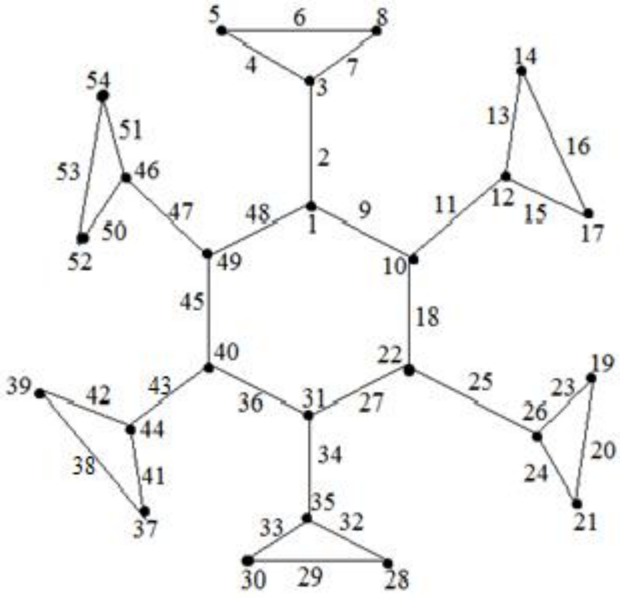

Figure 10

Theorem 2.11

A graph obtained by attaching $K_{1,2}$ at each vertex of the cycle C_n is a Super Contra Harmonic mean graph. (ie) $(C_n\odot K_1)\odot K_{1,2}$ is a Super Contra Harmonic mean graph.

Proof:

Let C_n be the cycle $u_1,u_2,\ldots,u_n$. Let v_i, x_i, y_i, z_i be the vertices of i^{th} copy of $K_{1,2}$ in which v_i is the central vertex. Identify z_i with u_i, $1\le i\le n$. Let the resultant graph be G .

Let $G=(C_n\odot K_1)\odot K_{1,2}$

Define $f : V(G) \rightarrow \{1,2,\ldots,p+q\}$ as

$f(u_1)=1,\ f(u_2)=9,\ f(u_i)=8i-1,\ 3\le i\le n$
$f(v_1)=3,\ f(v_2)=11,\ f(v_i)=8i-6,\ 3\le i\le n$
$f(w_1)=5,\ f(w_2)=13,\ f(w_i)=8i-8,\ 3\le i\le n$
$f(x_1)=7,\ f(x_2)=15,\ f(x_i)=8i-2,\ 3\le i\le n-1$, $f(x_n)=8n$

with edge labels

$f(u_1u_2)=8,\ f(u_iu_{i+1})=8i+3,\ 2\le i\le n-1$, $f(u_nu_1)=8n-2$,
$f(u_iv_i)=8i-6,\ 1\le i\le 2$, $f(u_iv_i)=8i-4$, $3\le i\le n$
$f(w_iv_i)=8i-4,\ 1\le i\le 2$, $f(w_iv_i) = 8i-7,\ 3\le i\le n$
$f(v_ix_i)=8i-2,\ 1\le i\le 2,\ f(v_ix_i)=8i-3,\ 3\le i\le n$

Clearly, f is a Super Contra Harmonic mean labeling of G .

A Super Contra Harmonic mean labeling of $(C_6 \odot K_1) \odot K_{1,2}$ is

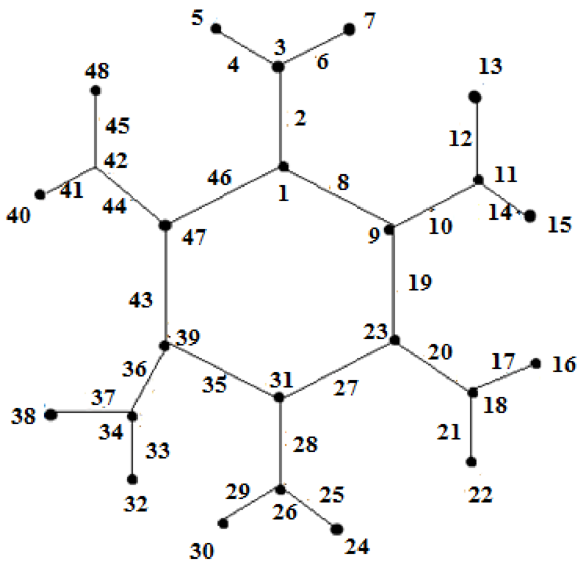

Figure 11

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