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TWO DIMENSIONAL OFFSET FRACTIONAL FOURIER TRANSFORM OF SOME SIGNALS

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Abstract

The Fractional Fourier transform (FRFT) is the generalization of classical Fourier transform. It can analyze the signal in between the time and frequency domain. The most possible application of the FRFT are optical signal processing, quantum mechanics, optimal filtering. We defined two-dimensional Offset Fractional Fourier transform.The Offset Fractional Fourier transform is the space shifted frequency modulated version of original one. Aim of this paper this paper to present two-dimensional Offset Fractional Fourier transform of some signals.

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Keywords:

Fourier transform;
Fractional Fourier Transform;
Two-dimensional Offset Fractional Fourier transform;
Generalized Function;
Testing function Space.

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1. Introduction

Fourier-related transforms are of central importance in diverse application of science, engineering and technology. It is well known that Fourier transform are powerful integral transform and have innumerous application in variety of disciplines not only in engineering side like signal processing [2], optics communication [3] but also in music, economics and geography like subjects.

FRFT has received much attention in recent years. Several applications if FRFT have been suggested. In particular, many signal and image processing applications have been developed on the basis of the FRFT [4, 5, and 6]. Several two-dimensional optical implementations have been discussed previously by different researchers. Thus, FRFT is very useful tool for signal processing and has many applications such as optical filter design, signal synthesis, solving differential equations, phase retrieval, and pattern recognition, quantum mechanics, fractional convolution and correlation, beam forming etc.

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Offset Fourier transform‘s are similar to original Fourier transform‘s, except that the kernel e^{-iwx} is replace by$e^{-i[(w-\tau)(x-\eta)}$. That is the kernel is generalization by appending a space shifted term and a frequency-modulated term [10]. Offset FRFT's are useful in optics. It is especially useful for analyzing optical systems with prisms or shifted lenses. In our previous work we have defined the two dimensional Offset Fractional Fourier transform, testing function space E and E^*as follows

A. Two-Dimensional Offset Fractional Fourier Transform*:*

Two-Dimensional Offset Fractional Fourier Transform $\left[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)\right](s,u)$ of function f (t, x) through an angle α *is* defined as

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)\right](s,u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,x)k_\alpha(t,s-\eta,x,u-\gamma)dtdx$$

Where, $K_\alpha(t,s-\eta,x,u-\gamma)$

$$= \sqrt{\frac{1-i\,cot\alpha}{2\pi}}\, e^{i(s\tau+u\zeta)} e^{\frac{i}{2\sin\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha\, -2((s-\eta)t+(u\ -\gamma)x)]}$$

B. Testing function space E*:*

An infinitely deferential complex valued smooth function on $\phi(R^n)$ belongs to $E(R^n)$, if for each contact $I \subset S_{a,b}$

Where,

$$S_{a,b} = \{t;x{:}\,t\;x \in R^n\}\;;\;\; |t| \le a, |x| \le b, a < 0, b < 0\}\,, I \in R^n$$

$$\gamma_{I,l,q}(\phi) = \sup_{t,x,\in I}\left|D_{t,x}^{l,q}\phi(t,x)\right|$$

$$< \infty, \qquad l,q = 0,1,2\ldots\ldots\ldots$$

Thus $E(R^n)$ will denote the space of all $\phi \in E(R^n)$ with support contained in $S_{a,b}$.

Note that the space E is complete and therefore a Frechet space. Moreover, we say that f is Offset Fractional Fourier transformable if it is a member ofE^*, the dual space of E .

In the present work, Two-Dimensional Offset Fractional Fourier transform is extended in distributional generalized sense. Two-dimensional Offset Fractional Fourier transform of some functions are obtained.

