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MHD FLOW AND HEAT TRANSFER OF A NON-NEWTONIAN LAW FLUID PAST A STRETCHING SHEET WITH HEAT GENERATION/ABSORPTION

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Abstract

The present study is to investigate the effects of heat source/sink on convection heat transfer of an electrically conducting, non-Newtonian power-law stretched sheet with surface heat flux. The effects of suction/injection at the surface are considered. The governing equations are transformed into non linear ordinary differential equations using similarity transformation. The set of non linear ordinary differential equations are first linearized by using Quasi-linearization technique and then solved numerically by using implicit finite difference scheme. The solution is found to be dependent on various governing parameters. Velocity and Temperature profiles drawn for different controlling parameters reveal the tendency of the solution.

Keywords: Non-Newtonian power-law fluid, suction/injection, Surface heat flux and Heat Source/sink parameter.

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Introduction

The study of flow and heat transfer problems due to stretching boundary has many practical applications in technological processes, particularly in polymer processing systems involving drawing of fibers and films or thin sheets, etc. Sometimes the polymer sheet is stretched while it is extruded from a die. Usually the sheet is pulled through the viscous liquid with desired characteristics. The moving sheet may introduce a motion in the neighboring fluid or alternatively, the fluid may have an independent forced convection motion which is parallel to that of the sheet. Sakiadis [1] was the first to investigate the flow due to sheet issuing with constant speed from a slit into a fluid at rest. Schowalter [2] has introduced the concept of the boundary layer in the theory of non-Newtonian power-law fluids.

For the non-Newtonian power-law fluids, the hydrodynamic problem of the MHD boundary layer flow over a continuously moving surface has been by Mahmoud and Mahmoud [3]. Chiam [4] studied the boundary layer flow of a Newtonian fluid over a stretching plate in the presence of a transverse magnetic field. Pop and Na [5] performed an analysis for the MHD flow past a stretching permeable surface. Recently Chien-Hsin Chen [6] has studied the magneto-hydrodynamic flow and heat transfer of an electrically conducting, non-Newtonian power-law fluid past a stretching sheet in the presence of a transverse magnetic field by considering suction/injection.

Kishan and kavitha [7] studied the MHD heat transfer to non-Newtonian power-law fluids over a wedge with heat source/sink in the presence of viscous dissipation. MHD flow and heat transfer of a non-Newtonian power-law fluid past a stretching sheet with suction/injection and viscous dissipation was studied by Kishan and Shashidar [8]. Kishan and Shashidar Reddy [9] has studied the MHD Effects on non-Newtonian power-law fluid past a continuously moving porous flat plat with heat flux and viscous dissipation. Saritha et al [10] studied Quasi-linearization approach to effects of heat source/sink on MHD flow of non-Newtonian power-law fluid past a continuously moving porous flat plate with heat flux and viscous dissipation.

The present work deals with the flow and heat transfer of electrically conducting, non-Newtonian power-law fluids past a continuously stretching sheet under the action of a transverse magnetic field with suction/injection by taking into account the effect of heat source/sink.

2. Mathematical Formulation

To Construct the model, consider a steady two-dimensional flow of an incompressible, electrically conducting fluid obeying the power-law model past a permeable stretching sheet. The origin is located at the slit through which the sheet is drawn through the fluid medium, the x-axis is chosen along the sheet and y-axis is taken normal to it. This continuous sheet is assumed to move with a velocity according to a power-law form, i.e. = C x^p, and be subject to a surface heat flux. Also, a magnetic field of strength B is applied in the positive y-direction, which produces magnetic effect in the x-direction. Under the foregoing assumptions and invoking the usual boundary layer approximations, the problem is governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial U}{\partial y}=\frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)-\frac{\sigma B^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho C_p}(T-T_\infty) \tag{3}$$

Where u and v are the velocity components, T is the temperature, B is the magnetic field strength, K is the consistency coefficient, *n* is the flow behavior index, ρ is the density, σ is the electrical conductivity and α is the thermal diffusivity, C_p is the specific heat at a constant pressure and Q is the heat generation constant. The appropriate boundary conditions are given by

$u_w(x) = C.x^p$, $\quad v = v_w$, $\quad \frac{\partial T}{\partial y}=-\frac{q_w}{k}$ $\quad$ at y = 0, x > 0 (4)

$u \to 0$, $\quad T \to T_\infty$ as $y \to \infty$ (5)

where v_w is the surface mass flux and q_w is the surface heat flux. It should be noted that positive p indicates that the surface is accelerated while negative p implies that the surface is decelerated from the slit. Also note that positive v_w is for fluid injection and negative for fluid suction at the sheet surface.

3. Method of Solution

We shall further transform equations (2) & (3) into a set of partial differential equations amenable to a numerical solution. For this purpose we introduce the variables

$$\eta=\left(\frac{C^{2-n}}{K/\rho}\right)^{1/(n+1)} x^{[p(2-n)-1]/(n+1)}y \tag{6}$$

$$\psi=\left(\frac{C^{1-2n}}{K/\rho}\right)^{-1/(n+1)} x^{[p(2n-1)+1]/(n+1)}f \tag{7}$$

$$\theta=\frac{(T-T_\infty)\mathrm{Re}_x^{1/(n+1)}}{q_w x/k} \tag{8}$$

Where the dimensionless stream function *f* satisfies the continuity equation with $u=\frac{\partial\psi}{\partial y}=u_w f'$ and

$$v=-\frac{\partial\psi}{\partial x}=-u_w\,\mathrm{Re}_x^{-1/(n+1)}\left(\frac{p(2n-1)+1}{n+1}f+\frac{p(2-n)-1}{n+1}\eta f'\right).$$

