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BIPOLAR (S, T) SMOOTH FUZZY SOFT NORMAL SUBGROUPOIDS OVER SMOOTH FUZZY SOFT COSETS

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ABSTRACT:

In this paper, we study bipolar smooth fuzzy soft subgroups and investigate relations with bipolar smooth fuzzy soft cosets and normal soft subgroups. Then we define normalizer and give some theorems relative to these concepts.

Keywords:

Bipolar smooth fuzzy set, soft subgroup, smooth fuzzy right cosets and left cosets, self conjugate, bipolar smooth fuzzy soft normalizer.

1. Introduction:

The concept of fuzzy set was introduced by Zadeh [17] as a new mathematical tool for dealing with uncertainties. There are many kinds of fuzzy set extensions in the fuzzy set theory, such as intuitionistic fuzzy sets, interval-valued fuzzy sets, rough fuzzy set, soft fuzzy set, vague sets, etc. [4, 9, 10]. Bipolar-valued fuzzy set is another extension of fuzzy set whose membership degree range extends from the interval [0, 1] to the interval [−1, 1]. K.M.Lee [14, 15] introduced the idea of bipolar valued fuzzy set as a generalization of the notion of fuzzy set. Thus, the theory of bipolar valued fuzzy sets has become a vigorous area of research in various disciplines such as algebraic structure, medical science, graph theory, decision making, machine theory and so on [2, 3, 5, 6, 7]. The notion of a fuzzy sub near-ring was introduced by S. Abou-Zaid [1] and also studied fuzzy left (resp. right) ideals of a near-ring and has given some properties of fuzzy prime ideals of a near-ring. A method of parameter reduction on soft set by using knowledge reduction of rough sets was developed by Chen et.al [8]. The definitions of soft set operations was modified by Cagman and Enginoglu [7] and gave a decision making method called uni-int decision making method. Sezgin and Atagun [16] has also studied on soft set operations which was defined by Ali et.al [3]. The soft BCK/BCI-algebras [11] and the applied soft sets in ideal theory of BCK/BCI-algebras were defined by Jun in 2008. The soft order semi group was also defined by Jun et.al [12]. Cagman et.al [7] defined a new structure called soft int-group and obtained some properties of this new structure. Aygunoglu and Aygun [6] defined the concept of fuzzy soft group structure and Karaaslan et.al [13] introduced the intuitionistic fuzzy soft groups. Zhang introduced the bipolar fuzzy set as a generalization of a fuzzy set. The bipolar fuzzy set is an extension of a fuzzy set whose membership degree interval is [−1, 1]. This research paper, studies the bipolar smooth fuzzy soft subgroups and investigate relations with bipolar smooth fuzzy soft cosets and normal soft subgroups. Thus, it defines the normalizer and gives some of the theorems relative to these concepts.

2. BIPOLAR SMOOTH FUZZY SOFT SUBGROUPS:

Definition 2.1 Let X be a set. Then a mapping $\delta : X \to S^*([-1,1])$ is called bipolar smooth fuzzy soft subset of X, where $S^*([-1,1])$ denotes the combination of non empty subsets [-1, 0] and [0, 1].

Example1 Consider the table BFS set $X = \{a,b,c,d,e\}$

δ^+	0.2	0.4	0.5	0.7	0.9
δ^-	− 0.1	− 0.3	− 0.2	− 0.6	− 0.7

defined by $\delta^+(a) = 0.2$, $\delta^+(b) = 0.4$, $\delta^+(c) = 0.5$, $\delta^+(d) = 0.7$, $\delta^+(e) = 0.9$ and $\delta^-(a) = -0.1$, $\delta^-(b) = -0.3$, $\delta^-(c) = -0.2$, $\delta^-(d) = -0.6$, $\delta^-(e) = -0.7$. Clearly, we can check that (δ^+, δ^-) is bipolar smooth fuzzy soft set.

Definition 2.2 Let X be a non empty set and δ, Δ be two bipolar smooth fuzzy soft subsets of X. Then the intersection of δ and Δ is denoted by $\delta \cap \Delta$ and defined by $(\delta \cap \Delta)(x) = \{\min\{p,q\}, p \in \delta(x), q \in \Delta(x) \text{ for all } x \in X\}$. The union of δ and Δ is denoted by $\delta \cup \Delta$ and defined by $(\delta \cup \Delta)(x) = \{\max\{p,q\}, p \in \delta(x), q \in \Delta(x) \text{ for all } x \in X\}$.

Definition 2.3 Let X be a groupoid (ie) a set which is closed under a binary relation denoted multiplicatively. A mapping $\delta : X \to S^*([-1,1])$ is called a bipolar smooth fuzzy soft groupoid if for all $x, y \in X$, the following conditions hold

(i) $\inf\left(\delta^P(x\,y)\right) \geq T\{\inf \delta^P(x), \inf \delta^P(y)\}$

(ii) $\sup\left(\delta^P(x\,y)\right) \geq T\{\sup \delta^P(x), \sup \delta^P(y)\}$

(iii) $\inf\left(\delta^N(x\,y)\right) \leq S\{\inf \delta^N(x), \inf \delta^N(y)\}$

(iv) $\sup\left(\delta^N(xy)\right) \le S\left\{\sup\delta^N(x), \sup\delta^N(y)\right\}$

Definition 2.4 Let G be a group. A mapping $\delta : G \to S^*([-1,1])$ is called a bipolar smooth fuzzy soft subgroup of G if for all $x, y \in G$, the following conditions hold

(i) $\inf\left(\delta^P(xy)\right) \ge T\left\{\inf\delta^P(x), \inf\delta^P(y)\right\}$

(ii) $\inf\left(\delta^N(xy)\right) \le S\left\{\inf\delta^N(x), \inf\delta^N(y)\right\}$

(iii) $\sup\left(\delta^P(xy)\right) \ge T\left\{\sup\delta^P(x), \sup\delta^P(y)\right\}$

(iv) $\sup\left(\delta^N(xy)\right) \le S\left\{\sup\delta^N(x), \sup\delta^N(y)\right\}$

(v) $\inf\delta^P(x^{-1}) \ge \inf\delta^P(x)$, $\inf\delta^N(x^{-1}) \le \inf\delta^N(x)$

(vi) $\sup\delta^P(x^{-1}) \ge \sup\delta^P(x)$, $\sup\delta^N(x^{-1}) \le \sup\delta^N(x)$

Proposition 2.1

If δ is a bipolar smooth fuzzy soft subgroupoid of a finite group G, then δ is a bipolar smooth fuzzy soft subgroup of G.