2. Distributional two-dimensional Offset Fractional Fourier transform

The Two-Dimensional Offset Fractional Fourier Transform $\left[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)\right](s,u)$of generalization function $f(t,x)$ through an angle α is defined as,

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)\right](s,u) \;=\; \langle \mathrm{f}\,(\mathrm{t},\mathrm{x})\;\; K_\alpha(t,s-\eta,x,u-\gamma)\rangle$$

where$K_\alpha(t,s-\eta,x,u-\gamma) = C_{1\alpha}e^{i(s\tau+u\zeta)}e^{C_{2\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha\, -2((s-\eta)t+(u-\gamma)x)]}$

And$C_{1\alpha} = \sqrt{\frac{1-i\,cot\alpha}{2\pi}}$ and $C_{2\alpha} = \frac{i}{2\sin\alpha}$

3. Examples on Generalized two-dimensional Offset Fractional Fourier transform

3.1. Result

Prove that $\left[F_\alpha^{\tau,\eta,\zeta,\gamma}(1)\right](s,u) = \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)} e^{\frac{\pi i}{2}} e^{\frac{i}{2}[\frac{3+\cos 2\alpha}{sin\alpha}][(s-\eta)^2+(u-\gamma)^2]}$

Proof:

$\left[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)\right](s,u)$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha - 2((s-\eta)t+(u-\gamma)x)]} f(t,x)\,dtdx$

$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}(1)\right](s,u)$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{1\alpha} e^{i(s\tau+u\zeta)} e^{\frac{i}{2\sin\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha - 2((s-\eta)t+(u-\gamma)x)]}.1\,dtdx$

$= C_{1\alpha} e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\,cot\,\alpha}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha - 2((s-\eta)t+(u-\gamma)x)cosec\alpha}\, dtdx$$

Using formula $\int_{-\infty}^{\infty} e^{iap^2+ibp}\,dp = \frac{e^{\frac{\pi i}{4}}\sqrt{\pi}}{\sqrt{a}} e^{\frac{ib^2}{4a}}$ ………………… (3.1)

where$a = \frac{cot\alpha}{2}$ $b = (-(s-\eta)cosec\alpha)$

$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}(1)\right](s,u)$

$= \sqrt{1-i\,cot\alpha}\, e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\cot\alpha} \left[\frac{e^{\frac{\pi i}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}} e^{\frac{i[-(s-\eta)cosec\;\alpha]^2}{\frac{4\cot\alpha}{2}}}\right]\left[\frac{e^{\frac{\pi i}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}} e^{\frac{i[-(u-\gamma)cosec\;\alpha]^2}{\frac{4\cot\alpha}{2}}}\right]$

$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)} e^{\frac{i\pi}{2}} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2][\frac{cos\alpha}{sin\alpha}+\frac{1}{sin\alpha\;cos\alpha}]}$

$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)} e^{\frac{i\pi}{2}} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2][\frac{\cos^2\alpha+1}{sin\alpha\;cos\alpha}]}$

$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)} e^{\frac{i\pi}{2}} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2][\frac{3+\cos 2\alpha}{\sin 2\alpha}]}$

3.2. Result

Prove that

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}\delta(x-a,y-b)\right](s,u)$$

$$= C_{1\alpha}e^{i(s\tau+u\zeta)}e^{\frac{i}{2\sin\alpha}[((s-\eta)^2+a^2+(u-\gamma)^2+b^2)\cos\alpha-2((s-\eta)a+(u-\gamma)b)]}$$

Proof:

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}f(t,x)\right](s,u)$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}C_{1\alpha}e^{i(s\tau+u\zeta)}e^{iC_{2\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha-2((s-\eta)t+(u-\gamma)x)]}f(t,x)dtdx$$

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}f(t,x)\right](s,u)$$

$$=C_{1\alpha}e^{i(s\tau+u\zeta)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2\sin\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha-2((s-\eta)t+(u-\gamma)x)]}f(t,x)dtdx$$

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}f(t,x)\right](s,u)$$

$$=C_{1\alpha}e^{i(s\tau+u\zeta)}e^{[(s-\eta)^2+(u-\gamma)^2]cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{i}{2}(t^2+x^2)cot\alpha-i[t(s-\eta)^2+x(u-\gamma)^2]cosec\alpha}f(t,x)dtdx$$

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}\delta(x-a,y-b\right](s,u)$$

$$=C_{1\alpha}e^{i(s\tau+u\zeta)}e^{[(s-\eta)^2+(u-\gamma)^2]cot\alpha}e^{\frac{i}{2}(a^2+b^2)cot\alpha-i[a(s-\eta)^2+b(u-\gamma)^2]cosec\alpha}$$

$$=C_{1\alpha}e^{i(s\tau+u\zeta)}e^{\frac{i}{2\sin\alpha}[((s-\eta)^2+a^2+(u-\gamma)^2+b^2)\cos\alpha-2((s-\eta)a+(u-\gamma)b)]}$$

