International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, **Email: ijesmj@gmail.com**
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

NUMERICAL STUDY OF MAGNETO-HYDRODYNAMIC THREE-DIMENSIONAL COUETTE FLOW THROUGH A MEDIUM IN THE PRESENCE OF HEAT SOURCE

Kotagiri Srihari
Department of Mathematics,Mahatma Gandhi Institute of Technology
Gandipet, Hyderabad, Telangana State, India, 500075.

1 Abstract

In the present article numerical simulation is made using an implicit finite difference scheme on Couette flow of a viscous incompressible electrically conducting fluid between two horizontal parallel porous flat plates in the presence of heat source.A magnetic field is applied normal to the fluid flow.The plate in uniform motion and stationary plate are subjected to constant suction and the transverse sinusoidal injection of the fluid respectively. This type of sinusoidal injection velocity makes the flowthree-dimensional. Obtaining exact solution of thisproblem is very difficult, so the governing non-linear equations with boundary conditions are transformed to ordinary and partial differential equations of zeroth and first order respectively, using perturbation method. Exact solutions correspondingto the zeroth order differential equations are obtained in the case of $\varepsilon = 0$. Thefirst order differential equations in the case of $\varepsilon \neq 0$ are reduced to ordinary differential equationsusing appropriate substitutions. Then, these resultant equations are transformed to system of equations using implicit-finite difference method, for which simulation is carried out by coding inC-Program**.** The results obtained for Nusselt number are discussed graphically and compared with previously published work in the absence of heat source parameter.These comparisons have shown a good agreement between the results. From this study, it is observed that the temperature and Nusselt number increasein the presence of heat source parameter.

Key words
Three-dimensional, Couette flow, magnetic field, heat source, permeability, Finite-difference method;

Introduction

In recent years the fluid flow through porous media have attracted the attention of a number of researchers because of their applications in many branches of science and technology. In fact a porous material containing the fluid is a non-homogeneous medium. For the sake ofanalysis it is also considered as a homogeneous medium, byconsidering its dynamical properties to be equal to the local averages of the original non-homogeneous continuum. So a difficult problem of the fluid flow through a porous medium gets reduced to the flow problem of a homogeneous fluid with additional resistance. A series of investigation have been made by different scholars where the porous medium is either bounded by horizontal or vertical surfaces. Unsteady flow of a conducting fluid through porous medium in the presence of magnetic field was studied by Gulub Ram and Mishra

[1]. Oscillatory flow through porous medium bounded by a horizontal porous plate with variable suction velocitywas analysed by Varshney [2].Unsteady free-convection flow through a porous medium bounded by an infinite porous plate with variable temperatureand constant suction was investigated by Raptis [3].Further, Raptis and Perdikis [4] discussed the oscillatory free convective flow through a porous medium bounded by a vertical porous plate with constant suction when the velocity of free stream oscillates about a mean in time. Gersten and Gross [6] studied the three dimensional convection flow and heat transfer through a porous medium, later this study was extended by Singh *et al* [5] by considering effect periodic suction velocity into account. The variation of such suction velocity on different heat transfer flow problems along flat and vertical porous plates have also been studied by Singh [7, 8].

The study of channel flows through porous medium play a vital role in many scientific and engineering investigations like geophysical applications, for example, In chemical engineering for filtration and purification processes; to study the underground water resourcesin agriculture engineering; to study the movement of natural gas, oil and water through the oil channels/reservoirsin petroleum technologies. Due to these applications, Couette flow with transpiration cooling for ordinary medium has been studied bySingh [9]. Singh and Sharma [10] later investigated magneto hydrodynamic three-dimensional Couette flow with transpiration cooling. Recently Singh et al [11]analysed the effect of permeability of the porous medium on three-dimensional couette flow and heat transfer.Three-dimensional Couette flow andheat transfer through a porous medium with variable permeabilitywas discussed by Choudhary and Sharma [12]. Singh[13] studied the influence of a moving magnetic field on three-dimensional Couette flow. Theeffect of magnetohydrodynamics on three-dimensional viscous incompressible heat transfer flow between two horizontal parallel porous plates with periodic injection/suction was discussed byPawan Kumar Sharma and Chaudhary [14]. Sharma and Yadav [15] analyzed the effects of heat transfer on three dimensional flow between a stationary porous plate and a moving porous plate. Three Couette flow between two parallel plates with transpiration cooling through a porous medium in the slip flow regime has been investigatedby Jain et al. [16]. Govindarajan et al. [17] discussed the three-dimensional Couette flow of a viscous incompressible dusty fluid with transpiration cooling.

Das et al. [18] analyzed the three dimensional Couette flow and heat transfer in the presence of a transverse magnetic field. The effect of viscous dissipation onCouette flow of a viscous incompressible fluid through a porous medium between two infinite horizontal parallel porous flat plates in the presence of transverse magnetic field is studied by Srihari et al [19]. Das et al. [20] studied the effects of constant suction and sinusoidal injection on hydromagnetic three dimensional Couette flow of a viscous incompressible fluid between two infinite horizontal parallel porous flat plates.An analytical solution for three-dimensional mixed convective mass transfer flow along an infinite vertical porous plate by the presence of a magnetic field wasobtainedAhmed [21]. Ahmed and Talukdar [22] discussed the effect of chemical reaction in the presence of heat sink on oscillatory three-dimensional flow with mass transfer past a vertical plate with thermal diffusion. Recently, the effects ofan exponential injection/suction parameter onthree-dimensional

International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, **Email: ijesmj@gmail.com**
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Couette flow of a dusty fluid with heat transferwas analysedby Gireesha [23].Hayat et al. [24] investigated the three-dimensional flow of viscous fluid with convective boundary conditions and heat generation/absorption.Due to the practical applications of Magneto hydrodynamicsin the field of power generation,geophysics and astrophysics, Shehzad et al [26] studied the effect of magnetic field on three-dimensional flow of an Oldroyd-B nanofluid over a radiative surface.Baag and Mishra [25] made an analysis of heat and mass transfer on three-dimensional hydromagnetic nano-fluid flow.

The effect of heat sourcehas not been consideredin most of the previous investigations but it plays an important role in maintaining heat transfer at desired level in the fields ofNuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles. So, in the present paper an attempt has been made to study the effect of heat source on couette flow of a viscous incompressible fluid through a porous medium between two horizontal parallel porous flat plates.A magnetic field is introducedperpendicular to the flow.In order to describe the physics of the problem, first the non-linear boundary value problem is transformed to ordinary and partial differential equations of zeroth and first order respectively, usingperturbation method. Obtaining exact solution for first order partial differential equations with boundary conditions is very difficult so,using appropriate substitutions; theseequations are reduced to coupled non-linear ordinary differential equations, then they are solved numerically using finite difference method.