Proof:

Let $x \in G$. Since G is finite, x has a finite order, say n. then $x^n = e$, where e is the identity of G. Thus $x^{-1} = x^{n-1}$. Now using the definition 2.4, we have

$\inf\delta^P(x^{-1}) = \inf\delta^P(x^{n-1}) = \inf\delta^P(x^{n-2}x) \ge T\left\{\inf\delta^P(x^{n-2}), \inf\delta^P(x)\right\}$ and

$\inf\delta^N(x^{-1}) = \inf\delta^N(x^{n-1}) = \inf\delta^N(x^{n-2}x) \le S\left\{\inf\delta^N(x^{n-2}), \inf\delta^N(x)\right\}$

Again, $\inf\delta^P(x^{n-2}) = \inf\delta^P(x^{n-3}x) \ge T\left\{\inf\delta^P(x^{n-3}), \inf\delta^P(x)\right\}$ and

$\inf\delta^N(x^{n-2}) = \inf\delta^N(x^{n-3}x) \le S\left\{\inf\delta^N(x^{n-3}), \inf\delta^N(x)\right\}$.

So applying definition 2.4 respectively, we have that $\inf\delta^P(x^{-1}) \ge \inf\delta^P(x)$ and $\inf\delta^N(x^{-1}) \le \inf\delta^N(x)$. Similarly, we have $\sup\delta^P(x^{-1}) \ge \sup\delta^P(x)$ and $\sup\delta^N(x^{-1}) \le \sup\delta^N(x)$.

Hence δ is a bipolar smooth fuzzy soft subgroup G.

3. Bipolar Smooth Fuzzy Soft Cosets

We now define bipolar smooth fuzzy soft left cosets and bipolar smooth fuzzy soft right cosets.

Definition 3.1 Let δ be a bipolar smooth fuzzy soft subgroup of G. For any $x \in G$, define a mapping $\delta_{L(x)} : G \to S^*([-1,1])$ by $\delta_{L(x)}(g) = \delta(x^{-1}g)\ \forall\ g \in G$ and also define a mapping

$\delta_{R(x)}: G \rightarrow S^*([-1,1])$ by $\delta_{R(x)}(g) = \delta(g\,x^{-1})\ \forall\ g \in G$ then $\delta_{L(x)}, \delta_{R(x)}$ are respectively called bipolar smooth fuzzy soft left coset and bipolar smooth fuzzy soft right coset of G determined by x and δ.

In crisp concept, a subgroup H of a group G for which $aH = Ha$ holds for $a \in G$ (ie) left coset equals to corresponding right coset, is called a normal subgroup of G.

Here we extend this concept for bipolar smooth fuzzy soft subset. A bipolar smooth fuzzy soft subgroup of G is called normal if

$\inf \delta_{L(x)}(g) = \inf \delta_{R(x)}(g)$ and $\sup \delta_{L(x)}(g) = \sup \delta_{R(x)}(g)$.

$\inf \delta_{L(x)}(x^{-1}g) = \inf \delta_{R(x)}(g\,x^{-1})$ and $\sup \delta_{L(x)}(x^{-1}g) = \sup \delta_{R(x)}(g\,x^{-1})$.

Proposition 3.1

If δ is a bipolar smooth fuzzy soft subgroup of a group G having the identity e, then for all $x \in X$

(i) $\inf \delta^P(x^{-1}) = \inf \delta^P(x)$ and $\sup \delta^P(x^{-1}) = \sup \delta^P(x)$

$\inf \delta^N(x^{-1}) = \inf \delta^N(x)$ and $\sup \delta^N(x^{-1}) = \sup \delta^N(x)$

(ii) $\inf \delta^P(e) \geq \inf \delta^P(x)$, $\sup \delta^P(e) \geq \sup \delta^P(x)$ and

$\inf \delta^N(e) \leq \inf \delta^N(x)$, $\sup \delta^N(e) \leq \sup \delta^N(x)$

Proof:

(i) Assume δ is a bipolar smooth fuzzy soft subgroup of a group G, then $\inf \delta^P(x^{-1}) \geq \inf \delta^P(x)\ \forall\ x \in G$ again $\inf \delta^P(x) = \inf \delta^P((x^{-1})^{-1}) \geq \inf \delta^P(x^{-1})$.

So $\inf \delta^P(x^{-1}) = \inf \delta^P(x)$. Similarly we can prove that $\inf \delta^N(x^{-1}) = \inf \delta^N(x)$ and $\sup \delta^P(x^{-1}) = \sup \delta^P(x)$, $\sup \delta^N(x^{-1}) = \sup \delta^N(x)$ is proved.

(ii) $\inf \delta^P(e) = \inf \delta^P(x\,x^{-1}) \geq T\left\{\inf \delta^P(x), \inf \delta^P(x^{-1})\right\} = \inf \delta^P(x)$ and

$\sup \delta^P(e) = \sup \delta^P(x\,x^{-1}) \geq T\left\{\sup \delta^P(x), \sup \delta^P(x^{-1})\right\} = \sup \delta^P(x)$.

Similarly, $\inf \delta^N(e) \leq \inf \delta^N(x)$ and $\sup \delta^N(e) \leq \sup \delta^N(x)$ is proved.