3.3. Result

Prove that

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}e^{i(at^2+bx^2)}\right](s,u)$$

$$=\sqrt{\frac{2\pi(1-i\,cot\alpha)}{(cot\alpha+2a)(cot\alpha+2b)}}e^{i(s\tau+u\zeta)}e^{\frac{\pi i}{2}}e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]}e^{i\,cosec^2\alpha[\frac{(s-\eta)^2}{2cot\alpha+4a}+\frac{(u-\gamma)^2}{2cot\alpha+2b}]}$$

Proof:

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}f(t,x)\right](s,u)$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}C_{1\alpha}e^{i(s\tau+u\zeta)}e^{iC_{2\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha-2((s-\eta)t+(u-\gamma)x)]}f(t,x)dtdx$$

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma}e^{i(at^2+bx^2)}\right]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}C_{1\alpha}e^{i(s\tau+u\zeta)}$$

$$e^{\frac{i}{2\sin\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha-2((s-\eta)t+(u-\gamma)x)]}e^{i(at^2+bx^2)}dtdx$$

$$= C_{1\alpha} e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]cot\alpha} \int_{-\infty}^{\infty} e^{it^2\left[\frac{cot\alpha}{2}+a\right]-i(s-\eta)tcosec\alpha}\, dt$$

$$\int_{-\infty}^{\infty} e^{ix^2\left[\frac{cot\alpha}{2}+b\right]-i(u-\gamma)xcosec\alpha}\, dx$$

Taking $a = \left[\frac{cot\alpha}{2} + a\right]$, b = $-(s-\eta)cosec\alpha$

Therefore using equation (1)

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma} e^{i(at^2+bx^2)}\right](s,u)$$

$$= \sqrt{\frac{1-i\,cot\alpha}{2\pi}} e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]cot\alpha} \left[\frac{e^{\frac{\pi i}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}+a}} e^{\frac{i[-(s-\eta)cosec\ \alpha]^2}{4\left(\frac{cot\ \alpha}{2}+a\right)}}\right]$$

$$\left[\frac{e^{\frac{\pi i}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}+b}} e^{\frac{i[-(u-\gamma)cosec\ \alpha]^2}{4\left(\frac{cot\ \alpha}{2}+b\right)}}\right]$$

$$= \sqrt{\frac{1-i\,cot\alpha}{2\pi}} e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]cot\alpha} \frac{e^{\frac{i\pi}{2}}\pi}{\sqrt{\frac{cot\alpha+2a}{2}}\sqrt{\frac{cot\alpha+2b}{2}}} \qquad e^{icosec^2\alpha\left[\frac{(s-\eta)^2}{2cot\alpha+4a}+\frac{(u-\gamma)^2}{2cot\alpha+2b}\right]}$$

$$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{(cot\alpha+2a)(cot\alpha+2b)}} e^{i(s\tau+u\zeta)} e^{\frac{\pi i}{2}} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]} e^{i\,cosec^2\alpha\left[\frac{(s-\eta)^2}{2cot\alpha+4a}+\frac{(u-\gamma)^2}{2cot\alpha+2b}\right]}$$

3.4. Result

Prove that

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma} e^{i(at+bt)}\right](s,u)$$

$$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}} e^{i(s\tau+u\zeta)} e^{\frac{\pi i}{2}} e^{\frac{i}{2}\left\{\frac{(a^2+b^2)2cosec\alpha\ [a(s-\eta)+b(u-\gamma)]}{cot\alpha}+\frac{3+\cos 2\alpha}{\sin 2\alpha}[(s-\eta)^2+(u-\gamma)^2]\right\}}$$

Proof:

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)\right](s,u)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}\left[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha\ -2((s-\eta)t+(u-\gamma)x)\right]} f(t,x)dtdx$$

$$\left[F_\alpha^{\tau,\eta,\zeta,\gamma} e^{i(at+bt)}\right](s,u)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}\left[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha\ -2((s-\eta)t+(u-\gamma)x)\right]} e^{i(at+bt)}dtdx$$

