HEURISTIC APPROACH TO FUZZY TRANSPORTATION PROBLEMS

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Fuzzy logic is shown to be a very promising mathematical approach to modelling the transportation processes characterized by subjectivity, ambiguity, and uncertainty and imprecision. The basic premises of fuzzy logic systems are also presented as a detailed analysis of fuzzy logic systems developed to solve various transportation problems. Fuzzy logic systems are found to be important as universal approximates in solving transportation problems. Studies regarding further applications of fuzzy logic have been done. Zadeh was the first to introduce the concept of fuzzy set theory. For further discussions about fuzzy set theory, the reader may refer to Kaufmann. Zimmermann was the first to apply the concept of fuzzy set theory with suitable choices of membership functions, and to derive a fuzzy linear program identical to the maxim in program. Dyson has claimed that Zimmermann's fuzzy linear programming model is an innovation in the field of multicriteria decision making. Zimmermann applied fuzzy linear programming approaches to the linear vector maximum problem. It is shown by Zimmermann that solutions obtained using fuzzy linear programming are always efficient and that they also give an optimal compromise solution. Chanas et al. investigated transportation problems with fuzzy demands and supplies, and solved them using the parametric programming technique, in terms of the Bellman-Zadeh criterion. Stefan Chanas and Kuchta [8] discussed transportation problems with fuzzy cost coefficients and transformed the problem into a bicriteria transportation problem with crisp objective function. Stefan Chanas and Dorota Kuchta also proposed an algorithm which solves the transportation problem with fuzzy supply and demand values along with the integration condition imposed on the solution. Samir A. Abbas and Omar M.Saad proposed the concept of parametric study on transportation problems. In this paper, a solution is presented as an algorithm for solving fuzzy transportation problems having fuzzy parameters in the constraints. A parametric study is carried out for this problem. Shiang-Tai Liu and Chiang Kao have discussed an idea based on Zadeh's extension principle, to transform the fuzzy transportation problem into a pair of mathematical programs. When the cost coefficients or the demand and supply quantities are fuzzy numbers, the total transportation cost will be fuzzy as well. Nagoor Gani and Abdul Razak investigated the transportation problems with fuzzy parameters in the constraints, and discussed the utilization of Kuhn-Trucker conditions corresponding to the parametric problem. Nagoor Gani and Abdul Razak also discussed a two stage cost minimizing fuzzy transportation problem in which demands and suppUes are trapezoidal fuzzy numbers. A Parametric approach was used to obtain a fuzzy solution. Nagoor Gani and Abdul Razak have solved transportation problems with fuzzy demand and supply values and also with an integration condition imposed on the solution, using a procedure. Nagoor Gani and Abdul Razak also presented fuzziness in pre-emptive goal programming formulation of a multi-objective unbalanced transportation problem with budgetary constraints in which the demand and budget are specified imprecisely. Similar solutions were proposed in.

FUZZY TRANSPORTATION PROBLEM

Here, a transportation problem with m supply nodes and n demand nodes is considered, where s_i (greater than 0) units are supplied by a supply node i and *dj* (greater than 0) units are requested by a demand node j. Associated with each link (i,j) i.e., from supply node i to demand node j, is a unit shipping cost c_{ij}, for transportation. The problem here is to determine a feasible way of shipping the available amount to satisfy the demand in a way that minimizes the total transportation cost.

Here, x_{ij} denotes the number of units to be transported from supply i to demand j. The mathematical description of the conventional transportation problem is given below:

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

subject to,

$$\sum_{j=1}^{n} x_{ij} \le s_i \quad , \quad i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j \quad , \quad j = 1,2,\ldots,n$$

$$x_{ij} \ge 0 \qquad , \forall\ i,j$$

Clearly, if any of the parameters c_{ij}, s_i , or *dj* is fuzzy, the total transportation cost z is fuzzy as well. The conventional transportation problem defined above then turns into a fuzzy transportation problem.