Proposition 3.2

A bipolar smooth fuzzy soft subset δ of a group of G is a bipolar smooth fuzzy soft subgroup if and only if for all $x, y \in G$, the following are hold

(i) $\inf \delta^P(x\,y^{-1}) \geq T\left\{\inf \delta^P(x), \inf \delta^P(y)\right\}$ and

$\sup \delta^P(x\,y^{-1}) \geq T\left\{\sup \delta^P(x), \sup \delta^P(y)\right\}$

(ii) $\inf \delta^N(x\,y^{-1}) \leq S\left\{\inf \delta^N(x), \inf \delta^N(y)\right\}$ and

$$\sup\delta^N(xy^{-1}) \le S\{\sup\delta^N(x),\sup\delta^N(y)\}$$

Proof:

Let δ be a bipolar smooth fuzzy soft subgroup of G and $x, y \in G$. Then

$$\inf\delta^P(xy^{-1}) \ge T\{\inf\delta^P(x),\inf\delta^P(y^{-1})\}$$
$$= T\{\inf\delta^P(x),\inf\delta^P(y)\}$$
$$\sup\delta^P(xy^{-1}) \ge T\{\sup\delta^P(x),\sup\delta^P(y^{-1})\}$$
$$= T\{\sup\delta^P(x),\sup\delta^P(y)\}$$

Similarly,

$$\inf\delta^N(xy^{-1}) \le S\{\inf\delta^N(x),\inf\delta^N(y^{-1})\}$$
$$= S\{\inf\delta^N(x),\inf\delta^N(y)\}$$
$$\sup\delta^N(xy^{-1}) \le S\{\sup\delta^N(x),\sup\delta^N(y^{-1})\}$$
$$= S\{\sup\delta^N(x),\sup\delta^N(y)\}$$

Conversely, let δ be a bipolar smooth fuzzy soft subset of G and given conditions hold.

Then for all $x \in G$, we have to prove that δ is a bipolar smooth fuzzy soft subgroup of G.

$$\inf\delta^P(e) = \inf\delta^P(xx^{-1}) \ge T\{\inf\delta^P(x),\inf\delta^P(x)\} = \inf\delta^P(x) \qquad \ldots\ldots\ldots\ldots\ldots(1)$$
$$\sup\delta^P(e) = \sup\delta^P(xx^{-1}) \ge T\{\sup\delta^P(x),\sup\delta^P(x)\} = \sup\delta^P(x) \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

So,

$$\inf\delta^P(x^{-1}) = \inf\delta^P(ex^{-1}) \ge T\{\inf\delta^P(e),\inf\delta^P(x)\} = \inf\delta^P(x) \text{ by (1) and}$$
$$\sup\delta^P(x^{-1}) = \sup\delta^P(ex^{-1}) \ge T\{\sup\delta^P(e),\sup\delta^P(x)\} = \sup\delta^P(x) \text{ by (2)}$$

Similarly,

$$\inf\delta^N(x^{-1}) = \inf\delta^N(ex^{-1}) \le S\{\inf\delta^N(e),\inf\delta^N(x^{-1})\} = \inf\delta^N(x)$$
$$\sup\delta^N(x^{-1}) = \sup\delta^N(ex^{-1}) \le S\{\sup\delta^N(e),\sup\delta^N(x^{-1})\} = \sup\delta^N(x)$$

Again,

$$\inf\delta^P(xy^{-1}) \ge T\{\inf\delta^P(x),\inf\delta^P(y^{-1})\}, \text{ using given condition}$$
$$\ge T\{\inf\delta^P(x),\inf\delta^P(y)\} \text{ and}$$
$$\sup\delta^P(xy^{-1}) \ge T\{\sup\delta^P(x),\sup\delta^P(y^{-1})\}, \text{ using given condition}$$
$$\ge T\{\inf\delta^P(x),\inf\delta^P(y)\}.$$

Similarly,

$$\inf \delta^N(x y^{-1}) \leq S\left\{\inf \delta^N(x), \inf \delta^N(y^{-1})\right\}, \text{ using given condition}$$

$$\leq S\left\{\inf \delta^N(x), \inf \delta^N(y)\right\} \text{ and}$$

$$\sup \delta^N(x y^{-1}) \leq S\left\{\sup \delta^N(x), \sup \delta^N(y^{-1})\right\}, \text{ using given condition}$$

$$\leq S\left\{\sup \delta^N(x), \sup \delta^N(y)\right\}.$$

Hence δ is a bipolar smooth fuzzy soft subgroup of G.

Proposition 3.3

Intersection of any two bipolar smooth fuzzy soft subgroups of G is also a bipolar smooth fuzzy soft subgroup of G.

Proof:

Let δ and Δ be any two bipolar smooth fuzzy soft subgroups of G and $x, y \in G$.

Then

$$\inf (\delta^P \cap \Delta^P)(xy^{-1}) = T\left\{\inf \delta^P(xy^{-1}), \inf \Delta^P(xy^{-1})\right\} \text{ by definition}$$

$$\geq T\left\{T\left\{\inf \delta^P(x), \inf \delta^P(y)\right\}, T\left\{\inf \Delta^P(x), \inf \Delta^P(y)\right\}\right\} \text{ by proposition 3.2}$$

$$\geq T\left\{T\left\{\inf \delta^P(x), \inf \Delta^P(x)\right\}, T\left\{\inf \delta^P(y), \inf \Delta^P(y)\right\}\right\}$$

$$= T\left\{\inf (\delta^P \cap \Delta^P)(x), \inf (\delta^P \cap \Delta^P)(y)\right\}$$

Similarly,

$$\inf (\delta^N \cap \Delta^N)(xy^{-1}) = S\left\{\inf \delta^N(xy^{-1}), \inf \Delta^N(xy^{-1})\right\} \text{ by definition}$$

$$\leq S\left\{S\left\{\inf \delta^N(x), \inf \delta^N(y)\right\}, S\left\{\inf \Delta^N(x), \inf \Delta^N(y)\right\}\right\} \text{ by proposition 3.2}$$

$$\leq S\left\{S\left\{\inf \delta^N(x), \inf \Delta^N(x)\right\}, S\left\{\inf \delta^N(y), \inf \Delta^N(y)\right\}\right\}$$

$$= S\left\{\inf (\delta^N \cap \Delta^N)(x), \inf (\delta^N \cap \Delta^N)(y)\right\}$$