2Mathematical analysis

Couette flow of an incompressible viscous electrically conducting fluid between two parallel porous flat plates is considered. A co-ordinate system is introduced with the origin at the lower stationary plate, lying horizontally on $x^* - z^*$ plane. The upper plate, which is in uniform motion U, is separated by a distance'd' from the lower plate. The y^* - axis is taken perpendicular to the planes of the plates. A magnetic field is applied normal to the fluid flow.The lower and the upper plates are assumed to be at constant temperatures T_0 and T_1, respectively, with $T_1 > T_0$. The upper plate is subjected to a constant suction V whereas the lower plate to a transverse sinusoidal injection velocity distribution of the form:

$$v^*(z^*) = V(1 + \varepsilon \cos \pi z^* / d). \qquad (1)$$

where $\varepsilon(<<1)$ is a positive constant. Without loss of generality, the distance'd'is considered as wavelength of the injection velocity.The velocity components u∗, v∗ and w∗ are considered in the directions of x∗, y∗ and z∗, respectively.The governing equations are derived with the assumption that the flow is steady and laminar, and is of a finitely conducting fluid.For this fully developed laminar flowproblem, all physical quantities are independent of x^* but the flow remains three-dimensional due to the periodic injectionvelocity (1). Thus, under these assumptions, the problem is governed by the following equations:

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0 \qquad (2)$$

$$v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=\upsilon\left(\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{\partial^2 u^*}{\partial z^{*2}}\right)-\frac{\upsilon u^*}{K^*}-\frac{\sigma B_0^2}{\rho}u^* \qquad (3)$$

$$v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\upsilon\left(\frac{\partial^2 v^*}{\partial y^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right)-\frac{\upsilon}{K^*}v^* \qquad (4)$$

$$v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}+\upsilon\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right)-\frac{\upsilon}{K^*}w^*-\frac{\sigma B_0^2}{\rho}w^* \qquad (5)$$

$$\rho C_p\left(v^*\frac{\partial T^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}\right)=k\left(\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right)+Q(T^*-T_0) \qquad (6)$$

The boundary conditions of the problem are:

$$\left.\begin{array}{l} y^*=0:\ u^*=0, v^*(z^*)=V(1+\varepsilon\cos\pi z^*/d), w^*=0, T^*=T_0^* \\ y^*=d: u^*=U, v^*=V,\ w^*=0,\ T^*=T_1 \end{array}\right\} (7)$$

Substitution the following non dimensional quantities:

$$y=\frac{y^*}{d},\ z=\frac{z^*}{d},\ u=\frac{u^*}{U},\ v=\frac{v^*}{U},$$

$$w=\frac{w^*}{U},\ p=\frac{p^*}{\rho U^2},\ \theta=\frac{T^*-T_0}{T_1-T_0}, \qquad (8)$$

into equations (2) to (7), implies the following

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (9)$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{u}{\mathrm{Re}\,K}-\frac{M^2}{\mathrm{Re}}u \qquad (10)$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{v}{\mathrm{Re}\,K} \qquad (11)$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{w}{\mathrm{Re}\,K}-\frac{M^2}{\mathrm{Re}}w \qquad (12)$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)+\frac{S}{\mathrm{Re}\,\mathrm{Pr}}\theta \qquad (13)$$

with boundary conditions in non-dimensional form

$$\left.\begin{array}{l} y=0:\ u=0,\ v(z)=\alpha(1+\varepsilon\cos\pi z),\ w=0,\ \theta=0 \\ y=1:\ u=1,\ v=\alpha,\ w=0,\ \theta=1 \end{array}\right\} (14)$$

where,

$S = \frac{Qd^2}{k}$ (Heat source parameter), $\mathrm{Re} = \frac{Ud}{\upsilon}$ (Injection/suction parameter),

$P_r = \frac{\mu C_p}{k}$ (Prandtl number), $K = \frac{K^*}{d^2}$ (Permeability parameter),

$M = B_0 d\sqrt{\frac{\sigma}{\mu}}$ (Magnetic parameter),

3. Method of solution

In order to solve these non-linear differential equations, the solutions of the following form are assumed because the amplitude $\varepsilon, (\langle\langle 1)$ of the injection velocity is very small:

$$f(y, z) = f_0(y) + \varepsilon f_1(y,z) + \varepsilon^2 f_2(y,z) + \dots \quad (15)$$

where f stands for u, v, w, p and θ.

Substituting the equation (15) into equations(9) to (13) and equating the like power of ε,we get the following zeroth and first order equationsfor the case of $\varepsilon = 0$, and $\varepsilon \neq 0$**,** respectively.

3.1 Zeroth order equations for the caseε=0

$$\frac{dv_0}{dy} = 0 \quad (16)$$

$$\frac{d^2u_0}{dy^2} - \alpha \mathrm{Re}\frac{du_0}{dy} - \left(M^2 + \frac{1}{K}\right)u_0 = 0 \quad (17)$$

$$\frac{d^2\theta_0}{dy^2} - \alpha \mathrm{Re}\,\mathrm{Pr}\frac{d\theta_0}{dy} + S\theta_0 = 0 \quad (18)$$

The corresponding boundary conditions reduces to

$$y = 0;\ u_0 = 0,\ v_0 = \alpha,\ w_0 = 0,\ \theta_0 = 0 \quad (19)$$

$$y = 1;\ u_0 = 1,\ v_0 = \alpha,\ w_0 = 0,\ \theta_0 = 1$$

The solutions of the above equations are given by

$$u_0(y) = \frac{e^{m_1 y} - e^{m_2 y}}{e^{m_1} - e^{m_2}}, \quad (20)$$

$$\theta_0(y) = \frac{e^{a_2 y} - e^{m_1 y}}{e^{a_2} - e^{a_1}} \quad (21)$$

$$v_0 = \alpha = 1,\quad w_0 = 0,\quad p_0 = \text{constant} \quad (22)$$

where

$$m_1 = \frac{1}{2}\left[\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4\left(M^2 + \frac{1}{K}\right)}\right], \quad m_2 = \frac{1}{2}\left[\mathrm{Re} - \sqrt{\mathrm{Re}^2 + 4\left(M^2 + \frac{1}{K}\right)}\right]$$