A fuzzy set $\tilde{A}$ is convex if $\mu_A(\lambda p + (1 - \lambda)q) \ge \min(\mu_A(P), \mu_A(q))$, for all p, q $\in R^n$ and $\lambda \in [0, 1]$ [50].

Here, $\mu\tilde{c}_{ij}$, , $\mu\tilde{s}_j$ and $\mu\tilde{d}_j$ denote their membership functions. So, the following is obtained:

$$\tilde{c}_{ij} = \{(c_{ij}, \mu_{c_{ij}}(c_{ij})) \mid c_{ij} \in s(\tilde{c}_{ij})\},$$

$$\tilde{s}_i = \{(s_i, \mu_{s_i}(s_i)) \mid s_i \in s(\tilde{s}_i)\},$$

$$\tilde{d}_i = \{(d_j, \mu_{d_j}(d_j)) \mid d_j \in s(\tilde{d}_j)\},$$

Where s($\tilde{c}_{ij}$), s($\tilde{s}_i$) and $s(\tilde{d}_j)$ are the supports of $\tilde{c}_{ij}$, $\tilde{s}_I$ and $\tilde{d}_j$ which denote the universal sets of the unit shipping cost, the quantity supplied by ith supplier and the quantity required by jth customer, respectively.

The fuzzy transportation problem is written in the following form:

$$\text{Minimize } \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$$

subject to,

$$\sum_{j=1}^{n} x_{ij} \leq \tilde{s}_i \quad , \quad i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \geq \tilde{d}_j \quad , \quad j = 1,2,\ldots,n$$

$$x_{ij} \geq 0 \qquad \forall\ i,j.$$

In this model, all the unit shipping costs, supply quantities and demand quantities are assumed to be fuzzy numbers. Crisp values can be represented by degenerated membership functions, which only have one value in their domains.

Two new fuzzy transportation models for solving the fuzzy transportation problems are discussed in this study, namely Fuzzy Cost Deviation Vector (FCbV) method and Next to Next Minimum Penalty (NNMP) method. Both algorithms are found to produce efficient initial solution in terms of minimum transportation cost, compared to other existing algorithms. However, for a few problems, an optimal solution can be obtained, equal to the solution given by MOOT method. Numerical examples are used to illustrate them.

A comparison between the two proposed methods is made and, it is proved that Next to Next Minimum Penalty method provides a better solution in terms of minimum transportation

Transportation problem having demand and supply as fuzzy triangular numbers, have been solved by using North West comer rule, Row minima method, Column minima method, Matrix minima method, and the Vogel Approximation Method. In this chapter, algorithms namely, Next to Next Minimum Penalty (NNMP) method and Fuzzy Cost Deviation Vector (FCDV) method, are proposed for solving fuzzy transportation problems. These methods are found to be more efficient than other existing algorithms they are found to require, least iterations to reach optimality. The procedure for the solution is illustrated with a numerical example.

The proposed methods namely, NNMP method and FCDV method are found to have the following major advantages:

- they are very easy to understand
- they are better than other existing methods and,
- Their solution is close to the optimum solution.

NEXT TO NEXT MINIMUM PENALTY METHOD

For each row or column, a "penalty" is computed, that is the difference between the first and third smallest costs in the row or column. The row or column with the largest penalty is then found. The variable that has the smallest shipping cost in this row or column is chosen as the first basic variable. As described in the Northwest Comer method and minimum cost method, this variable is made as large as possible and a row or column is crossed out, and then, the supply or demand quantity associated with the basic variable is changed. New penalties are recalculated only using cells which do not lie in a crossed-out row or column. This procedure is repeated until only one uncrossed cell remains. This variable is set equal to the supply or demand associated with the variable and the variable's row and column are crossed-out. A basic feasible solution is thus obtained.

The following terms are used in the Next to Next Minimum Penalty Method for solving fuzzy transportation problems.