Again,

$$\sup (\delta^P \cap \Delta^P)(xy^{-1}) = T\left\{\sup \delta^P(xy^{-1}), \sup \Delta^P(xy^{-1})\right\} \text{ by definition}$$

$$\geq T\left\{T\left\{\sup \delta^P(x), \sup \delta^P(y)\right\}, T\left\{\sup \Delta^P(x), \sup \Delta^P(y)\right\}\right\} \text{ by proposition}$$

3.2

$$= T\left\{T\left\{\sup \delta^P(x), \sup \Delta^P(x)\right\}, T\left\{\sup \delta^P(y), \sup \Delta^P(y)\right\}\right\}$$

$$= T\left\{\sup (\delta^P \cap \Delta^P)(x), \sup (\delta^P \cap \Delta^P)(y)\right\}$$

Similarly,

$$\sup (\delta^N \cap \Delta^N)(xy^{-1}) = S\left\{\sup \delta^N(xy^{-1}), \sup \Delta^N(xy^{-1})\right\} \text{ by definition}$$

$$\leq S\left\{S\left\{\sup \delta^N(x), \sup \delta^N(y)\right\}, S\left\{\sup \Delta^N(x), \sup \Delta^N(y)\right\}\right\} \text{ by proposition}$$

3.2

$$= S\left\{ S\left\{ \sup \delta^N(x), \sup \Delta^N(x) \right\}, S\left\{ \sup \delta^N(y), \sup \Delta^N(y) \right\} \right\}$$

$$= S\left\{ \sup(\delta^N \cap \Delta^N)(x), \sup(\delta^N \cap \Delta^N)(y) \right\}$$

$\therefore\ \delta \cap \Delta$ is a bipolar fuzzy soft subgroup of G.

4. Bipolar smooth fuzzy soft normal subgroups

Definition 4.1 Let δ be a bipolar smooth fuzzy soft subgroup of G. Then δ is called a bipolar smooth fuzzy soft normal subgroup of G if $\inf \delta^P(xy) = \inf \delta^P(yx)$ and $\inf \delta^N(xy) = \inf \delta^N(yx)$. $\sup \delta^P(xy) = \sup \delta^P(yx)$ and $\sup \delta^N(xy) = \sup \delta^N(yx)$ for all $x, y \in G$.

Proposition 4.1

The intersection of any two bipolar smooth fuzzy soft normal subgroups of G is a bipolar smooth fuzzy soft normal subgroup of G.

Proof:

Let δ and Δ be any two bipolar smooth fuzzy soft normal subgroups of G and $x, y \in G$. Then

$$\inf(\delta^P \cap \Delta^P)(xy) = T\left\{ \inf \delta^P(xy), \inf \Delta^P(xy) \right\}$$

$$= T\left\{ \inf \delta^P(yx), \inf \Delta^P(yx) \right\} \text{ by definition 4.1}$$

$$= \inf(\delta^P \cap \Delta^P)(yx).$$

$$\sup(\delta^P \cap \Delta^P)(xy) = T\left\{ \sup \delta^P(xy), \sup \Delta^P(xy) \right\}$$

$$= T\left\{ \sup \delta^P(yx), \sup \Delta^P(yx) \right\} \text{ by definition 4.1}$$

$$= \sup(\delta^P \cap \Delta^P)(yx) \text{ and}$$

$$\inf(\delta^N \cap \Delta^N)(xy) = S\left\{ \inf \delta^N(xy), \inf \Delta^N(xy) \right\}$$

$$= S\left\{ \inf \delta^N(yx), \inf \Delta^N(yx) \right\} \text{ by definition 4.1}$$

$$= \inf(\delta^N \cap \Delta^N)(yx).$$

$$\sup(\delta^N \cap \Delta^N)(xy) = S\left\{ \sup \delta^N(xy), \sup \Delta^N(xy) \right\}$$

$$= S\left\{ \sup \delta^N(yx), \sup \Delta^N(yx) \right\} \text{ by definition 4.1}$$

$$= \sup(\delta^N \cap \Delta^N)(yx)$$

$\therefore\ \delta \cap \Delta$ is a bipolar smooth fuzzy soft normal subgroup of G.

Note 1

The intersection of any arbitrary collection of bipolar smooth fuzzy soft normal subgroups of G is also a bipolar smooth fuzzy soft normal subgroup of G.

Theorem 4.1

Let δ be a bipolar smooth fuzzy soft subgroup of G and $m \in G$. Then the bipolar smooth fuzzy soft subset $\Delta : G \to S^*([-1,1])$ defined by $\Delta(x) = \delta(m^{-1} x m) \quad \forall\ x \in G$, is a bipolar smooth fuzzy soft subgroup of G.