Under the transformations (6), (7) and (8), the differential equations (2) and (3) reduce to

$$\left(\left|f''\right|^{n-1} f''\right)' + \frac{p(2n-1)+1}{n+1} ff'' - p(f')^2 - Mf' = 0 \qquad (9)$$

$$\frac{1}{\Pr}\theta'' + \frac{p(2n-1)+1}{n+1} f\theta' + \frac{p(2-n)-1}{n+1} f'\theta + S\theta = 0 \qquad (10)$$

Where primes indicate the differentiation with respective to η.
Subject to the boundary conditions

$$\left.\begin{array}{l} f'(0)=1,\ f(0)=\dfrac{n+1}{p(2n-1)+1} f_w,\ \theta'(0)=-1 \\ f'(\infty)=0,\ \theta(\infty)=0 \end{array}\right\} \qquad (11)$$

Where

$M = \dfrac{\sigma B^2 x}{\rho u_w}$ is the magnetic parameter,

$f_w = -\dfrac{v_w}{u_w}\mathrm{Re}_x^{1/(n+1)}$ is the suction/injection parameter,

$\Pr = \dfrac{x u_w}{\alpha}\mathrm{Re}_x^{-2/(n+1)}$ is the Prandtl number and

$S = \dfrac{Qx}{U\rho C_p}$ is the Heat Source/Sink.

Where $\mathrm{Re}_x = \dfrac{\rho u_w^{2-n} x^n}{K}$ is the local Reynolds number. Note here that the magnetic field strength B should be proportional to x to the power (p-1)/2 to eliminate the dependence of M on x, i.e. $B(x)=B_0 x^{(p-1)/2}$ where B_0 is a constant.

The wall shear stress is given by

$$\tau_w = \left[K\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right]_{y=0} = \rho u_w^2 \mathrm{Re}_x^{-1/(n+1)}\left|f''(0)\right|^{n-1} f''(0)$$

The physical quantities of engineering interest in this problem are the local skin friction coefficient and the local Nusselt number and local Sherwood number, which are defined respectively by

$$C_f = \frac{\tau_w}{\rho u_w^2/2} = 2\mathrm{Re}_x^{-1/(n+1)}\left|f''(0)\right|^{n-1} f''(0) \qquad (12)$$

and

$$Nu_x = \frac{hx}{k} = \frac{\mathrm{Re}_x^{1/(n+1)}}{\theta(0)} \qquad (13)$$

where $h = \dfrac{q_w}{T_w - T_\infty} = \dfrac{k\,\mathrm{Re}_x^{1/(n+1)}}{\theta(0)}$

4. Numerical Procedure:

The system of coupled, non linear ordinary differential equations (9) and (10) with the boundary conditions (11) are solved numerically. The numerical solutions can be obtained in the following steps:

- Linearize Eq (9) using Quasi Linearization method [11].
- Write the difference equations using implicit finite difference scheme.
- Linearize the algebraic equations by Newton's method, and express them in matrix-vector form and
- Solve the linear system by Gauss Seidal Iteration method.

Since the equations governing the flow are nonlinear, iteration procedure is followed. For the sake of brevity, further details of the solution process are not presented here. The numerical computations were carried out C programming. The numerical solutions of f are considered as $(n+1)^{th}$ order iterative solutions and F are the n^{th} order iterative solutions. After each cycle of iteration the convergence check is performed, and the process is terminated when $|F - f| < 10^{-4}$.

6. Results and Discussions

The parametric study is performed to explore the effects of magnetic field parameter M, power-law fluid index n, velocity sheet exponent p and suction/ injection parameter f_w on the velocity distribution. And the effects of Heat source/sink parameter S, velocity sheet exponent p, power-law fluid index n, and the Prandtl number P_r on the temperature distribution were studied.

Velocity profiles f' are shown in figs.1–4 for different parameters n, M, f_w and p. It is observed from fig. 1 that the power-law fluid index n increases as f' increases near the wall and the reverse phenomenon is observed away from the wall. Figs. 2 and 3 show that the effect of magnetic field M and suction parameter f_w decelerates the fluid motion for both the cases of pseudo plastic and dilatant fluids. It can be noticed from fig. 4 that the velocity distribution f' decreases as velocity exponent p increases for the both the cases of pseudo plastic and dilatant fluids.

Temperature distributions presented for various values of n, p and Pr are shown in figures 5- 7. Figs. 5(a) and 5(b) represent the temperature profiles for various values of the power-law index n, respectively for an accelerated stretching surface (p = 1) and for a decelerated stretching surface (p = -0.3). It can be observed from the figures that for the accelerated stretching case fluid temperature decreases as the power-law index n increases, whereas an opposite behavior exist for an decelerated stretching surface. Here, it can be seen that the influence of power-law index n on the wall temperature is more significant for an accelerated stretching surface.

The influence of sheet velocity exponent p on the temperature distributions for n = 0.5 (pseudo plastic fluid) and n = 1.5 (dilatant fluid) are shown in figs. 6(a) and 6(b) respectively. It is clear from the figures that for a pseudo plastic fluid, a considerable increase in the temperature distribution and the surface temperature are caused by increasing the value of p, but reverse to that increase in p reduces the temperature for a dilatant fluid.

Fig. 7 reveals the effect of generalized Prandtl number Pr on the temperature distribution for shear thinning fluid (n = 0.5). It is obvious from the figure that an increase in Prandtl number will produce a decrease in the thermal boundary layer thickness, associated with the reduction in the temperature profiles. The effects of suction/injection parameter and Prandtl number on temperature distribution for dilatant fluids are similar to those for the pseudo plastic fluids.

Figs.8 shows the effect of Heat source/sink parameter on the temperature profiles for both pseudo – plastic and dilatants fluids. From the figures it can be noticed that an increase in the heat source strength, the temperature also increases whereas the temperature decreases with the increase in the heat sink strength.

