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GENERALIZED TWO DIMENSIONAL FOURIER-LAPLACE TRANSFORM AND TOPOLOGICAL PROPERTIES

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Abstract

Due to wide spread applicability in integral transform for partial differential equations involving distributional conditions, many of integral transform extended to generalized function and in last few years, the theory of generalized integral transforms have been of ever increasing interest due to its application in physics especially in quantum field theory, engineering and pure as well as applied mathematics. It provided new aspect to many mathematical disciplines such as ordinary and partial differential equation, operational calculus transformation theory and functional analysis.

This paper presents the generalization of two dimensional Fourier-Laplace transform in the distributional sense. The testing function spaces using Gelfand-Shilov techniques are defined. Also some topological properties of S-type spaces for distributional generalized two dimensional Fourier-Laplace transform are proved.

Keywords:

Fourier Transform;
Laplace Transform;
Two dimensional Fourier-Laplace Transform ;
Generalized Function;

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1. Introduction

More recently, a number of results on multidimensional Laplace transforms were proposed by Dahiya. Even with these enormous numbers of useful applications of Laplace transform a systematic account of general theory of operational calculus using Laplace transforms of double, triple and higher dimensions need serious attention. The two dimensional Laplace transformation is useful in the solution of non-homogeneous partial differential equations [1]. Mathematically the two dimensional Laplace transform is defined and denoted by the formula given below:

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The Two Dimensional Laplace transform with the parameters $p, v,$ of function $f(x,y)$ denoted by $L[f(x,y)] = F(p,v)$ and is given by,

$$L[f(x,y)] = F(p,v) = \int_0^\infty \int_0^\infty e^{-px-vy} f(x,y)\,dxdy$$

(1.1)

The two dimensional Fourier Transform has applied in image filtering in the frequency domain and in signal processing. The Fourier transform itself is translation invariant and its conversion to log polar co-ordinates converts the scale and rotation differences to vertical and horizontal offsets that can be measured [10]. And the formula for this two dimensional Fourier transform is defined and denoted below:

The Two Dimensional Fourier transform with the parameters $s, u,$ of function $f(t,z)$ denoted by $F[f(t,z)] = F(s,u)$ and is given by,

$$F[f(t,z)] = F(s,u) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(st+uz)} f(t,z)\,dtdz$$

(1.2)

The Fourier and Laplace transforms play a predominant role in the theory of signals and systems. Mechanical networks consisting of springs, masses and dampers, for the production of shock absorbers for example, processes to analyze chemical components, optical systems, and computer programs to process digitized sounds or images, can all be considered as system for which one can use Fourier and Laplace transform as well [3]. Many authors studied on various integral transforms separately. B. N.Bhosale and M.S.Choudhari [4] and S.M.Khairnar et.al. [6] has discussed double transform. Motivated by this we have also defined a combination of integral transform in distributional generalized sense namely two dimensional Fourier-Laplace transform which is given by the formula:

The Two Dimensional Fourier-Laplace transform with parameters $s, u, p, v,$ of $f(t,z,x,y)$ is defined as,

$$FL\{f(t,z,x,y)\} = F(s,u,p,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^\infty \int_0^\infty f(t,z,x,y) e^{-i\{(st+uz)-i(px+vy)\}}\,dtdzdxdy$$

(1.3)

where the kernel $K(s,u,p,v) = e^{-i\{(st+uz)-i(px+vy)\}}$.

Along with this we have developed some testing function spaces and proved some topological properties.

This paper is summarized as follows: Some testing function spaces which are useful for proving topological properties are given in section 2. In section 3, we have given definition of Distributional generalized Two dimensional Fourier-Laplace transform. Some topological properties of S-type spaces are proved in section 4. And in section 5, some results on strict inductive limit spaces are given. Finally we conclude the paper.

The notations and terminologies are as per Zemanian [11], [12].