The **row penalty** of a row in the fuzzy transportation table is computed as the difference between the smallest cost and the third smallest cost in each row.

The **column penalty** of a column in the fuzzy transportation table is computed as the difference between the smallest cost and the third smallest cost in each column.

The procedure for solving fuzzy transportation problems using the Next to Next Minimum Penalty method is given below.

Step 1: The smallest entry from the first row is chosen and it is subtracted from the third smallest entry. This value is written against the row on the right. This value is calculated as the penalty for the first row. Similarly, the penalty for each row is computed. Likewise, column penalties are calculated and they are written on the bottom of the cost matrix below their corresponding columns.

Step 2: The highest penalty is selected and the row or column for which this corresponds is verified. Min (ai,bj) allocation is made to the cell having the lowest cost from the selected row or column.

Step 3: The satisfied row or column is eliminated. Fresh penalties for the remaining sub matrix are calculated as in step 1 and allocations made as mentioned in step 2. This is continued until one row or column remains to be satisfied. Thus, last row or column is satisfied according to MMM.

RULES FOR A TIE SITUATION

In case of a tie for the largest penalty, the largest penalty is chosen by calculating the difference between the smallest cost and third smallest cost for the corresponding tied rows and columns. If there is a tie for the lowest cost cell, the cell which gives the minimum c^jX^j is selected for allocation.

SOLVED EXAMPLE

In the table below, a balanced fuzzy transportation problem involving three sources and four destinations is considered. The cost of fuzzy transportation per cell is represented by each cell entries.

	B_1	B_2	B_3	B_4	Supply
A_1	(1,5,9)	(7,9,11)	(11,13,15)	(1,2,3)	(25,50,75)
A_2	(9,11,13)	(15,18,21)	(10,20,30)	(1,3,5)	(20,50,80)
A_3	(11,14,17)	(10,13,16)	(12,16,20)	(2,3,4)	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

Step A: The difference between the smallest cost and the third smallest cost is written against each row in the penalty column. This difference is known as row penalty.

The difference between the smallest cost and the third smallest cost is written against each column in the penalty row. This difference is known as column penalty.

The resulting table is shown below.

	B_1	B_2	B_3	B_4	Supply	Penalties
A_1	(1,5,9)	(7,9,11)	(11,13,15)	(1,2,3)	(25,50,75)	(4,7,10)
A_2	(9,11,13)	(15,18,21)	(10,20,30)	(1,3,5)	(20,50,80)	(10,15,20)
A_3	(11,14,17)	(10,13,16)	(12,16,20)	(2,3,4)	(10,50,90)	(7,11,15)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)		
Penalties	(2,9,16)	(4,9,14)	(-5,7,19)	(-1,1,3)		

Step B: The maximum penalties are identified. In the table above, they are found in the second row. So, the maximum possible units i.e., (15, 25, 35) is allocated to the minimum cost cell, i.e., (2,4). The remaining stock is written in the second row. The fourth column is removed and step (a) is repeated. The resulting table is shown below:

	B_1	B_2	B_3		Supply	Penalties
A_1	(1,5,9)	(7,9,11)	(11,13,15)		(25,50,75)	(2,8,14)
A_2	(9,11,13)	(15,18,21)	(10,20,30)		(-15,25,65)	(-3,9,21)
A_3	(11,14,17)	(10,13,16)	(12,16,20)		(10,50,90)	(-4,3,10)
Demand	(10,30,50)	(20,40,60)	(50,55,60)			
Penalties	(2,9,16)	(4,9,14)	(-5,7,19)			

Step C: The maximum penalties are identified. In the table above, they are found in the second row and in the first and second columns. The second row is considered, and the maximum possible units, i.e., (-15, 25, 65) is allocated to the minimum cost cell, i.e., (2,1). The remaining stock is written in the first column. The second row is removed and step (a) is repeated. The table thus obtained is shown below:

	B_1	B_2	B_3		Supply	Penalties
A_1	(1,5,9)	(7,9,11)	(11,13,15)		(25,50,75)	(2,8,14)
A_3	(11,14,17)	(10,13,16)	(12,16,20)		(10,50,90)	(-4,3,10)
Demand	(-55,5,65)	(20,40,60)	(50,55,60)			
Penalties	(2,9,16)	(-1,4,9)	(-3,3,9)			

Step D: The maximum penalties are identified. In the table above, they found in the first column. So, the maximum possible units, i.e., (-55, 5, 65) is allocated to the minimum cost cell, i.e., (1,1). The remaining stock is written in the first row. The first column is removed and step (a) is repeated. The table thus obtained is shown below:

		B_2	B_3		Supply	Penalties
A_1		(7,9,11)	(11,13,15)		(-40,45,130)	(0,4,8)
A_3		(10,13,16)	(12,16,20)		(10,50,90)	(-4,3,10)
Demand		(20,40,60)	(50,55,60)			
Penalties		(-1,4,9)	(-3,3,9)			

Step E: The maximum penalties are identified. In the table above, they are found in the first row and in the second column. The second column is considering and the maximum possible units, i.e., (20, 40, 60) is allocated to the minimum cost cell, i.e., (1, 2). The remaining stock is written in the row. The second column is removed and step (a) is repeated. The table thus obtained is shown below:

			B_3		Supply	Penalties
A_1			(11,13,15)		(-100,5,110)	(11,13,15)
A_3			(12,16,20)		(10,50,90)	(12,16,20)
Demand			(50,55,60)			
Penalties			(-3,3,9)			

Step F: From the above table, the maximum penalties are identified. They are found in the third row. So, the maximum possible units, i.e., (10, 50, 90) is then allocated to the minimum cost cell, i.e., (3,3).

The sum is completed by allocating (-100, 5, 110) units to the cell (1,3). Thus, the allocation is completed.

	B_1	B_2	B_3	B_4	Supply
A_1	(1,5,9) **(-55,5,65)**	(7,9,11) **(20,40,60)**	(11,13,15) **(-100,5,110)**	(1,2,3)	(25,50,75)
A_2	(9,11,13) **(-15,25,65)**	(15,18,21)	(10,20,30)	(1,3,5) **(15,25,35)**	(20,50,80)
A_3	(11,14,17)	(10,13,16)	(12,16,20) **(10,50,90)**	(2,3,4)	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

Fuzzy transportation cost is calculated from the above table as follows:

(-55, 5 ,65) X (1,5,9) + (20,40,60) x (7,9,11) + (-100,5,110) x (11,13,15) + (-15,25,65) X (9,11,13) + (15,25,35) x (1,3,5) + (10,50,90) x (12,16,20) = (-1915, 1600,5715)

Thus, the Defuzzified Fuzzy Transportation Cost is 1700.

FUZZY COST DEVIATION VECTOR METHOD

The Fuzzy Cost Deviation Vector method uses the shipping costs to produce a basic feasible solution which does not have an extremely high total cost. Fewer pivots are predicted to be required to find the problem's optimal solution.

In the Fuzzy Cost Deviation Vector method, the row and column deviation of a cell for each row and column is computed to obtain an ordered pair. This ordered pair is equal to the fuzzy transportation cost of the cell minus the minimum of the corresponding row or column fuzzy transportation cost. The ordered pair (9, (j)) is said to be the cost deviation vector of a cell, if 9 is the row cost deviation and (j) is the column cost deviation of the cell. The row and column with the largest cost deviation is found. Then, the minimum cost deviation vector for the corresponding row and column is also found. Cells corresponding to the vectors are found and Xij is assigned with the min {Sj, dj}. In the Northwest Comer method, row i or column j is eliminated and the supply or demand of the non-eliminated row or column is reduced and the procedure is repeated until there is only one cell that can be chosen. The procedure for solving fuzzy transportation problems using the Fuzzy Cost Deviation Vector method is given below:

Step 1: The fuzzy cost deviation vector table is constructed for the given fuzzy transportation problem.