=

$$C_{1\alpha} e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]cot\alpha} \int_{-\infty}^{\infty} e^{it^2cot\alpha-i(s-\eta)tcosec\alpha+iat}\, dt \int_{-\infty}^{\infty} e^{ix^2cot\alpha-i(u-\gamma)xcosec\alpha+ibx}\, dx$$

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Taking $a = \frac{cot\alpha}{2}$, b = $a - (s-\eta)cosec\alpha$ etc,

Therefore using equation (1)

$$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{2\pi}}\, e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]cot\alpha} \left[\frac{e^{\frac{\pi i}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}} e^{\frac{i[a-(s-\eta)cosec\alpha]^2}{4(\frac{cot\,\alpha}{2})}}\right] \left[\frac{e^{\frac{\pi i}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}} e^{\frac{i[b-(u-\gamma)cosec\alpha]^2}{4(\frac{cot\,\alpha}{2})}}\right]$$

$$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]cot\alpha}\, e^{\frac{\pi i}{2}} e^{\frac{i}{2}[(s-\eta)^2 cot\alpha + \frac{[a-(s-\eta)cosec\,\alpha]^2}{cot\alpha}]}$$

$$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]cot\alpha}\, e^{\frac{\pi i}{2}} e^{\frac{i}{2}[(s-\eta)^2 cot\alpha + \frac{a^2-2a(s-\eta)cosec^2\alpha}{cot\alpha} + \frac{(s-\eta)^2 cosec^2\alpha}{cot\alpha}]}$$

$$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)}\, e^{\frac{i}{2}[\frac{a^2-2a(s-\eta)cosec^2\alpha}{cot\alpha}+(s-\eta)^2(\frac{3+\cos 2\alpha}{\sin 2\alpha})]}\, e^{\frac{i}{2}[\frac{b^2-2b(u-\gamma)cosec^2\alpha}{cot\alpha}+(u-\gamma)^2(\frac{3+\cos 2\alpha}{\sin 2\alpha})]}$$

$$= \sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)} e^{\frac{\pi i}{2}} e^{\frac{i}{2}\{\frac{(a^2+b^2)2cosec\alpha\,[a(s-\eta)+b(u-\gamma)]}{cot\alpha}+\frac{3+\cos 2\alpha}{\sin 2\alpha}[(s-\eta)^2+(u-\gamma)^2]\}}$$

Two-Dimensional Offset Fractional Fourier transform of special functions are represented in tabular form as follows:

S.N.	$f(t,x)$	$\{2D\ Offset\ FrFT f(t.x)\}(s,u)$
1	1	$\sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{i(s\tau+u\zeta)} e^{\frac{i\pi}{2}} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2][\frac{3+\cos 2\alpha}{\sin 2\alpha}]}$
2	$\delta(x-a, y-b)$	$C_{1\alpha} e^{i(s\tau+u\zeta)} e^{\frac{i}{2\sin\alpha}[((s-\eta)^2+a^2+(u-\gamma)^2+b^2)\cos\alpha - 2((s-\eta)a+(u-\gamma)b)]}$
3	$e^{i(at^2+bx^2)}$	$\sqrt{\frac{2\pi(1-i\,cot\alpha)}{(cot\alpha+2a)(cot\alpha+2b)}}\, e^{i(s\tau+u\zeta)} e^{\frac{\pi i}{2}} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]} e^{i\,cosec^2\alpha[\frac{(s-\eta)^2}{2cot\alpha+4a}+\frac{(u-\gamma)^2}{2cot\alpha+2b}]}$
4	$e^{i(at+bx)}$	$\sqrt{\frac{2\pi(1-i\,cot\alpha)}{cot\alpha}}\, e^{\frac{i\pi}{2}}.e^{\frac{i}{2\cot\alpha}}.$ $e^{\frac{i}{2}\{\frac{(a^2+b^2)2cosec\alpha\,[a(s-\eta)+b(u-\gamma)]}{cot\alpha}+\frac{3+\cos 2\alpha}{\sin 2\alpha}[(s-\eta)^2+(u-\gamma)^2]\}}$

4. Conclusion

In the present work generalization of two-dimensional Offset Fractional Fourier transform is presented. Some applications of two-dimensional Offset Fractional Fourier transform

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