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2. Testing Function Spaces

2.1. The Space $FL_{a,b,c,d,\alpha}$ (S_α -type space):

Let I be the open set in $R_+ \times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I, the space $FL_{a,b,c,d,\alpha}$ is given by

$$FL_{a,b,c,d,\alpha} = \{\phi : \phi \in E_+ / \gamma_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$
$$= \sup_{I_1} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y)\right]\right| \le C_{l,q,n,m} A^k k^{k\alpha} B^r r^{r\alpha} \Big\}$$

(2.1)

where the constants A, B and $C_{l,q,n,m}$ depend on the testing function ϕ.

2.2. The space $FL_{a,b,c,d,\gamma}$ (S_γ -type space):

This space is given by,

$$FL_{a,b,c,d,\gamma} = \{\phi : \phi \in E_+ / \xi_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$
$$= \sup_{I_1} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y)\right]\right| \le C_{k,r,l,n} A^q q^{q\gamma} B^m m^{m\gamma} \Big\}$$

(2.2)

where the constants A, B and $C_{k,r,l,n}$ depend on the testing function ϕ.

2.3. The space $FL_{a,b,c,d,\alpha,g,h}$:

It is defined as,

$$FL_{a,b,c,d,\alpha,g,h} = \{\phi : \phi \in E_+ / \gamma'_{a,b,c,d,k,r,q,m,l,n}\left[\phi(t,z,x,y)\right]$$

$$= \sup_{I_1} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \left[\phi(t,z,x,y)\right]\right| \le C_{l,q,n,m,\delta,\mu} (g+\delta)^k k^{k\alpha} (h+\mu)^r r^{r\alpha} \Big\}$$

(2.3)

For any $\delta > 0,\ \mu > 0$ and g, h are the constants depending on the function ϕ.

The space $FL_{a,b,c,d,\alpha}$ and $FL_{a,b,c,d,\alpha,g,h}$ are equipped with their natural Hausdorff locally convex topologies $T_{a,b,c,d,\alpha}$ and $T_{a,b,c,d,\alpha,g,h}$. These topologies are respectively generated by the total families of seminorms $\{\gamma_{a,b,c,d,k,r,q,m,l,n}\}$ and $\{\gamma'_{a,b,c,d,k,r,q,m,l,n}\}$ given by (2.1) and (2.3).

3. Distributional Generalized Two Dimensional Fourier-Laplace Transform (2DFLT):-

For $f(t,z,x,y) \in FL^*_{a,b,c,d,\alpha}$, where $FL^*_{a,b,c,d,\alpha}$ is the dual space of $FL_{a,b,c,d,\alpha}$. It contains all distributions of compact support. The distributional two dimensional Fourier-Laplace transform is a function of $f(t,z,x,y)$ is defined as,

$$FL\{f(t,z,x,y)\} = F(s,u,p,v) = \langle f(t,z,x,y), \phi(t,z,x,y,s,u,p,v)\rangle,$$

(3.1)

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Where $\phi(t,z,x,y,s,u,p,v)=e^{-i\{(st+uz)-i(px+vy)\}}$ and for each fixed $t(0<t<\infty)$, $z(0<z<\infty)$, $x(0<x<\infty)$ and $y(0<y<\infty)$. Also $s>0$, $u>0$, $p>0$ and $v>0$. The right hand side of (3.1) has a sense as an application of $f(t,z,x,y)\in FL^{*}_{a,b,c,d,\alpha}$ to $\phi(t,z,x,y,s,u,p,v)\in FL_{a,b,c,d,\alpha}$.

4. Topological Properties of S-type spaces:

4.1. Theorem: $\left(FL_{a,b,c,d,\alpha};T_{a,b,c,d,\alpha}\right)$ is a Frechet space.

Proof:- As the family $D_{a,b,c,d,\alpha}$ of seminorms $\left\{\gamma_{a,b,c,d,k,r,q,m,l,n}\right\}_{k,r,q,m,l,n=0}^{\infty}$ generating $T_{a,b,c,d,\alpha}$ is countable, it suffices to prove the completeness of the space $\left(FL_{a,b,c,d,\alpha};T_{a,b,c,d,\alpha}\right)$.