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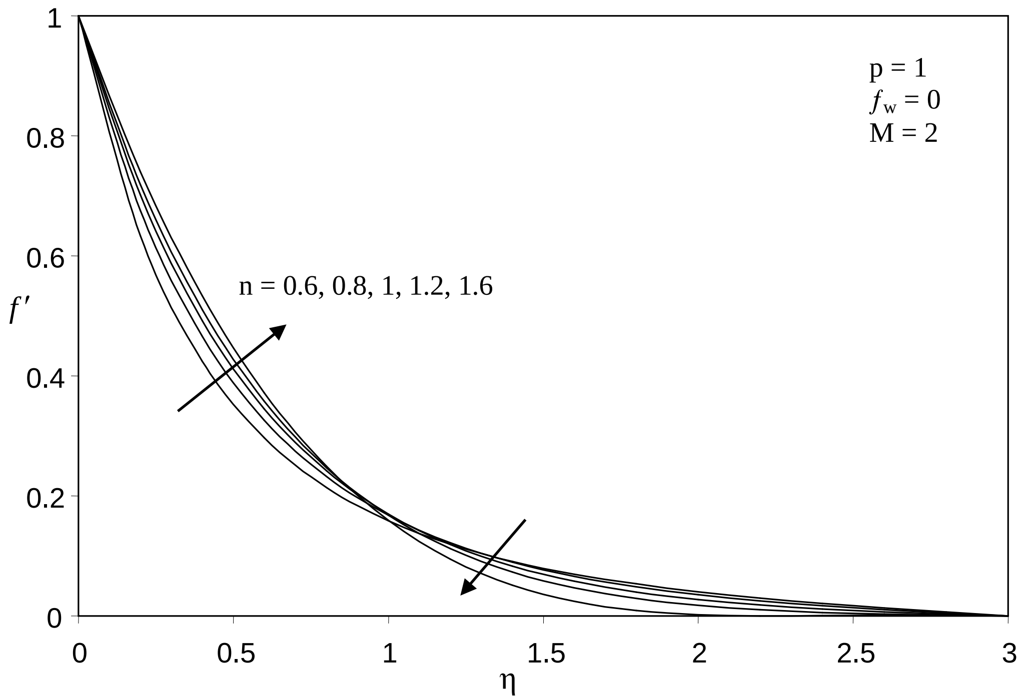
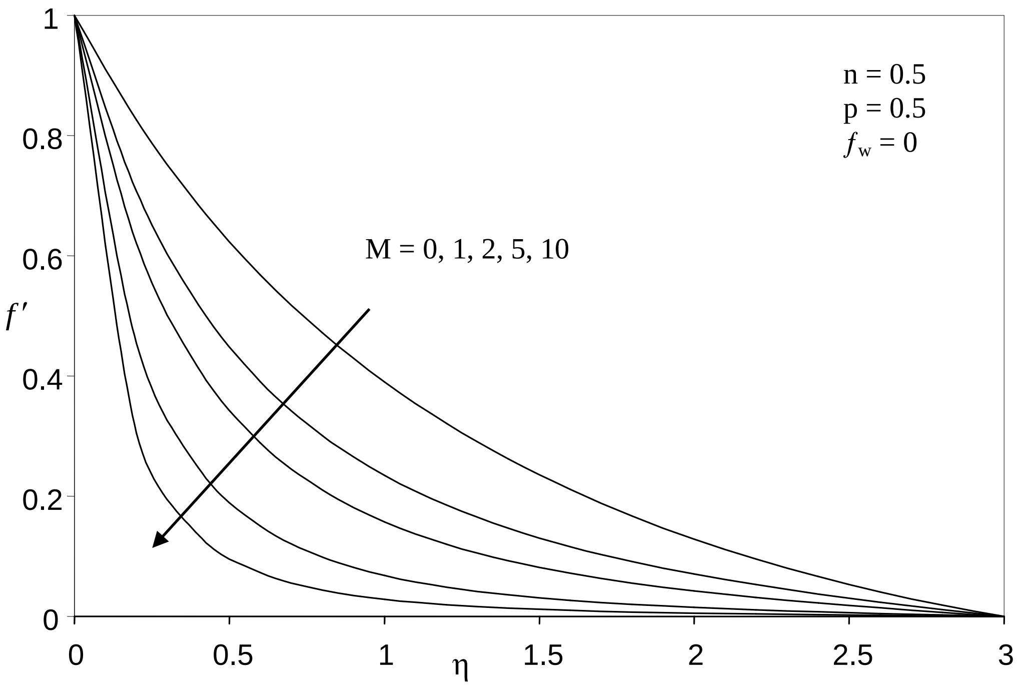
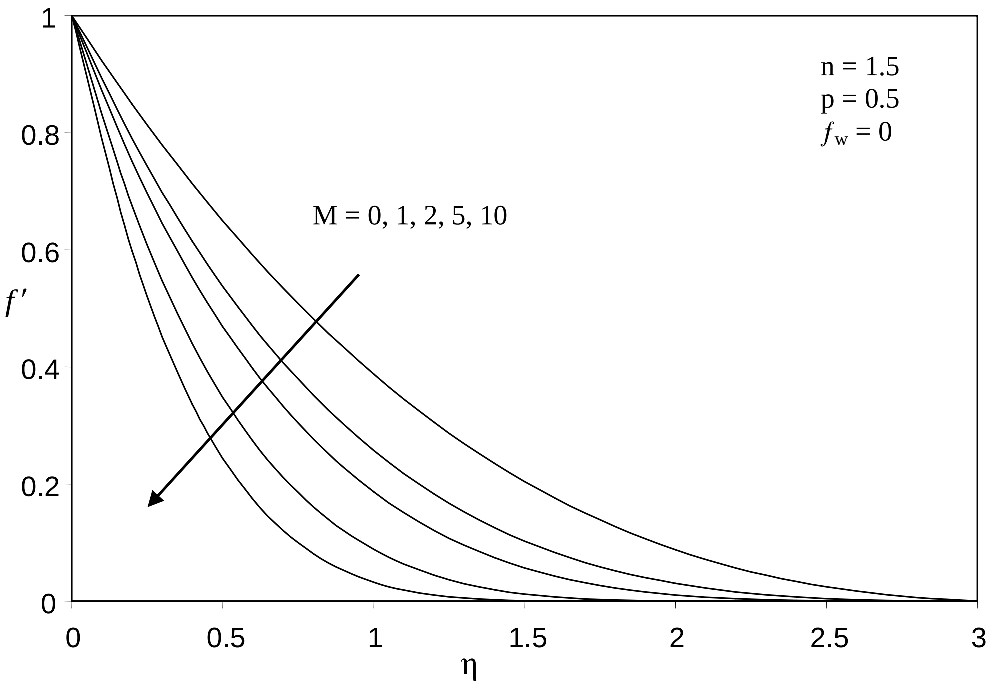