International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

$$a_1 = \frac{1}{2}\left[\mathrm{Re}\,\mathrm{Pr} + \sqrt{(\mathrm{Re}\,\mathrm{Pr})^2 - 4S}\right] \quad a_2 = \frac{1}{2}\left[\mathrm{Re}\,\mathrm{Pr} - \sqrt{(\mathrm{Re}\,\mathrm{Pr})^2 - 4S}\right],$$

3.2 First order equationsfor the case $\varepsilon \neq 0$

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{23}$$

$$v_1\frac{\partial u_0}{\partial y} + \frac{\partial u_1}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \frac{u_1}{\mathrm{Re}\,K} - \frac{M^2}{\mathrm{Re}}u_1 \tag{24}$$

$$\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{v_1}{\mathrm{Re}\,K} \tag{25}$$

$$\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{w_1}{\mathrm{Re}\,K} - \frac{M^2}{\mathrm{Re}}w_1 \tag{26}$$

$$v_1\frac{\partial \theta_0}{\partial y} - \frac{\partial \theta_1}{\partial y} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) + \frac{S}{\mathrm{Re}\,\mathrm{Pr}}\theta_1 \tag{27}$$

Thecorresponding boundary conditions reduced to the following form:

$$y = 0;\ u_1 = 0,\ v_1 = \cos\pi z,\ w_1 = 0,\ \theta_1 = 0$$
$$y = 1;\ u_1 = 0,\ v_1 = 0,\ w_1 = 0,\ \theta_1 = 0 \tag{28}$$

The equations, which are obtained forthe case,$\varepsilon \neq 0$ describe the three-dimensional flow through porous medium.

In order to solve the equations(23), (25) and (26),we assume v_1, w_1 and p_1 as follows:

$$v_1(y, z) = v_{11}(y)\cos\pi z \tag{29}$$

$$w_1(y, z) = -\frac{1}{\pi}\frac{dv_{11}(y)}{dy}\sin\pi z \tag{30}$$

$$p_1(y, z) = p_{11}(y)\cos\pi z \tag{31}$$

In the aboveequations (29) and (30) are chosen so that the equation of continuity (23) is satisfied. Substitution of the equations (29), (30) and (31) into (25) and (26), gives the following

$$\frac{d^2 v_{11}}{dy^2} - \mathrm{Re}\frac{dv_{11}}{dy} - \left(\pi^2 + \frac{1}{K}\right)v_{11} = \mathrm{Re}\frac{dp_{11}}{dy} \tag{32}$$

$$\frac{d^3 v_{11}}{dy^3} - \mathrm{Re}\frac{d^2 v_{11}}{dy^2} - \left(\pi^2 + M^2 + \frac{1}{K}\right)\frac{dv_{11}}{dy} = \mathrm{Re}\,\pi^2 p_{11} \tag{33}$$

Elimination of the terms $\frac{dp_{11}}{dy}$ and p_{11} in the equations (32) and (33), implies

$$\frac{d^4 v_{11}}{dy^4} - \mathrm{Re}\frac{d^3 v_{11}}{dy^3} - \left(M^2 + \frac{1}{K} + 2\pi^2\right)\frac{d^2 v_{11}}{dy^2} + \mathrm{Re}\,\pi^2 \frac{dv_{11}}{dy} + \left(\pi^4 + \frac{\pi^2}{K}\right)v_{11} = 0 \quad (34)$$

The corresponding boundary conditions given by

$$y = 0 : v_{11} = 0, \qquad (35)$$
$$y = 1 : v_{11} = 0,$$

In order to obtain the solution for main flow velocity (u_1) and temperature (θ_1),the following areassumed

$$u_1(y,z) = u_{11}(y)\cos\pi z \quad (36)$$
$$\theta_1(y,z) = \theta_{11}(y)\cos\pi z \quad (37)$$

Substituting (36) and (37) in to the partial differential equations (24) and (27) and in their corresponding boundary conditions, we get the following

$$\frac{d^2 u_{11}}{dy^2} - \mathrm{Re}\frac{du_{11}}{dy} - \left(M^2 + \frac{1}{K} + \pi^2\right)u_{11} = \mathrm{Re}\, v_{11}\frac{du_0}{dy} \quad (38)$$

$$\frac{d^2 \theta_{11}}{dy^2} - \mathrm{Re}\ \mathrm{Pr}\frac{d\theta_{11}}{dy} - (\pi^2 - S)\theta_{11} = \mathrm{Re}\,\mathrm{Pr}\, v_{11}\frac{d\theta_0}{dy} \quad (39)$$

with boundary conditions

$$y = 0 : \ u_{11} = 0, \theta_{11} = 0 \qquad (40)$$
$$y = 1 : u_{11} = 0, \theta_{11} = 0$$

3.3 Finite difference method

Obtaining exact solution for the above forth order differential equation is difficult, so substituting the following finite difference formulae

$$\frac{dv_{11}}{dy} = \frac{v_{11}(i+1) - v_{11}(i-1)}{2h}$$

$$\frac{d^2 v_{11}}{dy^2} = \frac{v_{11}(i+1) - 2v_{11}(i) + v_{11}(i-1)}{h^2}$$

$$\frac{d^3 v_{11}}{dy^2} = \frac{v_{11}(i+2) - 2v_{11}(i+1) + 2v_{11}(i-1) - v_{11}(i-2)}{2h^3}$$

$$\frac{d^4 v_{11}}{dy^2} = \frac{v_{11}(i+2) - 4v_{11}(i+1) + 6v_{11}(i) - 4v_{11}(i-1) + v_{11}(i-2)}{h^4}$$

inequation (34), the following system of equations are obtained

$$B_1 v_{11}(i+2) + B_2 v_{11}(i+1) + B_3 v_{11}(i) + B_4 v_{11}(i-1) + B_5 v_{11}(i-2) = 0 \quad (41)$$

with boundary conditionsin finite difference form:

$$v_{11}[i] = 0, \quad for\ \ i = 0$$

$$\boldsymbol{v_{11}[i] = 0,\ \ for\ \ i = 10}$$

where

$$B_1 = 2 - \mathrm{Re}\,h$$

$$B_2 = -8 + 2\,\mathrm{Re}\,h - 2h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) + \mathrm{Re}\,h^3\pi^2$$

$$B_3 = 12 + 4h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) + 2h^4\left(\pi^4 + \frac{\pi^2}{K}\right)$$