Let us consider a Cauchy sequence $\{\phi_n\}$ in $FL_{a,b,c,d,\alpha}$. Hence for a given $\varepsilon>0$, there exist an $N=N_{k,r,q,m,l,n}$ such that for $m,n\geq N$,

$$\gamma_{a,b,c,d,k,r,q,m,l,n}\left(\phi_m-\phi_n\right)=\sup_{I_1}\left|t^k z^r K_{a,b}(x)R_{c,d}(y)D_t^l D_x^q D_z^n D_y^m\left(\phi_m-\phi_n\right)\right|<\varepsilon$$

(4.1.1)

In particular for $k=r=q=m=l=n=0$, for $m,n\geq N$,

$$\sup_{I_1}\left|K_{a,b}(x)R_{c,d}(y)\left\{\phi_m(t,z,x,y)-\phi_n(t,z,x,y)\right\}\right|<\varepsilon$$

(4.1.2)

Consequently, for fixed (t,z,x,y) in I_1, $\left\{\phi_m(t,z,x,y)\right\}$ is a numerical Cauchy sequence.

Let $\phi(t,z,x,y)$ be the point-wise limit of $\left\{\phi_m(t,z,x,y)\right\}$ using (4.1.2) we can easily deduce that $\{\phi_m\}$ converges to ϕ uniformly on I_1. Thus ϕ is continuous.

Moreover, repeated use of (4.1.1) for different values of k,r,q,m,l,n yields that ϕ is smooth that is $\phi\in E_+$. Further from (4.1.1) we get,

$$\gamma_{a,b,c,d,k,r,q,m,l,n}\left(\phi_m\right)\leq\gamma_{a,b,c,d,k,r,q,m,l,n}\left(\phi_N\right)+\varepsilon,\quad\forall m\geq N$$
$$\leq C_{lqnm}A^k k^{k\alpha}B^r r^{r\alpha}+\varepsilon.$$

Taking $m\to\infty$ and ε is arbitrary we get,

$$\gamma_{a,b,c,d,k,r,q,m,l,n}(\phi)=\sup_{I_1}\left|t^k z^r K_{a,b}(x)R_{c,d}(y)D_t^l D_x^q D_z^n D_y^m\phi(t,z,x,y)\right|\leq C_{lqnm}A^k k^{k\alpha}B^r r^{r\alpha}$$

Hence $\phi\in FL_{a,b,c,d,\alpha}$ and it is the $T_{a,b,c,d,\alpha}$ limit of ϕ_m by (4.1.1) again. This proves the completeness of $FL_{a,b,c,d,\alpha}$ and our proof is completed. For the justification of our study of two dimensional Fourier-Laplace spaces, that is the non triviality of these spaces we have proved the following theorem.

4.2. Theorem: The space $D(I_1)$ is a subspace of $FL_{a,b,c,d,\alpha}$ such that the injection mapping from $D(I_1)$ to $FL_{a,b,c,d,\alpha}$ is continuous that is $T_{a,b,c,d,\alpha}/\mathrm{D}(I_1)\subset T(I_1)$.

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Proof: For $\phi(t,z,x,y)\in D(I_1)$, set

$$L=\sup_{I_1}\{t,z:(t,z,x,y)\in \operatorname{supp}\phi\} \text{ and } C_{lqnm}=\sup_{I_1}\{K_{a,b}(x)R_{c,d}(y)D_t^l D_x^q D_z^n D_y^m\phi(t,z,x,y)\}$$

then,

$$\gamma_{a,b,c,d,k,r,q,m,l,n}\phi(t,z,x,y)=\sup_{I_1}\left|t^k z^r K_{a,b}(x)R_{c,d}(y)D_t^l D_x^q D_z^n D_y^m\phi(t,z,x,y)\right|\le C_{lqnm}L^k N^r$$

$$=C_{lqnm}L^k N^r\frac{k^{k\alpha}}{k^{k\alpha}}.\frac{A^k}{A^k}.\frac{r^{r\alpha}}{r^{r\alpha}}.\frac{B^r}{B^r}$$

$$=C_{lqnm}\left(\frac{L}{Ak^{\alpha}}\right)^k\left(\frac{N}{Br^{\alpha}}\right)^r A^k k^{k\alpha}.B^r r^{r\alpha}$$