Step 2: A row r, which contains the maximum fuzzy row cost deviation is found.

Step 3: A column t, which contains the maximum fuzzy column cost deviation is found.

Step 4: The minimum fuzzy cost deviation vector in the r"' row is found, say (a,b).

Step 5: The minimum fuzzy cost deviation vector in the t*^ column is found, say (c,d). \

Step 6: The corresponding cells for (a,b) and (c,d) are found, say (x,y) and (a,P), respectively.

Step 7: (a) If x ≠ α and y ≠ β, both cells (x, y) and (α, β) are selected and the maximum possible is allocated to them.

(b) (i) If x ≠ α and y = β or x = α and y ≠ β, if (a,b) > (c,d), cell (x,y) is selected the maximum possible is allocated to it. Then, cell (α, β) is selected and the maximum possible is allocated to it.

(ii) If x ≠ α and y = β or x = α and y ≠ β, if (a,b) < (c,d), cell (α, β) is selected the maximum possible is allocated to it. Then, cell (x, y) is selected and the maximum possible is allocated to it.

(iii) If x ≠ α and y = β or X = α and y 7≠ β, if (a,b) = (c,d), any one of the cell is selected and the maximum possible is allocated to it. Then, another cell is selected the maximum possible is allocated to it.

(c) If x = a and y = β, cell (a,b) = (c,d) is selected and the maximum possible is allocated to it.

Step 8: The fuzzy transportation table is refined after deleting fully used fuzzy supply points and fully received fuzzy demand points. Incompletely used fully used fuzzy supply points and incompletely received fuzzy demand points are modified.

Step 9: The fuzzy cost deviation table is constructed for the reduced fuzzy transportation problem. Then, step 2 is executed.

Step 10: The above process is repeated until all fuzzy supply points are fully used or all fuzzy demand points are fully received.

Step 11: A fuzzy solution is resulted for the problem from the allotment.

SOLVED EXAMPLE

	B_1	B_2	B_3	B_4	Supply
A_1	(1,5,9)	(7,9,11)	(11,13,15)	(1,2,3)	(25,50,75)
A_2	(9,11,13)	(15,18,21)	(10,20,30)	(1,3,5)	(20,50,80)
A_3	(11,14,17)	(10,13,16)	(12,16,20)	(2,3,4)	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

The fuzzy row and column deviation of a cell (ordered pair) in the fuzzy transportation table is calculated to be the fuzzy transportation cost of the cell minus the minimum of the corresponding fuzzy row and column transportation costs.

	B_1	B_2	B_3	B_4	Supply
A_1	[(-2,3,8), (-8,0,8)]	[(4,7,10), (-4,0,4)]	[(8,11,14), (-4,0,4)]	[(2,0,2), (-2,0,2)]	(25,50,75)
A_2	[(4,8,12), (0,6,12)]	[(10,15,20), (4,9,14)]	[(5,17,29), (-5,7,19)]	[(-4,0,4), (-2,1,4)]	(20,50,80)
A_3	[(7,11,15) ,(2,9,16)]	[(6,10,14), (-1,4,9)]	[(8,13,18), (-3,3,9)]	[(-2,0,2), (-1,1,3)]	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

The maximum row cost deviation is identified, which is in cell (2,3) and the row minimum cost vector is selected from the second row, which is found to be in cell (2,4).