Proof:

Let $x, y \in G$. Then for all $m \in G$,

$\inf \Delta^P(xy^{-1}) = \inf \delta^P(m^{-1}\, x\, y^{-1} m)$ (by definition of $\Delta(x)$)

$= \inf \delta^P(m^{-1}\, x\, m\, m^{-1} y^{-1} m)$

$= \inf \delta^P((m^{-1}\, x\, m)(m^{-1} y\, m)^{-1})$

$\geq T\left\{\inf \delta^P(m^{-1}\, x\, m), \inf \delta^P(m^{-1} y\, m)\right\}$ ($\because \delta$ is a bipolar smooth fuzzy soft subgroup) $= T\left\{\inf \Delta^P(x), \inf \Delta^P(y)\right\}$

Again,

$\sup \Delta^P(xy^{-1}) = \sup \delta^P(m^{-1}\, x\, y^{-1} m)$ (by definition of $\Delta(x)$)

$= \sup \delta^P(m^{-1}\, x\, m\, m^{-1} y^{-1} m)$

$= \sup \delta^P((m^{-1}\, x\, m)(m^{-1} y\, m)^{-1})$

$\geq T\left\{\sup \delta^P(m^{-1}\, x\, m), \sup \delta^P(m^{-1} y\, m)\right\}$ ($\because \delta$ is a bipolar smooth fuzzy soft subgroup)

$= T\left\{\sup \Delta^P(x), \sup \Delta^P(y)\right\}$

Also

$\inf \Delta^N(xy^{-1}) = \inf \delta^N(m^{-1}\, x\, y^{-1} m)$ (by definition of $\Delta(x)$)

$= \inf \delta^N(m^{-1}\, x\, m\, m^{-1} y^{-1} m)$

$= \inf \delta^N((m^{-1}\, x\, m)(m^{-1} y\, m)^{-1})$

$\leq S\left\{\inf \delta^N(m^{-1}\, x\, m), \inf \delta^N(m^{-1} y\, m)\right\}$ ($\because \delta$ is a bipolar smooth fuzzy soft subgroup)

$= S\left\{\inf \Delta^N(x), \inf \Delta^N(y)\right\}$

Again,

$\sup \Delta^N(xy^{-1}) = \sup \delta^N(m^{-1}\, x\, y^{-1} m)$ (by definition of $\Delta(x)$)

$= \sup \delta^N(m^{-1}\, x\, m\, m^{-1} y^{-1} m)$

$$= \sup \delta^{N}((m^{-1} x m)(m^{-1} y m)^{-1})$$

$$\leq S\left\{\sup \delta^{N}(m^{-1} x m), \sup \delta^{N}(m^{-1} y m)\right\} \ (\because \delta \text{ is a bipolar smooth fuzzy soft subgroup})$$

$$= S\left\{\sup \Delta^{N}(x), \sup \Delta^{N}(y)\right\}$$

Hence by proposition 3.2, Δ is a bipolar smooth fuzzy soft subgroup of G.

Definition 4.2 Let δ and Δ be any two bipolar smooth fuzzy soft subgroups of G. We say that Δ is a conjugate to δ if for some $m \in G$,

$\inf \Delta^{P}(x) = \inf \delta^{P}(m^{-1} x m)$ and $\inf \Delta^{N}(x) = \inf \delta^{N}(m^{-1} x m)$,

$\sup \Delta^{P}(x) = \sup \delta^{P}(m^{-1} x m)$ and $\sup \Delta^{N}(x) = \sup \delta^{N}(m^{-1} x m)$ for all $x \in G$.

Proposition 4.2

For any bipolar smooth fuzzy soft subgroup δ of G and for all $x, y \in G$, the following equivalent

(i) $\inf \delta^{P}(x y x^{-1}) = \inf \delta^{P}(y)$ and $\sup \delta^{P}(x y x^{-1}) = \sup \delta^{P}(y)$,

$\inf \delta^{N}(x y x^{-1}) = \inf \delta^{N}(y)$ and $\sup \delta^{N}(x y x^{-1}) = \sup \delta^{N}(y)$.

(ii) $\inf \delta^{P}(x y) = \inf \delta^{P}(y x)$ and $\sup \delta^{P}(x y) = \sup \delta^{P}(y x)$,

$\inf \delta^{N}(x y) = \inf \delta^{N}(y x)$ and $\sup \delta^{N}(x y) = \sup \delta^{N}(y x)$.

(iii) $\inf \delta^{P}_{L(x)}(y) = \inf \delta^{P}_{R(x)}(y)$ and $\sup \delta^{P}_{L(x)}(y) = \sup \delta^{P}_{R(x)}(y)$,

$\inf \delta^{N}_{L(x)}(y) = \inf \delta^{N}_{R(x)}(y)$ and $\sup \delta^{N}_{L(x)}(y) = \sup \delta^{N}_{R(x)}(y)$.

Proof:

Let $x, y \in G$ and δ be a bipolar smooth fuzzy soft subgroup of a group G.

$(i) \Rightarrow (ii)$ $\inf \delta^{P}(y x) = \inf \delta^{P}(x^{-1} x y x) = \inf \delta^{P}(x y)$, using (i)

$\inf \delta^{N}(y x) = \inf \delta^{N}(x^{-1} x y x) = \inf \delta^{N}(x y)$, using (i)

and $\sup \delta^{P}(y x) = \sup \delta^{P}(x^{-1} x y x) = \sup \delta^{P}(x y)$, using (i)

$\sup \delta^{N}(y x) = \sup \delta^{N}(x^{-1} x y x) = \sup \delta^{N}(x y)$, using (i)

$(ii) \Rightarrow (iii)$ $\inf \delta^{P}_{L(x)}(y) = \inf \delta^{P}(x^{-1} y)$

$= \inf \delta^{P}(y x^{-1})$, using (ii)

$= \inf \delta^{P}_{R(x)}(y)$

$\text{Inf } \delta^{N}_{L(x)}(y) = \inf \delta^{N}(x^{-1} y)$

$= \inf \delta^{N}(y x^{-1})$, using (ii)

$$= \inf \delta^{N}_{R(x)}(y)$$

and $\quad \sup \delta^{P}_{L(x)}(y) = \sup \delta^{P}(x^{-1}y)$

$$= \sup \delta^{P}(y\,x^{-1}), \text{ using (ii)}$$
$$= \sup \delta^{P}_{R(x)}(y)$$
$$\sup \delta^{N}_{L(x)}(y) = \sup \delta^{N}(x^{-1}y)$$
$$= \sup \delta^{N}(y\,x^{-1}), \text{ using (ii)}$$
$$= \sup \delta^{N}_{R(x)}(y)$$