Fig.1. Velocity profiles f' for different values of power-law index **n** with $f_w = 0$, p = 1 and M = 2.

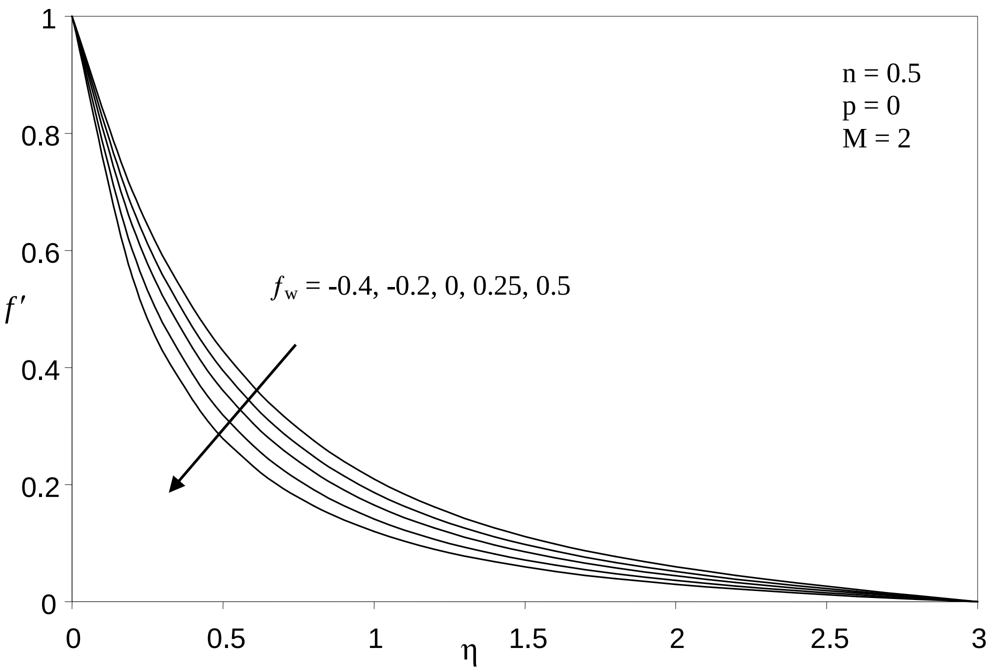


2 (a)

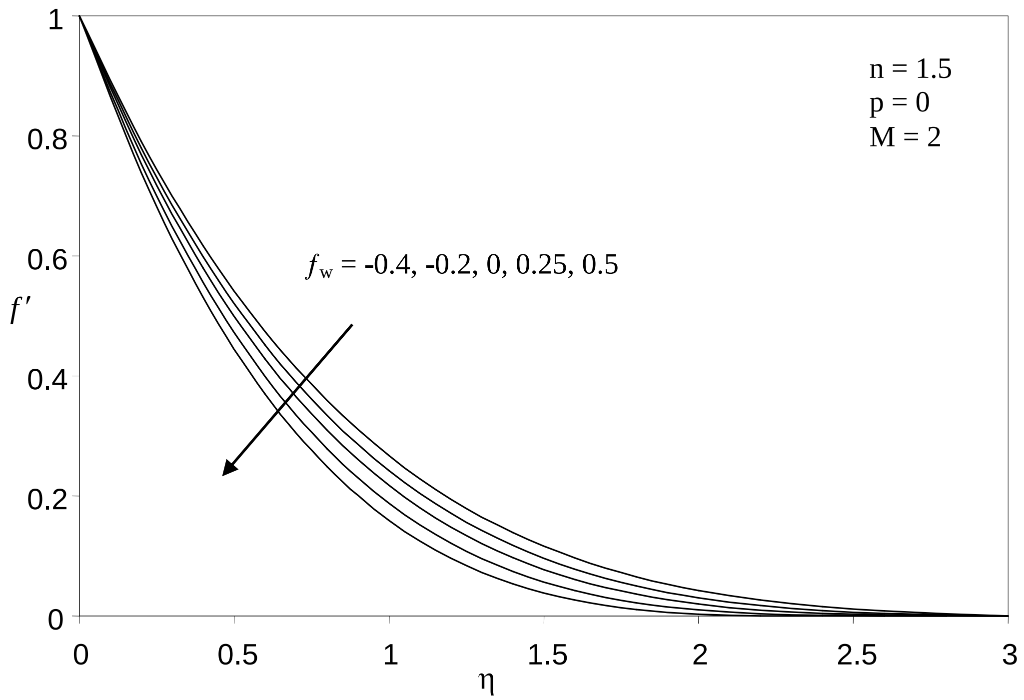


2 (b)