	B_1	B_2	B_3	B_4	Supply
A_1	[(-2,3,8), (-8,0,8)]	[(4,7,10), (-4,0,4)]	[(8,11,14), (-4,0,4)]	[(2,0,2), (-2,0,2)]	(25,50,75)
A_2	[(4,8,12), (0,6,12)]	[(10,15,20), (4,9,14)]	**[(5,17,29),** (-5,7,19)]	**[(-4,0,4),** (-2,1,4)]	(20,50,80)
A_3	[(7,11,15) ,(2,9,16)]	[(6,10,14), (-1,4,9)]	[(8,13,18), (-3,3,9)]	[(-2,0,2), (-1,1,3)]	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

Similarly, the maximum column cost deviation is identified, which is in cell (2,2) and the column minimum cost vector is selected from the second column, which is found to be in the cell (1,2).

	B_1	B_2	B_3	B_4	Supply
A_1	[(-2,3,8), (-8,0,8)]	[(4,7,10), **(-4,0,4)**]	[(8,11,14), (-4,0,4)]	[(2,0,2), (-2,0,2)]	(25,50,75)
A_2	[(4,8,12), (0,6,12)]	[(10,15,20), **(4,9,14)**]	[(5,17,29), (-5,7,19)]	[(-4,0,4), (-2,1,4)]	(20,50,80)
A_3	[(7,11,15) ,(2,9,16)]	[(6,10,14), (-1,4,9)]	[(8,13,18), (-3,3,9)]	[(-2,0,2), (-1,1,3)]	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

The selected cells are (2,4) and (1,2). The row and column values in each cell are equated with each other, and are found that, $2 \neq 1$ and $4 \neq 2$. According to the algorithm, maximum possible units are allocated to cells (2,4) and (1,2).

	B_1	B_2	B_3	B_4	Supply
A_1	[(-2,3,8), (-8,0,8)]	**[(4,7,10), (-4,0,4)]**	[(8,11,14), (-4,0,4)]	[(2,0,2), (-2,0,2)]	(25,50,75)
A_2	[(4,8,12), (0,6,12)]	[(10,15,20), (4,9,14)]	[(5,17,29), (-5,7,19)]	**[(-4,0,4), (-2,1,4)]**	(20,50,80)
A_3	[(7,11,15) ,(2,9,16)]	[(6,10,14), (-1,4,9)]	[(8,13,18), (-3,3,9)]	[(-2,0,2), (-1,1,3)]	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

After the allocation is made on both the cells, it results in the fuzzy transportation table shown below:

	B_1		B_3		Supply
A_1	(1,5,9)		(11,13,15)		(-35,10,55)
A_2	(9,11,13)		(10,20,30)		(-15,25,65)
A_3	(11,14,17)		(12,16,20)		(10,50,90)
Demand	(10,30,50)		(50,55,60)		

Similarly, the maximum row cost deviation is identified. It is found in the cell (2,3), i.e., at the intersection of the second row and the third column. The row minimum cost vector is selected from the second row, and it is found to be (2,1) cell.

	B_1		B_3		Supply
A_1	[(-8,0,8), (-8,0,8)]		[(2,8,14), (-4,0,4)]		(-35,10,55)
A_2	**[(-4,0,4),** (0,6,12)]		**[(-3,9,21),** (-5,7,19)]		(-15,25,65)
A_3	[(-6,0,6), (2,9,16)]		[(-5,2,9), (-3,3,9)]		(10,50,90)
Demand	(10,30,50)		(50,55,60)		

Similarly, the maximum column cost deviation is identified. It is found in the cell (3,1), i.e., at the intersection of the third row and the first column. The column minimum cost vector is selected from the first column, and is found to be in (1,1) cell.