$(iii) \Rightarrow (i)$ $\quad \inf \delta^{P}(x\,y\,x^{-1}) = \inf \delta^{P}_{R(x)}(x\,y)$

$$= \inf \delta^{P}_{L(x)}(x\,y), \text{ using (iii)}$$
$$= \inf \delta^{P}_{R(x)}(y)$$
$$\inf \delta^{N}(x\,y\,x^{-1}) = \inf \delta^{N}_{R(x)}(x\,y)$$
$$= \inf \delta^{N}_{L(x)}(x\,y), \text{ using (iii)}$$
$$= \inf \delta^{N}_{R(x)}(y)$$

and $\quad \sup \delta^{P}(x\,y\,x^{-1}) = \sup \delta^{P}_{R(x)}(x\,y)$

$$= \sup \delta^{P}_{L(x)}(x\,y), \text{ using (iii)}$$
$$= \sup \delta^{P}(x^{-1}x\,y)$$
$$= \sup \delta^{P}(y), \text{ using (i)}$$
$$\sup \delta^{N}(x\,y\,x^{-1}) = \sup \delta^{N}_{R(x)}(x\,y)$$
$$= \sup \delta^{N}_{L(x)}(x\,y), \text{ using (iii)}$$
$$= \sup \delta^{N}(x^{-1}x\,y)$$
$$= \sup \delta^{N}(y), \text{ using (i)}$$

Hence the proof.

Definition 4.3 A bipolar smooth fuzzy soft subgroup δ of a group G is called a self conjugate bipolar smooth fuzzy soft subgroup if for all $m, x \in G$,

$\inf \delta^{P}(x) = \inf \delta^{P}(m^{-1}\,x\,m)$ and $\inf \delta^{N}(x) = \inf \delta^{N}(m^{-1}\,x\,m)$,

$\sup \delta^{P}(x) = \sup \delta^{P}(m^{-1}\,x\,m)$ and $\sup \delta^{N}(x) = \sup \delta^{N}(m^{-1}\,x\,m)$.

Theorem 4.2

A bipolar smooth fuzzy soft subgroup δ of a group G is normal iff δ is self conjugate bipolar smooth fuzzy soft subgroup.

Proof:

Let a bipolar smooth fuzzy soft subgroup δ of a group G be normal. Then $\inf \delta^{P}(xy) = \inf \delta^{P}(yx)$ and $\inf \delta^{N}(xy) = \inf \delta^{N}(yx)$,

$\sup \delta^{P}(xy) = \sup \delta^{P}(yx)$ and $\sup \delta^{N}(xy) = \sup \delta^{N}(yx)$ for all $x, y \in G$. Then by the proposition 4.2, we have $\inf\, \delta^{P}(x\,y\,x^{-1}) = \inf\, \delta^{P}(y)$ and $\sup \delta^{P}(x\,y\,x^{-1}) = \sup \delta^{P}(y)$, $\inf\, \delta^{N}(x\,y\,x^{-1}) = \inf\, \delta^{N}(y)$ and $\sup \delta^{N}(x\,y\,x^{-1}) = \sup \delta^{N}(y)$ for all $x, y \in G$. So, δ is a self conjugate bipolar smooth fuzzy soft subgroup.

Conversely assume that δ is a self conjugate bipolar smooth fuzzy soft subgroup. To prove that δ is normal. Now, δ is a self conjugate bipolar smooth fuzzy soft subgroup.

$\therefore \inf\, \delta^{P}(x\,y\,x^{-1}) = \inf\, \delta^{P}(y)$ and $\sup \delta^{P}(x\,y\,x^{-1}) = \sup \delta^{P}(y)$,

$\inf\, \delta^{N}(x\,y\,x^{-1}) = \inf\, \delta^{N}(y)$ and $\sup \delta^{N}(x\,y\,x^{-1}) = \sup \delta^{N}(y)$.

Then again by previous proposition 4.2 we have

$\inf\, \delta^{P}(xy) = \inf\, \delta^{P}(yx)$ and $\sup \delta^{P}(xy) = \sup \delta^{P}(yx)$,

$\inf\, \delta^{N}(xy) = \inf\, \delta^{N}(yx)$ and $\sup \delta^{N}(xy) = \sup \delta^{N}(yx)$.

So, δ is normal.

5. Bipolar smooth fuzzy soft Normalizer

Definition 5.1 Let δ be a bipolar smooth fuzzy soft subgroup of G. Then normalizer of δ is defined by $N(\delta) = \left\{ \begin{array}{r} m \in G \,/\, \forall\; x \in G, \inf\, \delta(m^{-1} x\, m) = \inf\, \delta(x) \\ \sup \delta(m^{-1} x\, m) = \sup \delta(x) \end{array} \right\}$.

Theorem 5.1

Let δ be a bipolar smooth fuzzy soft normal subgroup of G. Then

(i) $N(\delta)$ is a soft subgroup of G.

(ii) If $\Delta : N(\delta) \to S^{*}([-1,1])$ is defined by $\Delta(x) = \delta(x)\ \forall\ x \in N(\delta)$, then Δ is a bipolar smooth fuzzy soft normal subgroup of $N(\delta)$.

Proof:

(i) Let $x, y \in N(\delta)$. Then for all $g \in G$

$$\begin{aligned} \inf\, \delta^{P}((x\,y)^{-1} g (x\,y)) &= \inf\, \delta^{P}(y^{-1} x^{-1} g\, x\, y) \\ &= \inf\, \delta^{P}(x^{-1} g\, x) \\ &= \inf\, \delta^{P}(g) \quad (\because\ y \in N(\delta)\ ,\ x^{-1} g\, x \in G). \end{aligned}$$

$$\begin{aligned} \inf\, \delta^{N}((x\,y)^{-1} g (x\,y)) &= \inf\, \delta^{N}(y^{-1} x^{-1} g\, x\, y) \\ &= \inf\, \delta^{N}(x^{-1} g\, x) \\ &= \inf\, \delta^{N}(g) \quad (\because\ y \in N(\delta)\ ,\ x^{-1} g\, x \in G). \end{aligned}$$

Since $x \in N(\delta)$, similarly

$\sup \delta^P((x\,y)^{-1} g(x\,y)) = \sup \delta^P(g)$ and

$\sup \delta^N((x\,y)^{-1} g(x\,y)) = \sup \delta^N(g)$.