$$B_4 = -8 - 2\,\mathrm{Re}\,h - 2h^2\left(M^2 + \frac{1}{K} + 2\pi^2\right) - \mathrm{Re}\,h^3\pi^2$$

$$B_5 = 2 + \mathrm{Re}\,h$$

Similarly substitution of the following finite difference formulae

$$\frac{dg}{dy} = \frac{g(i+1) - g(i-1)}{2h},$$

$$\frac{d^2g}{dy^2} = \frac{g(i+1) - 2g(i) + g(i-1)}{h^2},$$ where g stands u_{11} and θ_{11},

intoequations (38) and (39), gives the following system of equations

$$B_1 u_{11}(i+1) - B_6 u_{11}(i) + B_5 u_{11}(i-1) = B(i) \qquad (42)$$

$$D_1 \theta_{11}(i+1) - D_2 \theta_{11}(i) + D_3 \theta_{11}(i-1) = D(i) \qquad (43)$$

with conditions in finite difference form :

$$u_{11}[i] = 0,\ \theta_{11}[i] = 1,\ \ for\ \ i = 0$$

$$\boldsymbol{u_{11}[i] = 0,\ \theta_{11}[i] = 0,\ \ for\ i = 10}$$

where i stands plate divisions with step length h=0.1 and $y = i\,h$,

$$D_1 = 2 - R_e P_r h,\ D_2 = 4 + 2(\pi^2 - S)h^2$$

$$D_3 = 2 + R_e P_r h,\ B_6 = 4 + 2h^2\left(M^2 + \frac{1}{K} + \pi^2\right)$$

$$B(i) = R_e v_{11}(i)\left(\frac{m_1 e^{m_1 ih} - m_2 e^{m_2 ih}}{e^{m_1} - e^{m_2}}\right)$$

$$D(i) = \mathrm{Re}\,\mathrm{Pr}\,v_{11}(i)\left(\frac{a_2 e^{a_2 ih} - a_1 e^{a_1 ih}}{e^{a_2} - e^{a_1}}\right)$$

Equations (41), (42) and (43) have been solved using Gauss-seidel iteration method and to obtain the solution of this system a numerical code is executed using C-Program. In order to prove the convergence of finite difference scheme, computation is carried out for slightly changed value of h and the iterations on until a **tolerance** 10^{-8} is attained. No significant change was observed in the values of temperature profile (θ**)**, main and cross flow velocities w & θ respectively**.** Thus, it is concluded that the finite difference scheme is convergent and stable.

4 Nusselt-number:

From the temperature field the heat transfer coefficient in terms of Nusselt number Nu is given by

$$Nu = \frac{-d.q_w^*}{k(T_0 - T_1)} = \left(\frac{d\theta_0}{dy} + \varepsilon \frac{d\theta_{11}}{dy} \cos \pi z \right)_{y=0} \qquad (46)$$

Nomenclature

ρ	Density of the fluid
p^*	Pressure
K^*	Permeability of the porous medium
υ	Kinematic viscosity
k	Thermal conductivity
B_0	Magnetic field component along y^*-axis,
σ	Electrical conductivity
μ	Viscosity
C_p	Specific heat at constant pressure
S	Heat source parameter

5 Results and Discussion

In order to explore the importance of the various physical parameters involved in this study, a parametric study of the physical parameters is conductedin the presence of heat source.During the course of numerical calculations, to be realistic, the values of Prandtl number (Pr) are chosen to be 0.71, 7.0 and 11.4 representing air, water at 20^0 C and water at 4^0 C respectively.

Figure (1) shows the temperature profile for various values of heat generation parameter (S). It is evident from the figure that temperature of the fluid increases with increasing values of heat source parameter S. Figure (2) depicts the effect of Prandtl number Pr on temperature field (**θ**) in the presence of heat source. It is observed that the presence of heavier Prandtl number fluid is found to decelerate the temperature at all the points in the flow field.This is due to the fact thata fluid with higher Prandtl number has relatively lower thermal conductivity.Figure (3) is drawn for various selected values of injection/suction parameter Re on temperature profile (**θ**)in the presence of heat source. Acomparative study of the graph reveals that the temperature of the fluid increasesin the presence of heat source parameter.This result qualitatively agrees with the physical fact that heat generation is to increase the rate of heat transport to the fluid there by increasing the temperature of the fluid. Further it is noted that temperature decreases with increasing values injection/suction parameter (Re).

The main flow velocity profile is shown graphically in figure (6) for various values of the magnetic parameter M. It is noted from figure that the increasing values of magnetic parameter is to reduce the velocity of the flow. This is due to the fact that the introduction of magnetic field normal to the fluid flow has a tendency to give rise to a resistive-type force called the Lorentz force,which acts against the fluid flow and hence results in reducing thevelocity profiledue to this type of magnetic pull of Lorentz force.

Figure (7) depicts the main flow velocity profile for small and large values of permeability and for various smaller values of injection/suction parameter (Re).It is observed from figure thatan increase in Re leads to decrease in the velocity while an increase in permeability of the porous medium leads increase in the velocity of the fluid flow. Also it is observed that the dotted line in the figure shows a couette flow in ordinary medium as the value of Re is considered to be zero, it means that there is no injection and suction at lower and upper plates respectively. Figure (8) shows the effect of magnetic parameter M on cross flow velocity. It is observed from figure thatfor the increasing values magnetic parameter M , Cross flow velocity is found to increase near the lower and upper half of the plates. But a reverse phenomenon is observed in the middle of the channel.This is due to the fact that there is an injection at the stationary plate and the constant suction at the plate in uniform motions, which are two exactly opposite processes.

Figures(4), (9) and (5)are plotted for Nusselt number versus various values of injection/suction parameter Re in the case of Pr=0.71 and 7.0 respectively.It is observed thatNusselt number increases in the presence of heat sourcewhile it decreases with the increasing values of Re. Further,it is interesting to note that the effect of heat source on Nusselt number is more significant in the case of Pr=0.71, than in the case of Pr=7.0.The graphical results obtained for Nusselt number in the absence of heat source parameter, show a good agreement with previously published work [11]. Figure (10) shows that Nusselt number 'Nu' versus 'Re' for various values of Pr and K. It is observed that the results obtained in the absence of heat source and magnetic parameters are almost all similar to the results of K.D Singh [11]. The obtained results are in accordance with the physical realities which validate the correctness of our work presented here.