	B_1		B_3		Supply
A_1	[(-8,0,8), **(-8,0,8)**]		[(2,8,14), (-4,0,4)]		(-35,10,55)
A_2	[(-4,0,4), (0,6,12)]		[(-3,9,21), (-5,7,19)]		(-15,25,65)
A_3	[(-6,0,6), **(2,9,16)**]		[(-5,2,9), (-3,3,9)]		(10,50,90)
Demand	(10,30,50)		(50,55,60)		

The selected cells are (2,1) and (1,1). The row and column values in each cell are equated with each other, and is found that, 2^1, and 1=1. The maximum possible units is allocated to cell (2,1), i.e., (-15, 25, 65) units. This completes the allocation in the second row, so the other cells in this row are removed. The maximum possible units is allocated to cell (1,1), i.e., min [(-55, 5, 65), (-35, 10, 55)] = (-55, 5, 65) units. This completes the allocations in the first column, so cell (3,1) is removed from the table. This is shown below:

	B₁		B₃		Supply
A₁			(11,13,15)		(-100,5,110)
A₂					
A₃			(12,16,20)		(10,50,90)
Demand			(50,55,60)		

Cells (1,3) and (3,3) are made to remain in the table as shown above, and it is automatically allocated with (-100, 5, 110) and (10, 50, 90), respectively. Thus, with this information, the allocation table is completed as follows:

	B_1	B_2	B_3	B_4	Supply
A_1	(1,5,9) **(-55,5,65)**	(7,9,11) **(20,40,60)**	(11,13,15) **(-100,5,110)**	(1,2,3)	(25,50,75)
A_2	(9,11,13) **(-15,25,65)**	(15,18,21)	(10,20,30)	(1,3,5) **(15,25,35)**	(20,50,80)
A_3	(11,14,17)	(10,13,16)	(12,16,20) **(10,50,90)**	(2,3,4)	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

Using the above figures, fuzzy transportation cost is calculated to be:

(-55,5,65) X (1,5,9) + (20,40,60) x (7,9,11) + (-100,5,110) x (11,13,15) + (-15,25,65) X (9,11,13) -f- (15,25,35) x (1,3,5) + (10,50,90) x (12,16,20) = (-1915, 1600,5715)

Thus, the Defuzzified Fuzzy Transportation Cost = 1700

COMPARATIVE STUDY

Through the following examples, a comparative study is shown below among the fuzzy transportation algorithms namely matrix minima method (MMM), Vogel's Approximation Method (VAM), NNMP Method and FCDV Method.

EXAMPLE 1

					Supply
	(1,5,9)	(7,9,11)	(11,13,15)	(1,2,3)	(25,50,75)
	(9,11,13)	(15,18,21)	(10,20,30)	(1,3,5)	(20,50,80)
	(11,14,17)	(10,13,16)	(12,16,20)	(2,3,4)	(10,50,90)
Demand	(10,30,50)	(20,40,60)	(50,55,60)	(15,25,35)	

EXAMPLE 2

					Supply
	(1,2,3)	(1,3,5)	(9,11,13)	(5,7,9)	(1,6,11)
	(0,1,2)	(-1,0,1)	(5,6,7)	(0,1,2)	(0,1,2)
	(3,5,7)	(5,8,11)	(12,15,18)	(7,9,11)	(5,10,15)
Demand	(5,7,9)	(1,5,9)	(1,3,5)	(1,2,3)	

EXAMPLE 3

					Supply
	(17,19,21)	(25,35,45)	(40,50,60)	(7,10,13)	(4,7,10)
	(50,70,90)	(10,30,50)	(30,40,50)	(55,60,65)	(4,9,14)
	(37,40,43)	(6,8,10)	(50,70,90)	(10,20,30)	(16,18,20)
Demand	(3,5,7)	(5,8,11)	(4,7,10)	(10,14,18)	

The problems are solved using the above mentioned fuzzy transportation algorithms and the results obtained are tabulated as follows:

Problem	MMM	VAM	NNMPM / FCDVM	Optimum
1	2033	1832	1700	1700
2	121	112	107	107
3	869	814	814	778

From the results given in the table above, it is found that NNMP Method / FCDV Method is better than MMM and VAM for solving fuzzy transportation problems, as the former methods are found to provide nearly optimum solution for every fuzzy transportation problem, compared to the latter methods.

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