So $x\,y \in N(\delta)$. Again $g \in G$, $x \in N(\delta) \Rightarrow x\,g\,x^{-1} \in G$, then for all $g \in G$,

$\inf \delta^P(x\,g\,x^{-1}) = \inf \delta^P(x^{-1}(x\,g\,x^{-1}))$ and

$\inf \delta^N(x\,g\,x^{-1}) = \inf \delta^N(x^{-1}(x\,g\,x^{-1}))$. Since $x \in N(\delta)$, $x\,g\,x^{-1} \in G$

$\Rightarrow \inf \delta^P(x^{-1} x\,g\,x^{-1} x) = \inf \delta^P(g)$ and

$\inf \delta^N(x^{-1} x\,g\,x^{-1} x) = \inf \delta^N(g)$.

Similarly, $\sup \delta^P(x\,g\,x^{-1}) = \sup \delta^P(g)$ and $\sup \delta^N(x\,g\,x^{-1}) = \sup \delta^N(g)$.

So $x^{-1} \in N(\delta)$. Hence $N(\delta)$ is a soft subgroup of G.

(ii) Since δ is a bipolar smooth fuzzy soft normal subgroup of G and we have proved that $N(\delta)$ is a soft subgroup of G.

Then δ is a bipolar smooth fuzzy soft subgroup of $N(\delta)$. Hence Δ is a bipolar smooth fuzzy soft subgroup of $N(\delta)$.

Now we have to prove that Δ is normal. Since $N(\delta)$ is a soft subgroup of G, then $x, y \in N(\delta) \Rightarrow x^{-1} y\,x \in N(\delta)$.

Now, by definition of Δ, we have for all $x, y \in N(\delta)$,

$\inf \Delta^P(x^{-1} y\,x) = \inf \delta^P(x^{-1} y\,x)$ and $\inf \Delta^N(x^{-1} y\,x) = \inf \delta^N(x^{-1} y\,x)$.

Since $x^{-1} y\,x \in N(\delta)$.

$= \inf \delta^P(y)$ and $\inf \delta^N(y)$ since $x \in N(\delta)$.

$= \inf \Delta^P(y)$ and $\inf \Delta^N(y)$ since $x \in N(\delta)$.

Similarly, $\sup \Delta^P(x\,y\,x^{-1}) = \sup \Delta^P(y)$ and $\sup \Delta^N(x\,y\,x^{-1}) = \sup \Delta^N(y)$.

Hence Δ is self conjugate bipolar smooth fuzzy soft subgroup of $N(\delta)$.

Hence by theorem 4.2, Δ is a bipolar smooth fuzzy soft subgroup of $N(\delta)$.

Hence the proof.

Theorem 5.2

Let δ be a bipolar smooth fuzzy soft subgroup of a group G. Define

$H = \{ g \in G / \inf \delta(g) = \inf \delta(e) \text{ and } \sup \delta(g) = \sup \delta(e) \}$,

$K = \{ g \in G / \inf \delta_{R(g)}(x) = \inf \delta_{R(e)}(x) \text{ and } \sup \delta_{R(g)}(x) = \sup \delta_{R(e)}(x) \}$

be two soft subgroups of G. Then H=K.

Proof:

Let δ be a bipolar smooth fuzzy soft subgroup of a group G and $g, h \in H$. Then by proposition 3.2, we have

$$\begin{aligned}\inf \delta^P(g h^{-1}) &\geq T\left\{\inf \delta^P(g), \inf \delta^P(h)\right\}\\ &= T\left\{\inf \delta^P(e), \inf \delta^P(e)\right\}\\ &= \inf \delta^P(e) \quad \text{and}\end{aligned}$$

$$\begin{aligned}\inf \delta^N(g h^{-1}) &\leq S\left\{\inf \delta^N(g), \inf \delta^N(h)\right\}\\ &= S\left\{\inf \delta^N(e), \inf \delta^N(e)\right\}\\ &= \inf \delta^N(e)\end{aligned}$$

Again by proposition 3.1, we have

$\inf \delta^P(e) \geq \inf \delta^P(g h^{-1})$ and $\inf \delta^N(e) \leq \inf \delta^N(g h^{-1})$.

Hence $\inf \delta^P(g h^{-1}) = \inf \delta^P(e)$ and

$\inf \delta^N(g h^{-1}) = \inf \delta^N(e)$.

Similarly, by proposition 3.2 and 3.1, we have

$\sup \delta^P(g h^{-1}) = \sup \delta^P(e)$ and

$\sup \delta^N(g h^{-1}) = \sup \delta^N(e)$. So $g h^{-1} \in H$.

Hence H is a normal soft subgroup of G.

We now show that $H = K$. Let $h \in H$.

So, $\inf \delta^P(h) = \inf \delta^P(e)$ and $\sup \delta^P(h) = \sup \delta^P(e)$

$\inf \delta^N(h) = \inf \delta^N(e)$ and $\sup \delta^N(h) = \sup \delta^N(e)$.

Now for all $x \in G$,

$\inf \delta^P_{R(h)}(x) = \inf \delta^P(x h^{-1})$ and

$\inf \delta^N_{R(h)}(x) = \inf \delta^N(x h^{-1})$.

$$\begin{aligned}\inf \delta^P_{R(h)}(x) &= \inf \delta^P(x h^{-1})\\ &\geq T\left\{\inf \delta^P(x), \inf \delta^P(h)\right\}\\ &= T\left\{\inf \delta^P(x), \inf \delta^P(e)\right\} \text{ since } \delta \text{ is a BFSS of G and } h \in H.\\ &= \inf \delta^P(x) \text{ . Since by preposition 3.1,}\end{aligned}$$

$\inf \delta^P(e) \geq \inf \delta^P(x) = \inf \delta^P(x e^{-1}) = \inf \delta^P_{R(e)}(x)$.

$$\therefore \inf \delta^P_{R(h)}(x) = \inf \delta^P_{R(e)}(x) \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

$\inf \delta^N_{R(e)}(x) = \inf \delta^N(xe^{-1})$

$= \inf \delta^N(x)$

$= \inf \delta^N(xh^{-1}h)$

$\geq T\{\inf \delta^N(x\,h^{-1}), \inf \delta^N(h)\}$ since δ be a BFSS of G and $h \in H$.