Conclusions

Effect of heat source on Couetteflow of a viscous incompressible electrically conducting fluid between two horizontal parallel porous flat platesis analyzed.From this study the following conclusions are drawn.

(1) Temperature andNusselt number increase in the presence heat source.This due to the fact that effect of heat generation is to increase the rate of heat transport to the fluid.
(2) The effect of heat source on Nusselt number is more significantin the case Pr=0.71than in the case Pr=7.0
(3) A growing magnetic parameter reducesthe main flow velocity (u) due to the magnetic pull of the Lorentz force.But it accelerates the cross flow velocity near the lower and upper half of the plates.
(4) The results of the validation of this work in the absence of heat source and magnetic parameter agree significantly with the graphical results of K.D Singh [11].

International Journal of Engineering, Science and Mathematics
Vol. 7 Issue 1, January 2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, **Email: ijesmj@gmail.com**
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

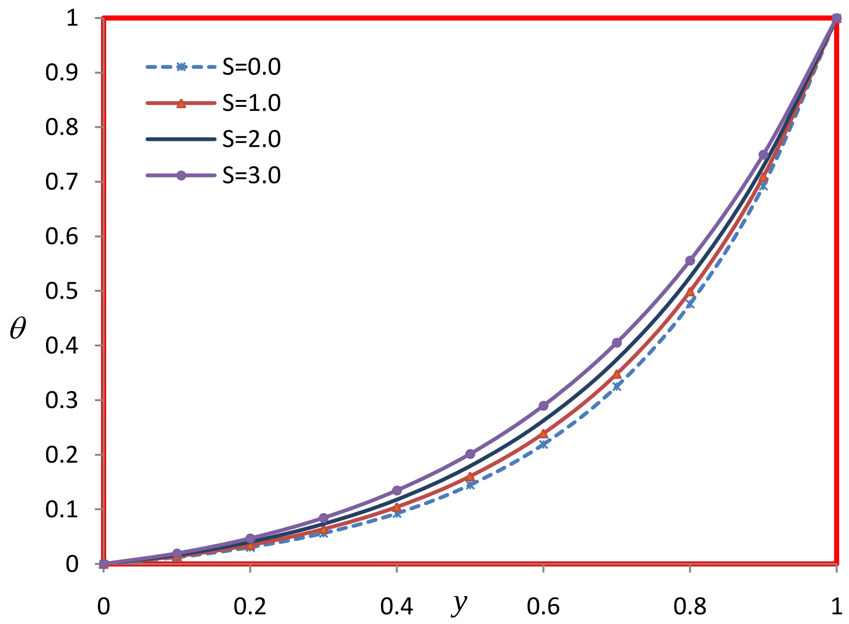

Fig1: Temperature profile for Re=5.0, Pr=0.71, M=1.0, Z=0.5, ε=0.2 and K=0.2

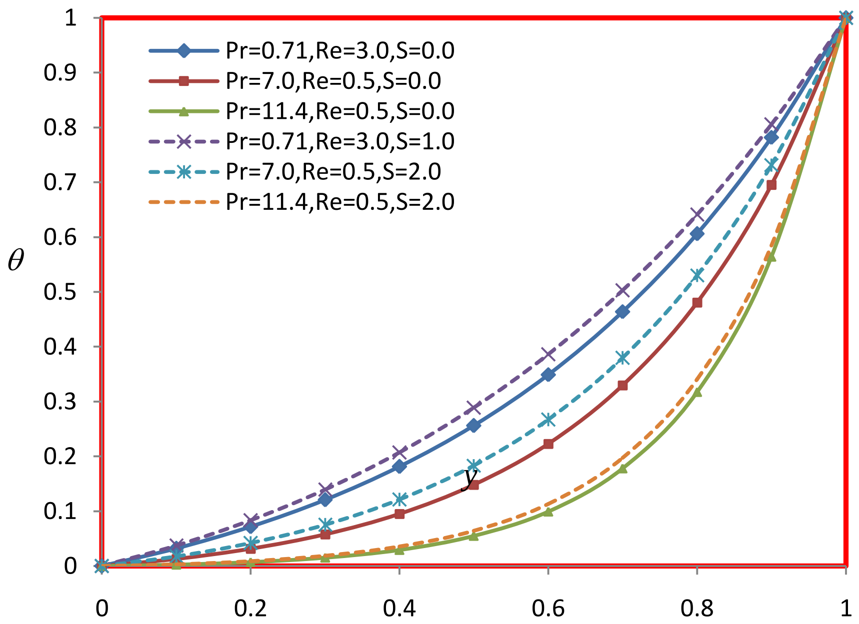

Fig2:Temperature profile forM=1.0, Z=0.5, ε=0.2 and K=0.2