Fig.2. Velocity profiles f' for different values of Magnetic parameter M with p = 0.5 and $f_w = 0$. (a) n = 0.5; (b) n = 1.5

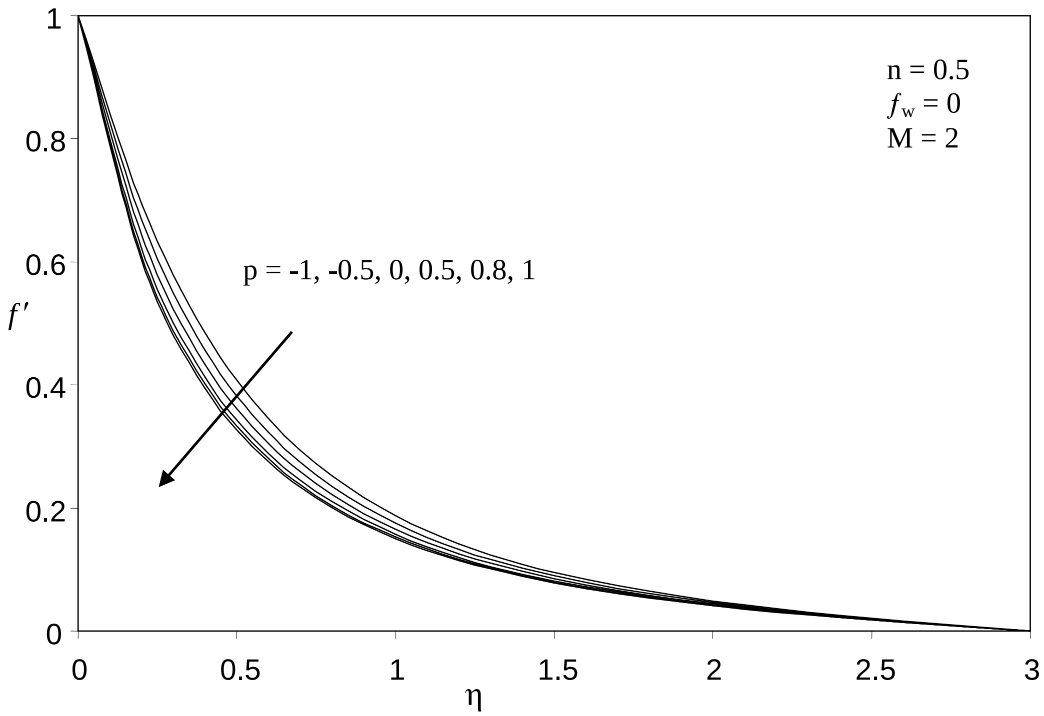


3 (a)

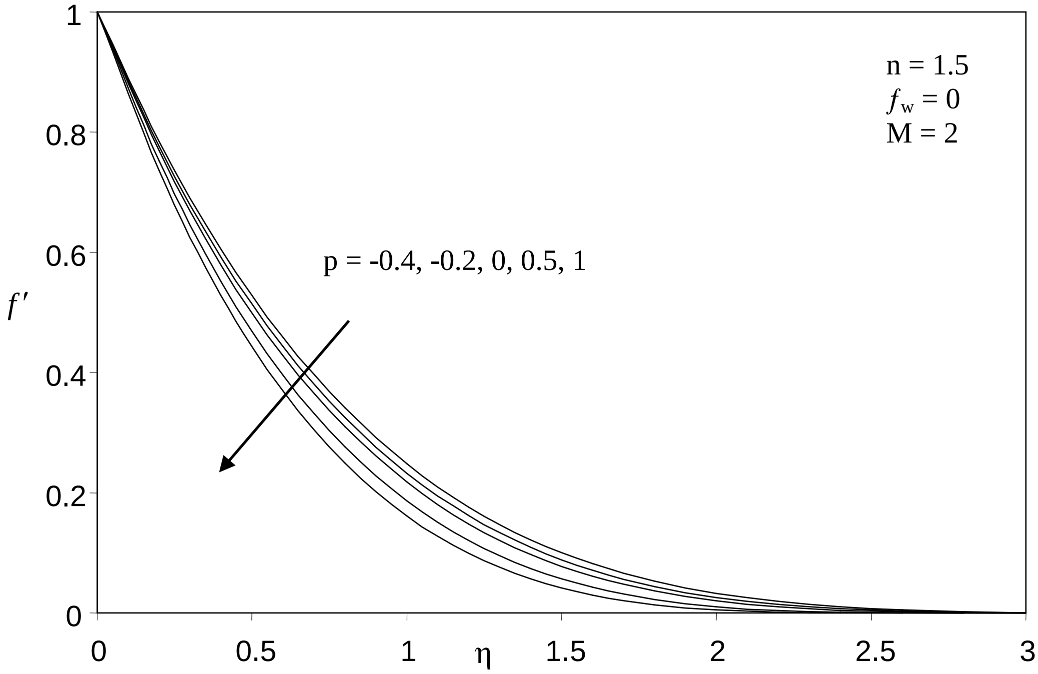


3 (b)

Fig.3. Velocity profiles f' for different values of suction/injection parameter $\mathbf{f_w}$ with p = 0 and M = 2. (a) n = 0.5; (b) n = 1.5