(4.2.1)

Since $\left(\frac{L}{Ak^{\alpha}}\right)\le 1$ and $\left(\frac{N}{Br^{\alpha}}\right)\le 1$ iff $k\ge\left(\frac{L}{A}\right)^{1/\alpha}$ and $r\ge\left(\frac{N}{B}\right)^{1/\alpha}$,

define $k_0=\left[\left(\frac{L}{A}\right)^{1/\alpha}\right]+1$ and $r_0=\left[\left(\frac{N}{B}\right)^{1/\alpha}\right]+1$, where $[t,z]$ denotes the Gaussian symbol,

that is the greatest integer not exceeding t,z . Therefore for $k>k_0$, $r>r_0$, we have

$$\gamma_{a,b,c,d,k,r,q,m,l,n}\phi(t,z,x,y)\le C_{lqnm}A^k k^{k\alpha}B^r r^{r\alpha}$$

(4.2.2)

If $k\le k_0$ and $r\le r_0$, let us write,

$$C=\max\left\{\left(\frac{L}{A}\right)\left(\frac{N}{B}\right),\left(\frac{L}{A_{2^{\alpha}}}\right)^2\left(\frac{N}{B_{2^{\alpha}}}\right)^2,\left(\frac{L}{A_{3^{\alpha}}}\right)^3\left(\frac{N}{B_{3^{\alpha}}}\right)^3,\ldots\ldots\ldots\ldots\ldots\left(\frac{L}{A_{k_0^{\alpha}}}\right)^{k_0}\left(\frac{N}{B_{r_0^{\alpha}}}\right)^{r_0}\right\}$$

Then again from (4.2.1)

$$\gamma_{a,b,c,d,k,r,q,m,l,n}\phi(t,z,x,y)\le CC_{lqnm}A^k k^{k\alpha}B^r r^{r\alpha}$$

(4.2.3)

Hence the inequalities (4.2.2) and (4.2.3) together yield,

$\gamma_{a,b,c,d,k,r,q,m,l,n}\phi(t,z,x,y)\le C'_{lqnm}A^k k^{k\alpha}B^r r^{r\alpha}$, $\forall k\ge 0$, $r\ge 0$, where $C'_{lqnm}=CC_{lqnm}$.

Implying that $\phi\in FL_{a,b,c,d,\alpha}$. Consequently, $D(I_1)\subset FL_{a,b,c,d,\alpha}$. To prove that continuity of the injective map, consider a sequence $\{\phi_n\}$ in $D(I_1)$, converging to zero. Then

$$\gamma_{a,b,c,d,k,r,q,m,l,n}(\phi_n)=\sup_{I_1}\left|t^k z^r K_{a,b}(x)R_{c,d}(y)D_t^l D_x^q D_z^n D_y^m\phi_n(t,z,x,y)\right|$$

(4.2.4)

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We find a compact subset Q_m (say) of I_1 such that $\phi_n \in D(Q_m)$ for each $n \geq 1$ and $\phi_n \to 0$ in $D(Q_m)$. If $P = \sup_{Q_m} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) \right|$; then from (4.2.4),

$$\gamma_{a,b,c,d,k,r,q,m,l,n}(\phi_n) \leq P \sup_{Q_m} \left| D^{l+q} D^{n+m} \phi_n(t,z,x,y) \right| \to 0 \text{ as } n \to \infty$$

Hence $\phi_n \to 0$ in $FL_{a,b,c,d,\alpha}$ and therefore $T_{a,b,c,d,\alpha} / \mathrm{D}(I_1) \subset T(I_1)$.