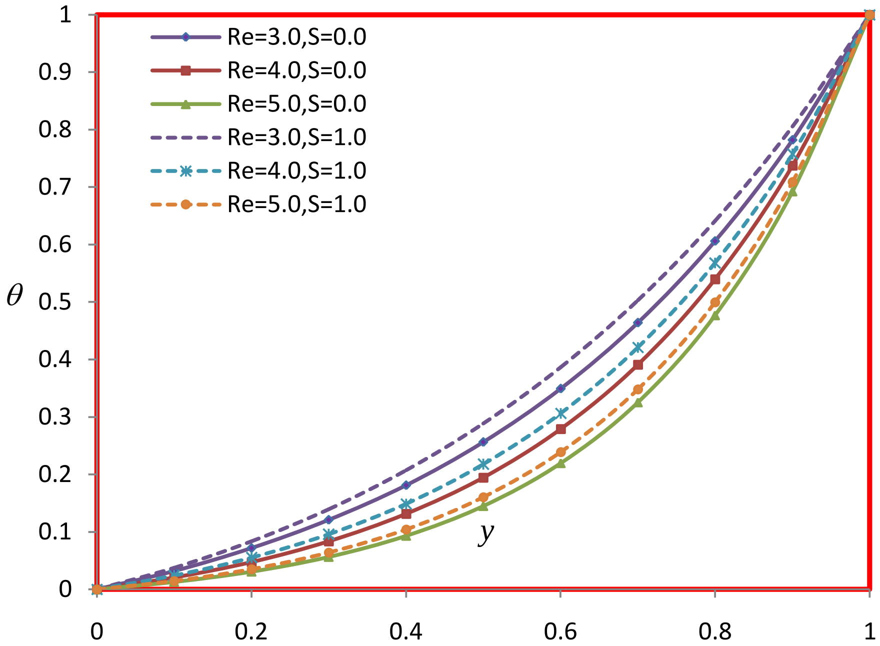

Fig3: Temperature profile for Pr=0.71, M=1.0, Z=0.5, ε=0.2 and K=0.2

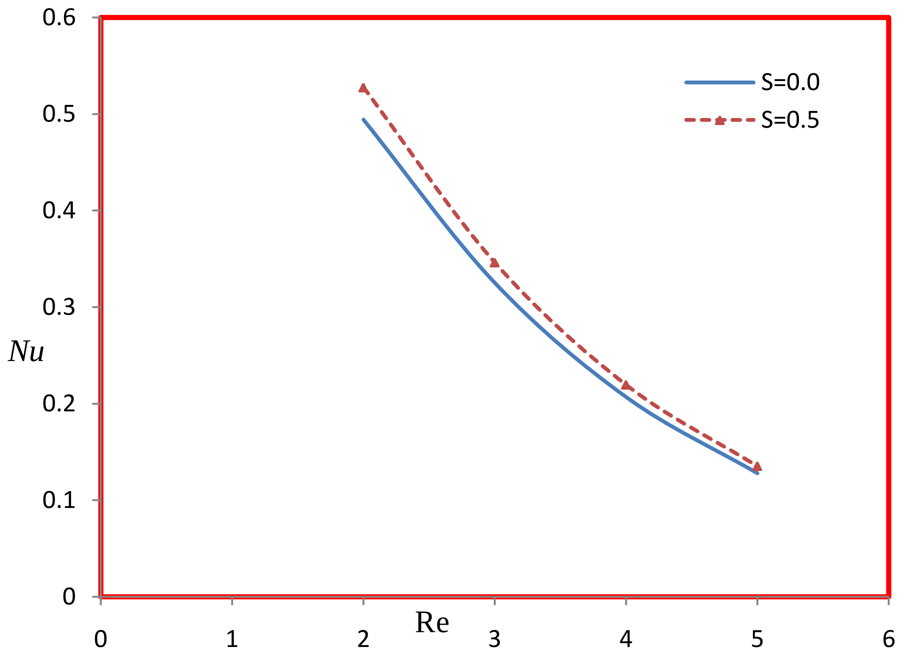

Fig4: Nusselt number for Pr=0.71, M=1.0, Z=0.5, ε=0.2 and K=0.2

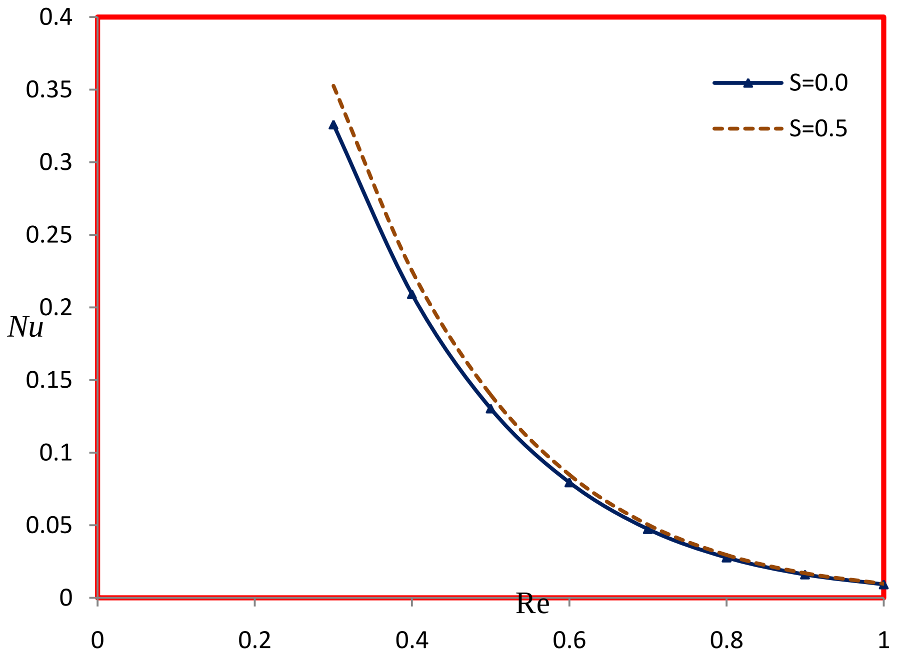

Fig5: Nusselt number for Pr=7.0, M=1.0, Z=0.5, ε=0.2 and K=0.2

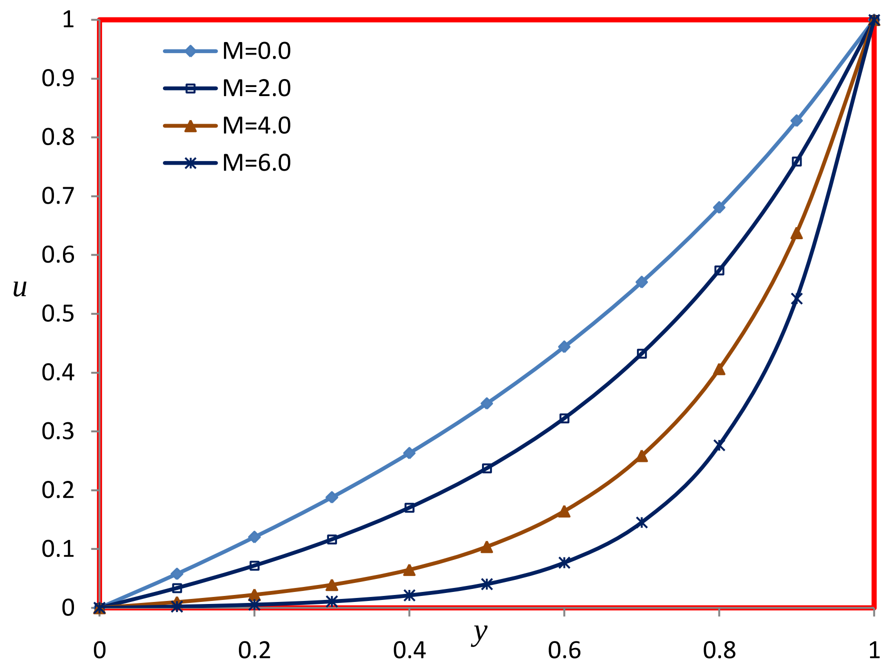

Fig 6: **Main flow velocity profile u forZ=0.0, ε=0.2 and K=0.2**

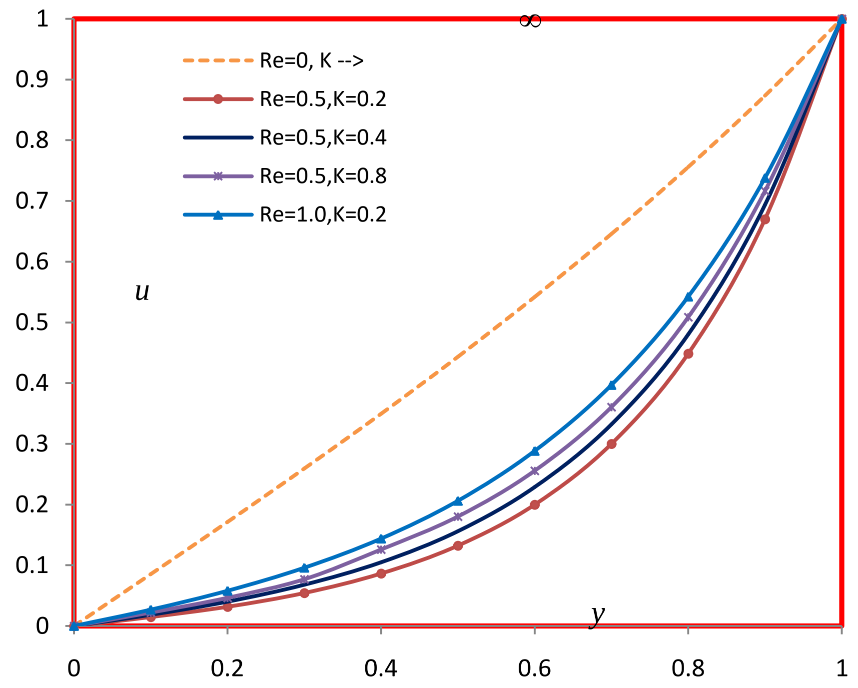

Fig 7: Main flow velocity profile u forZ=0.0, ε=0.2 and K=0.2

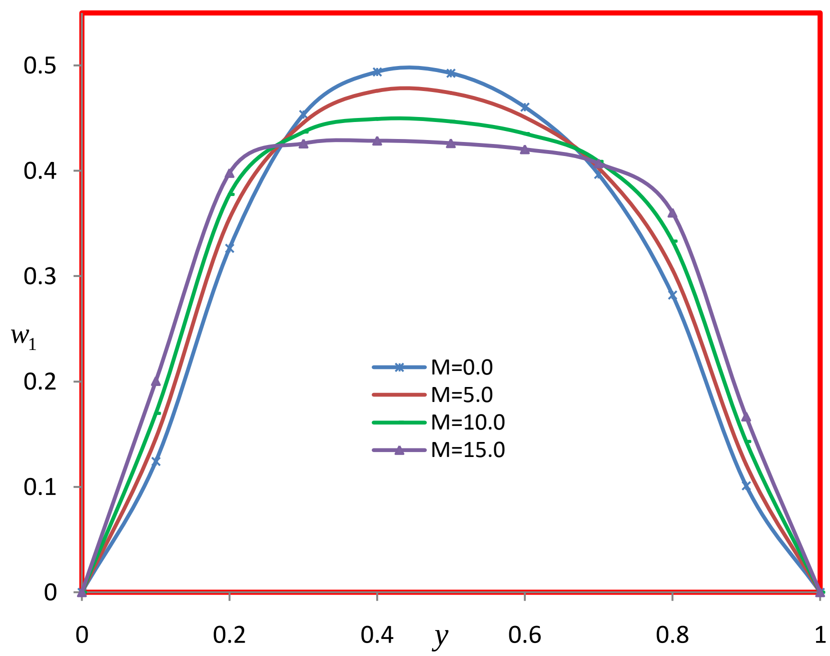

Fig 8: Effect of magnetic parameter M on cross flow velocity
(Z=0.0, ε=0.2,K=0.2,Re=0.5)