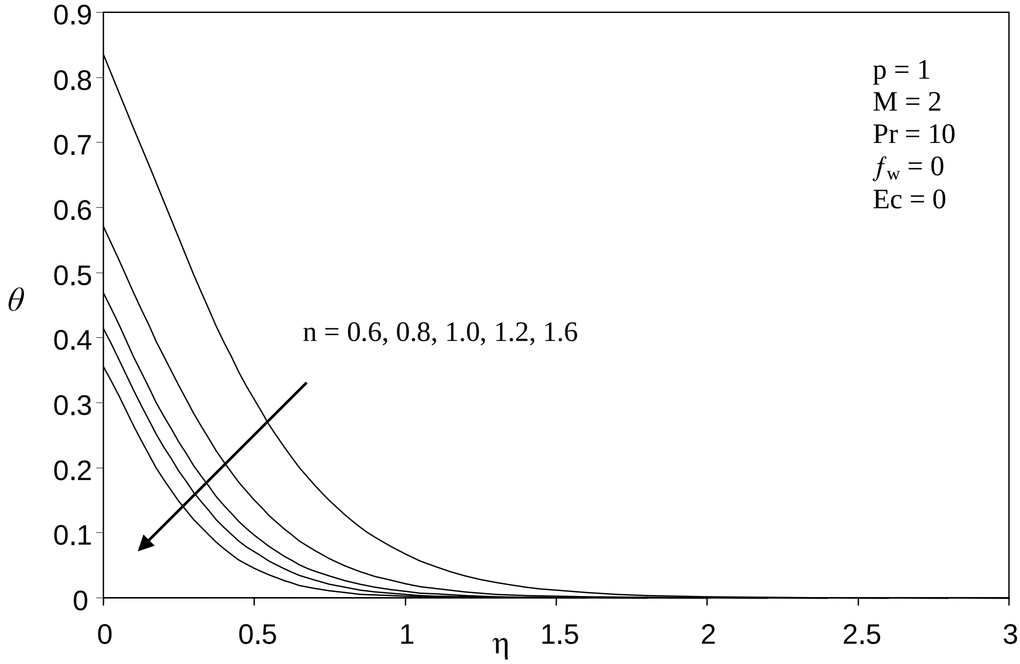


4 (a)

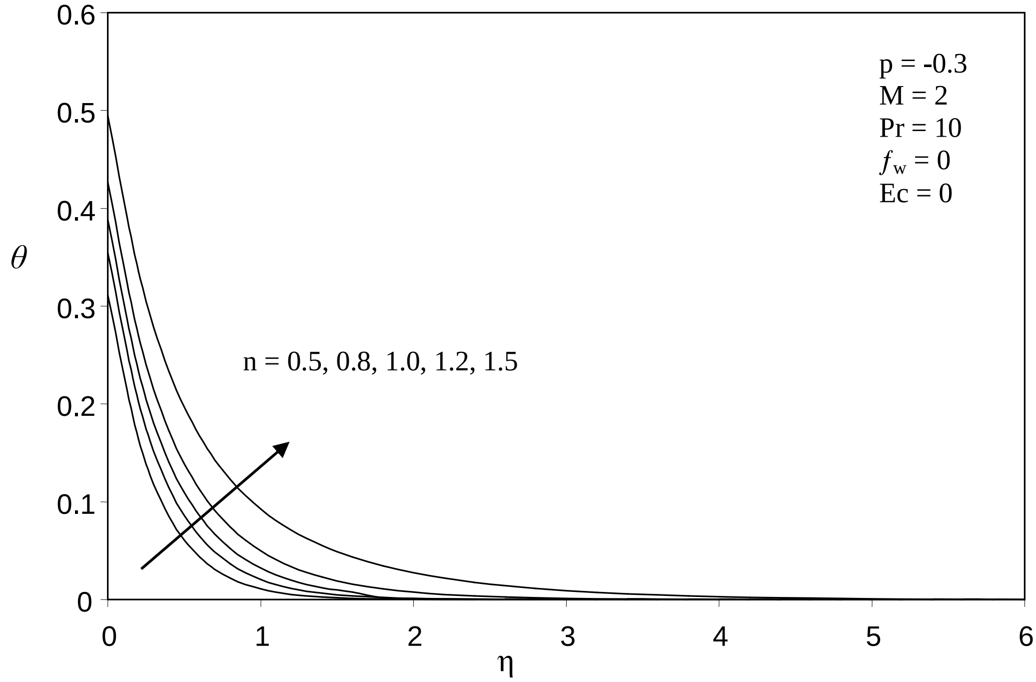


4 (b)

Fig.4. Velocity profiles f' for different values of sheet velocity exponent **p** with f_w = 0 and M = 2. (a) n = 0.5; (b) n = 1.5