5. Results on strict Inductive Limit Spaces:

5.1. Theorem: If $g_1 < g_2$, $h_1 < h_2$, then $FL_{a,b,c,d,\alpha,g_1,h_1} \subset FL_{a,b,c,d,\alpha,g_2,h_2}$. The topology of $FL_{a,b,c,d,\alpha,g_1,h_1}$ is equivalent to the topology induced on $FL_{a,b,c,d,\alpha,g_1,h_1}$ by $FL_{a,b,c,d,\alpha,g_2,h_2}$ i.e. $T_{a,b,c,d,\alpha,g_1,h_1} \square\ T_{a,b,c,d,\alpha,g_2,h_2} / FL_{a,b,c,d,\alpha,g_1,h_1}$.

Proof: For $\phi \in FL_{a,b,c,,d,\alpha,g_1,h_1}$

$$\gamma_{a,b,c,d,k,r,q,m,l,n}(\phi) \leq C_{lqnm\delta\mu} (g_1 + \delta)^k k^{k\alpha} (h_1 + \mu)^r r^{r\alpha}$$
$$\leq C_{lqnm\delta\mu} (g_2 + \delta)^k k^{k\alpha} (h_2 + \mu)^r r^{r\alpha}$$

Thus $FL_{a,b,c,d,\alpha,g_1,h_1} \subset FL_{a,b,c,d,\alpha,g_2,h_2}$.

The second part is clear from the definition of topologies of these spaces.

5.2. Theorem: $FL_{a,b,c,d,\alpha} = \bigcup_{i=1}^{\infty} FL_{a,b,c,d,\alpha,g_i,h_i}$ and if the space $FL_{a,b,c,d,\alpha}$ is equipped with the strict inductive limit topology $S_{a,b,c,d,\alpha,g,h}$ defined by the injective map from $FL_{a,b,c,d,\alpha,g_i,h_i}$ to $FL_{a,b,c,d,\alpha}$ then the sequence $\{\phi_n\}$ in $FL_{a,b,c,d,\alpha}$ converges to zero if $\{\phi_n\}$ is contained in some $FL_{a,b,c,d,\alpha,g_i,h_i}$ and converges to zero.

Proof: Once we show that

$$FL_{a,b,c,d,\alpha} = \bigcup_{i=1}^{\infty} FL_{a,b,c,d,\alpha,g_i,h_i}.$$

Clearly $\bigcup_{i=1}^{\infty} FL_{a,b,c,d,\alpha,g_i,h_i} \subset FL_{a,b,c,d,\alpha}$.

For proving the other inclusion, let $\phi \in FL_{a,b,c,,d,\alpha}$ then

$$\gamma_{a,b,c,d,k,r,q,m,l,n}(\phi) = \sup_{I_1} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m \phi(t,z,x,y) \right|$$
$$\leq C_{lqnm} A^k k^{k\alpha} B^r r^{r\alpha} \qquad ,$$

(5.2.1)

where A and B are some positive constants. Choose an integer $g = g_A$, $h = h_B$ and $\delta > 0$, $\mu > 0$ such that $C_{lqnm} A^k B^r \leq C_{lqnm} (g + \delta)^k (h + \mu)^r$

Then from (5.2.1) we immediately get $\phi \in FL_{a,b,c,d,\alpha,g_i,h_i}$, implying that

$$FL_{a,b,c,d,\alpha} \subset \bigcup_{i=1}^{\infty} FL_{a,b,c,d,\alpha,g_i,h_i} .$$

6. Conclusion

This paper is concerned with the generalization of two dimensional Fourier-Laplace Transform in the distributional sense. Testing function space using Gelfand-Shilov technique is developed. Topological properties are proved by using the testing function spaces. Also results on strict inductive limit spaces are proved.

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