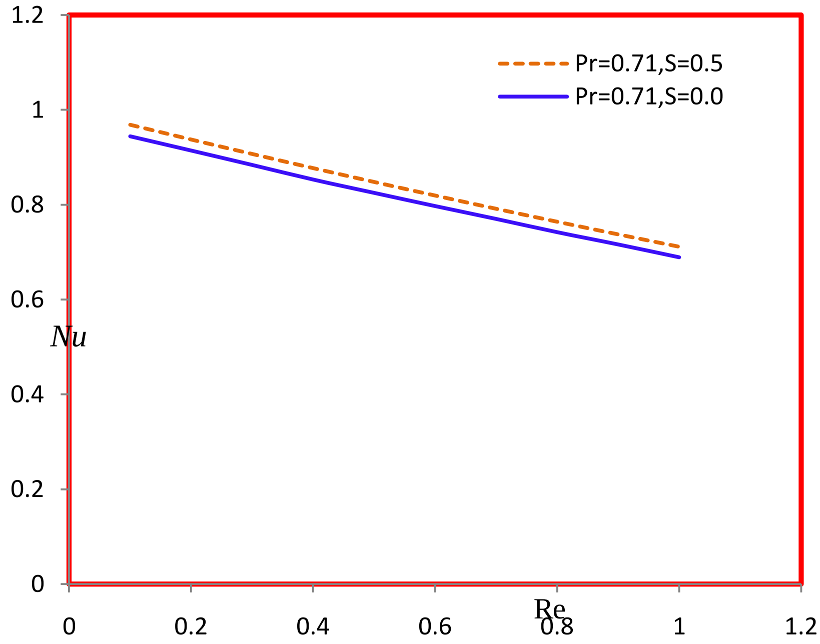

Fig 9: Nusselt number Nu **versus** Re **for M=1.0, K=0.2 and Z=0.5, ε=0.2**

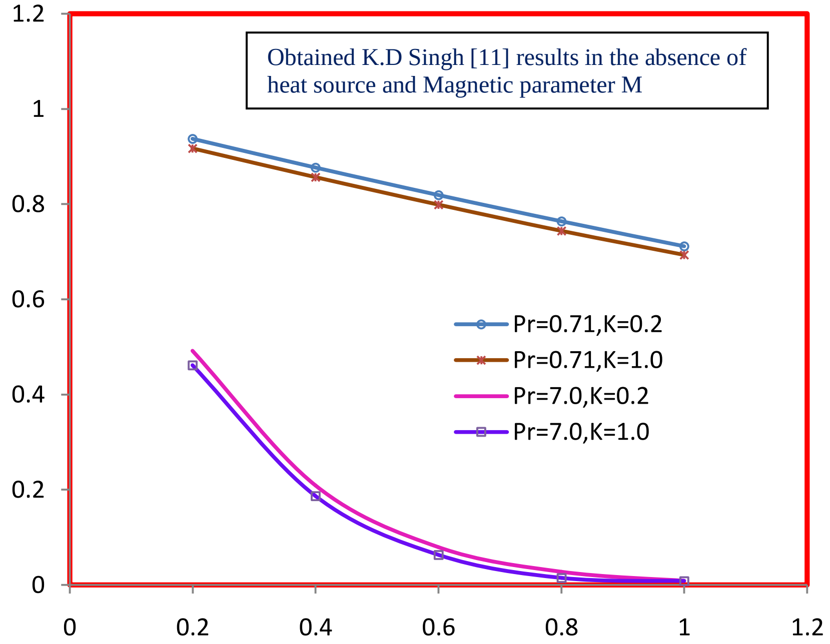

Fig 10: Nusselt number Nu **versus 'Re' for Z=0.5, ε=0.2**

REFERENCES

1. Gulab Ram and R S Mishra, MHD flow of conducting fluid through porous medium, *Indian J pure appl Math*, **8** (1977) 637-647.
2. C L Varshney, Oscillatory two dimensional flow through a porous medium with variable suction, *Indian J pure appl Math* **10** (1979) 1558-1564.
3. A.A Raptis, Unsteady free-convection flow through porous medium, *Int. J Engin Sci*,**21** (1983) 345-348.
4. A.A Raptis and C P Perdikis, Oscillatory flow through a porous medium by the presence of free-convection flow, *Int J Engin Sci* 23 (1985) 51-55.
5. P Singh, V P Sharma and U N Misra, Three dimensional free-convection flow and Heat transfer along a porous vertical wall, *Applied, Sci Res***34** (1978) 105-115.
6. K Gersten and J F Gross, Flow and Heat transfer along a plane wall with periodic suction, *ZAMP* **25** (1974) 399-408.
7. K.D.Singh, Three-dimensional MHD oscillatory flow past a porous plate, *ZAMM* **71**(1991) 3, 192-195.
8. K.D.Singh, Three-dimensional viscous flow and Heat transfer along a porous plate, *ZAMM***73**(1993) 1, 58-61
9. K D Singh, Three dimensional couette flow with transpiration cooling, *ZAMP***50** (1999) 661-668.
10. K D Singh and Rakesh Sharma, MHD Three dimensional couette flow with transpiration cooling, *ZAMM***81** (2001) 715-720
11. K D Singh and Rakesh Sharma, Three dimensional couette flow through a porous medium with Heat transfer, *Indian J.pure applied. Math,* **32**(12) (2001) 1819-1829
12. Chaudhary R.C and Pawan Kumar Sharma, Three dimensional Couette flow and heat transfer through a porous medium with variable permeability", J. Zhejiang Uni. Sci,vol 4 (2), 2003, 181–185.
13. Singh K.D,"Influence of moving magnetic field on three dimensional Couette flow", *Z. Angew. Math Phys*, vol.55, 2004, 894–902.
14. Pawan Kumar Sharma, Chaudhary R.C, "Magneto-hydrodynamics effect on three dimensional viscous incompressible flows between two horizontal parallel porous plates and heat transfer with periodic injection/suction", *Int. J. Math.Sci*,vol 62, 2004, 3357–3368.
15. Sharma P. R. and Yadav G. R, Heat transfer through three dimensional Couette flow between astationary porous plate bounded by porous medium and a moving porous plate, *Ultra Sci. Phys. Sci*, l 7(3), 2005, pp. 351-360.
16. Jain N. C, Gupta P and Sharma B, Three dimensional Couette flow with transpiration cooling through porous medium in slip flow regime, *AMSE. Journal. Mod. Meas. Cont*. B, 75(5) (2006) 33-52.
17. Govindarajan A, Ramamurthy V and Sundarammal K, 3-D Couette flow of dusty fluid with transpiration cooling, *J. Zhejiang Univ. Sci. A*, 8 (2), 2007, pp. 313–322.
18. Das S. S, Mohanty M, Panda J. P and Sahoo S. K, Hydromagnetic three dimensional couette flow and heat transfer, *J. Naval Architecture and Marine Engineering*, l 5(1), 2008, 1-10.

19. Srihari K, Kishan N, Anand Rao J,MHD Three-dimensional Couette flow through porous medium with heat transfer and viscous dissipation, *Journal of Indian Acad.Maths*,l30 (2), 2008, pp.371-388.
20. Das S.S, Effect of suction and injection on MHD three dimensional Couette flow and heat transfer through a porous medium, *Journal of Naval Architecture andMarine Engineering*, 6, 2009, 41-51.
21. Ahmed N,Magnetic field effect on a three-dimensional mixed convective flow with mass transfer along an infinite vertical porous plate, *Int. J. Eng. Sci.Tech*. 2(2), 2010, 117–135.
22. N.Ahmed, S.Talukdar, Oscillatory Three-dimensional flow past an infinite vertical porous plate with thermal diffusion and chemical reaction in the presence of heat sink, **33,** 333-350(2011).
23. Gireesha B.J, Chamkha A.J, Vishalakshi C.S,Bagewadi C.S,Three-dimensional Couette flow of a dusty fluid with heat transfer, *Applied Mathematical modeling*, 36, 2012, 683-701.
24. Taswar Hayat,Sabir Alishehzad,MohmmadQasim,SaleemAsgar, Three dimensionalstretched flowvia convective boundary condition and heat generation/absorption, *International Journal of Numerical methods for Heat and fluid flows*,24(2014) 42-358.
25. S Baag, S R Mishra, Heat and mass transfer analysis on MHD 3-D based nanofluid. *Journal of nanofluids*, 4, 2015, 1-10.
26. S A Shehzad, Z Abdullah, F M Abbasi, T. Hayat, A Alsaedi, Magnetic field effect in Three-dimensional flow of an Oldroyd-B nano-fluid over a radiative surface, *Journal of Magentism and magnetic materials*, 399, 2016, 97-108.
1. T Hayat, T Muhammad, S A Shehzad, A Alsaedi, On three dimensional boundary layer flow of Siskonano fluid with magnetic field effects, *Advanced power technology*, 27(2), 2016, 504-512.