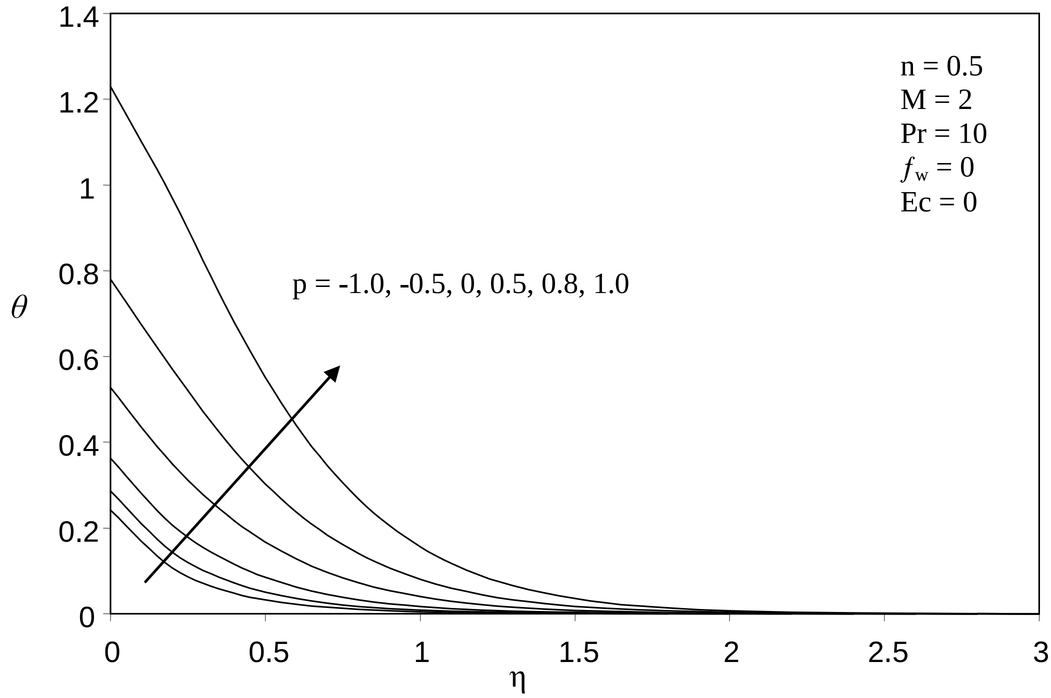


5 (a)

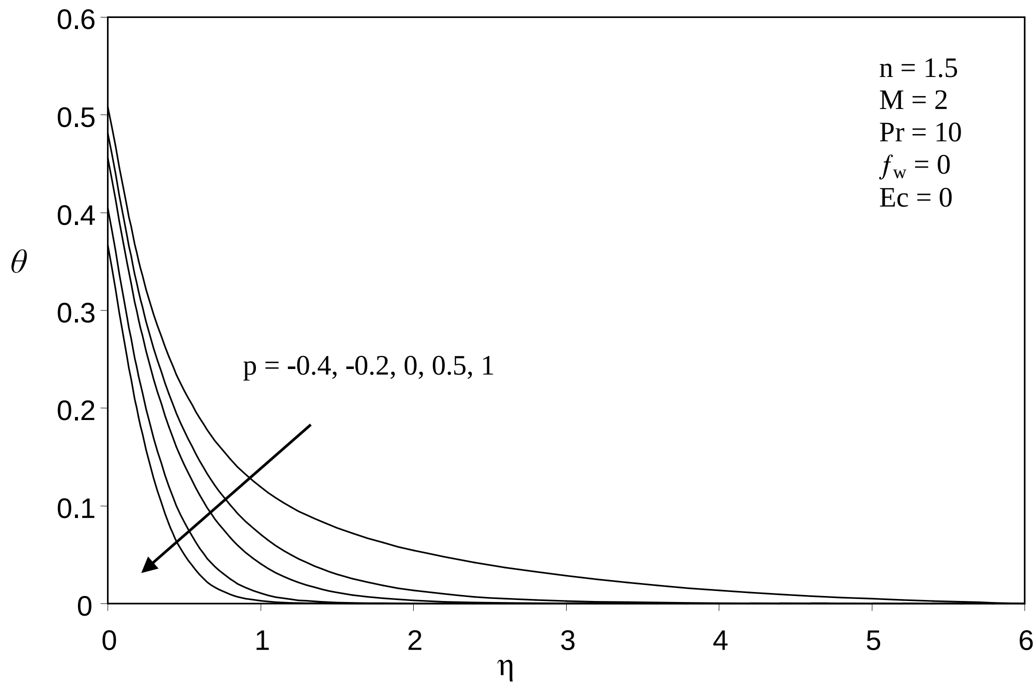


5 (b)

Fig.5. Temperature profiles for different values of power-law index **n** with M = 2, Pr = 10, f_w = 0 and S=0. (a) p = 1.0; (b) p = -0.3



6 (a)



6 (b)

Fig.6. Temperature profiles for different values of sheet velocity exponent **p** with M = 2, Pr = 10, f_w = 0and S=0. (a) n = 0.5; (b) n = 1.5