$= T\{\inf \delta^N(x\,h^{-1}), \inf \delta^N(e)\}$.

$= \inf \delta^N(x\,h^{-1})$. Since by preposition (3.1) ,

$\inf \delta^N(e) \geq \inf \delta^N(xh^{-1}) = \inf \delta^N_{R(h)}(x)$.

$\therefore \inf \delta^N_{R(e)}(x) = \inf \delta^N_{R(h)}(x)$ ....(4)

Hence by (3) and (4), we have

$\inf \delta^P_{R(h)}(x) = \inf \delta^P_{R(e)}(x)$ and $\inf \delta^N_{R(h)}(x) = \inf \delta^N_{R(e)}(x)$

Similarly, we can prove

$\sup \delta^P_{R(h)}(x) = \sup \delta^P_{R(e)}(x)$ and $\sup \delta^N_{R(h)}(x) = \sup \delta^N_{R(e)}(x)$

This implies that $h \in K$.

$\therefore \; h \in H \Rightarrow h \in K$

Hence $H \subseteq K$ ....(5)

Now suppose $h \in K$, then for all $x \in G$,

$\inf \delta^P_{R(k)}(x) = \inf \delta^P_{R(e)}(x)$ and $\sup \delta^P_{R(k)}(x) = \sup \delta^P_{R(e)}(x)$.

$\inf \delta^N_{R(k)}(x) = \inf \delta^N_{R(e)}(x)$ and $\sup \delta^N_{R(k)}(x) = \sup \delta^N_{R(e)}(x)$.

This implies that

$\inf \delta^P(xR^{-1}) = \inf \delta^P(x)$ and $\sup \delta^P(xR^{-1}) = \sup \delta^P(x)$

$\inf \delta^N(xR^{-1}) = \inf \delta^N(x)$ and $\sup \delta^N(xR^{-1}) = \sup \delta^N(x)$

Choosing $x = e$, we obtain

$\inf \delta^P(R^{-1}) = \inf \delta^P(e)$ and $\sup \delta^P(R^{-1}) = \sup \delta^P(e)$

$\inf \delta^N(R^{-1}) = \inf \delta^N(e)$ and $\sup \delta^N(R^{-1}) = \sup \delta^N(e)$.

Hence $h^{-1} \in H$. Since H is a subgroup. So $h \in H$.

$\therefore \; h \in K \Rightarrow h \in H$

Thus, we have $K \subseteq H$ ....(6)

$\therefore \; H = K$ (from equations (5) and (6))

Hence the proof

Remark:

In theorem 5.2, if

$K=\left\{g\in G/\forall\ x\in G,\ \inf\ \delta_{L(g)}(x)=\inf\ \delta_{L(e)}(x)\ \text{and}\ \sup\delta_{L(g)}(x)=\sup\delta_{L(e)}(x)\right\}$, then we have the following theorem.

Theorem 5.3

Let δ be a bipolar smooth fuzzy soft subgroup of G and Δ be a bipolar smooth fuzzy soft normal subgroup of G. Then $\delta\cap\Delta$ is a bipolar smooth fuzzy soft normal subgroup of G.

Proof:

By applying theorem 5.2, H is a soft subgroup of G and proposition 3.3 , $\delta\cap\Delta$ is a bipolar smooth fuzzy soft subgroup of H. We now show that $\delta\cap\Delta$ is a bipolar smooth fuzzy soft normal subgroup of H.

Let $x, y\in H$. Since H is a soft subgroup, xy and $yx\in H$.

Now

$$\begin{aligned}\inf(\delta^P\cap\Delta^P)(x\,y) &= T\left\{\inf\ \delta^P(x\,y),\inf\ \Delta^P(x\,y)\right\}\\ &= T\left\{\inf\ \delta^P(x\,y),\inf\ \Delta^P(y\,x)\right\} \quad \because\ \Delta \text{ is a BSFSN}\\ &= T\left\{\inf\ \delta^P(y\,x),\inf\ \Delta^P(y\,x)\right\} \quad \because\ \delta \text{ is a BSFSN}\\ &= \inf(\delta^P\cap\Delta^P)(y\,x)\end{aligned}$$

Also

$$\begin{aligned}\inf(\delta^N\cap\Delta^N)(x\,y) &= S\left\{\inf\ \delta^N(x\,y),\inf\ \Delta^N(x\,y)\right\}\\ &= S\left\{\inf\ \delta^N(x\,y),\inf\ \Delta^N(y\,x)\right\}\\ &= S\left\{\inf\ \delta^N(y\,x),\inf\ \Delta^N(y\,x)\right\}\\ &= \inf(\delta^N\cap\Delta^N)(y\,x)\end{aligned}$$

Hence $\delta\cap\Delta$ is a bipolar smooth fuzzy soft normal subgroup of H.

Hence the proof.

Conclusion

The fundamental properties of smooth fuzzy soft normal subgroupoids have been discussed in this paper. Also, smooth fuzzy soft cosets and its smooth fuzzy soft normal subgroups are discussed. Finally, the bipolar smooth fuzzy soft normalizer has been investigated.

SCOPE FOR FUTURE RESEARCH:

Bipolar smooth fuzzy soft set theorems are applied to solve Reynold's equations. Researcher may further study this idea into Legendre equation and Legendre function to find stipulated points in the real line structure.

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