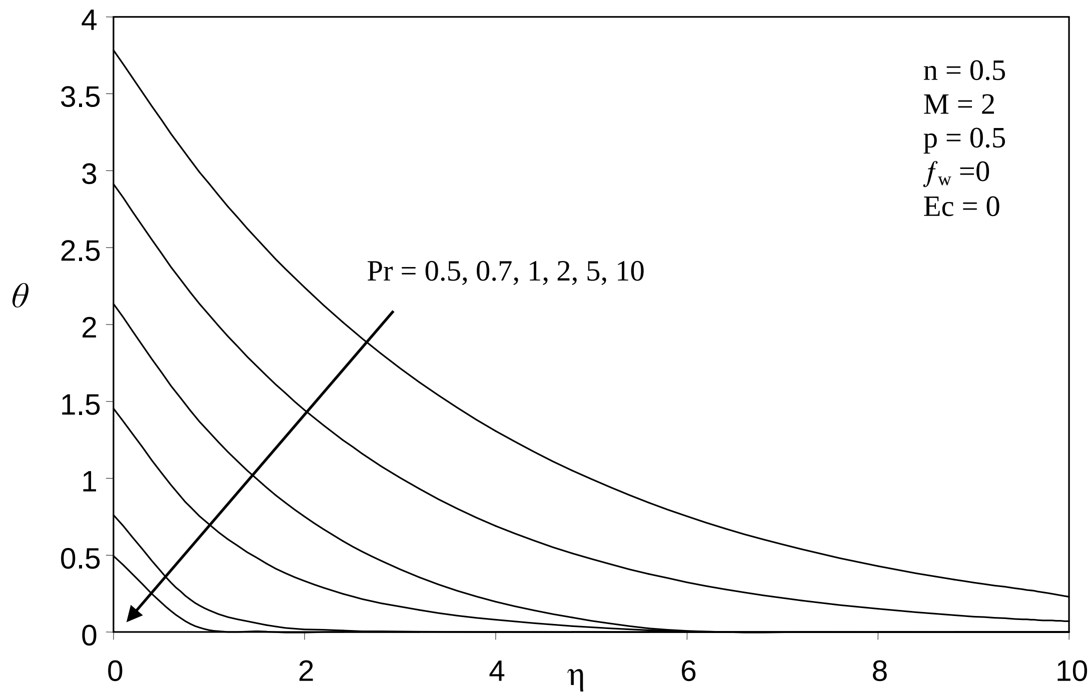
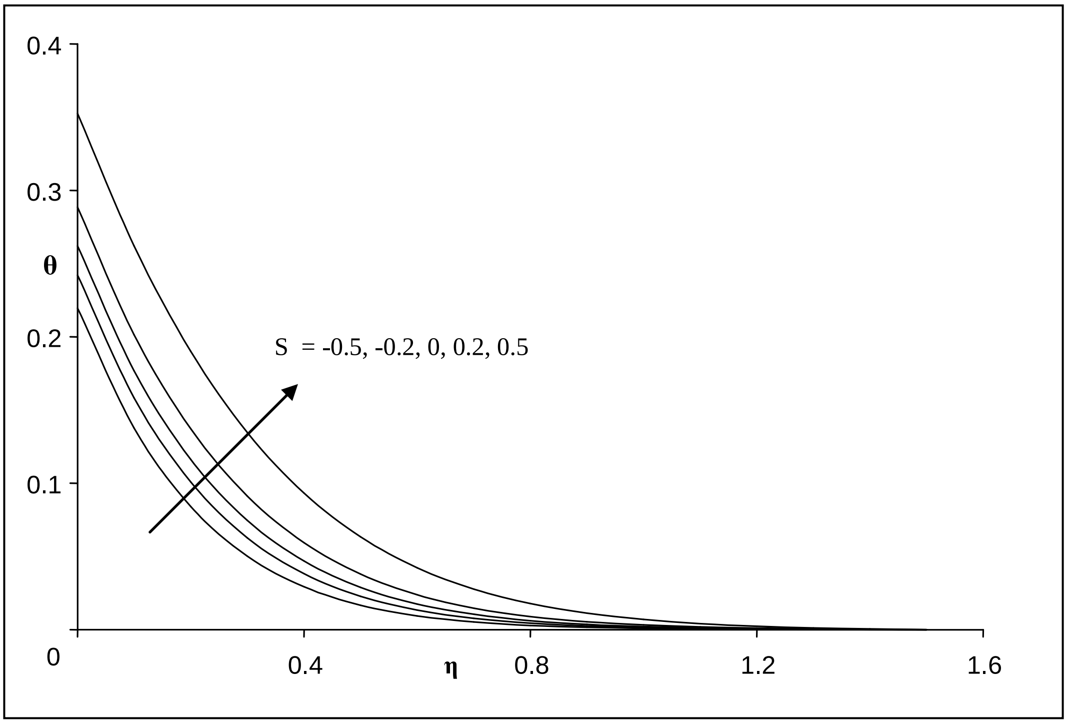
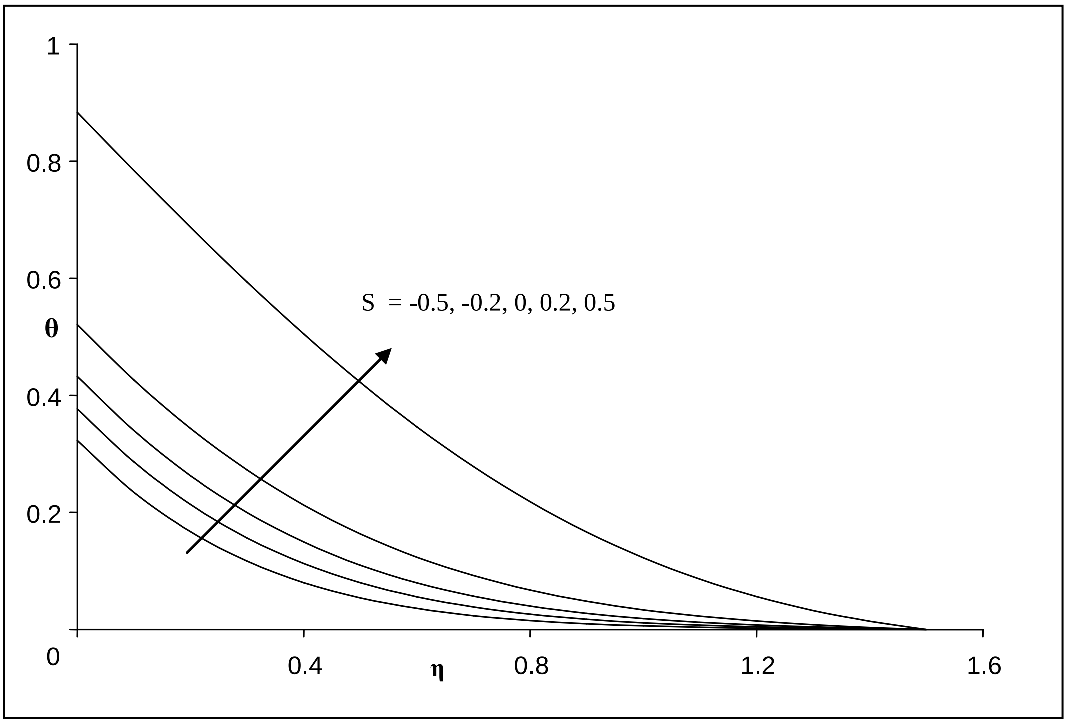


Fig.7. Temperature profiles for different values of Prandtl number Pr with n = 0.5, M = 2, p = 0.5, f_w =0 and S = 0.



8 (a)



8 (b)

Figure 8: Temperature profiles for different values of Heat Source/Sink **S** with M = 2, Pr = 10, f_w = 0. (a) n = 0.5; (b) n = 1.5