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GROUP ACTION ON BIPOLAR FUZZY SOFT $\Gamma-$NEAR RING OVER IDEAL STRUCTURES

[1]S.Anitha,[2]G.Subbiah[*]and [3]M.Navaneethakrishnan
[1]Assistant Professor, Department of Mathematics, M.I.E.T Engineering College, Trichy–620 007, Tamilnadu, India.
[2*]Associate Professor, Department of Mathematics, Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamilnadu, India.
[3]Associate Professor, Department of Mathematics, Kamaraj College, Thoothukudi-628 003, Tamilnadu, India.
[*]Corresponding Author: E-mail:subbiahkgs@gmail.com

ABSTRACT: In this paper, we have applied the concept of soft set theory into bipolar fuzzy soft substructures of $\Gamma-$near ring and study their properties. We have also constructed the bipolar fuzzy soft product, bipolar fuzzy soft characteristic function, bipolar fuzzy soft $\Gamma-$ideals of $\Gamma-$near ring and also the interrelations of them has been presented. Certain kind of $\Gamma-$near ring are characterized in terms of the bipolar fuzzy soft ideals of $\Gamma-$near ring. Thus, the bipolar fuzzy soft normal $\Gamma-$near ring is being defined and some characterizations of the $\Gamma-$near ring with soft normality are being given.

Keywords: *Soft $\Gamma-$near ring, $\Gamma-$ideal, Isomorphism, Normal $\Gamma-$ideal, Extremal, Soft set, Bipolar Fuzzy soft set.*

1. Introduction: It is well-known that a subject may feel at the same time a positive response as well as negative one for the same characteristic of an object in one's daily life. For example, a house, being close to downtown is both good (it is convenient) as well as bad (it is noisy). Thus, a recent trend in contemporary information processing focuses on the bipolar information, both as a knowledge representation point of view as well as a processing and reasoning one. Bipolarity is very important to distinguish between (i) positive information, which usually represents what is guaranteed to be possible, for instance, it has already been observed or experienced, and (ii) negative information that represents what is impossible or forbidden, or surely false. In 1999, the soft set theory was introduced by Molodtsov [19] as an alternative approach to fuzzy set theory which was defined by Zadeh [23] in 1965. After the study of Molodtsov [19] , many researchers have studied on the set theoretical approaches and the decision making applications of soft sets. For instance Maji et. al [18] has defined some new operations of soft sets and has given a decision making method based on soft sets. Chen et.al [7] has developed a method of parameters reduction on soft set by using knowledge reduction of rough sets. Cagman and Enginoglu [6] has modified the definitions of soft set operations and has given a decision making method called uni-int decision making method. Ali et.al [2] has defined some new operations on the soft set theory such as extended union and the intersection, restriction union and intersection. Sezgin and Atagun [22] studied on soft set operations as defined by Ali et.al [2]. The studies related to soft sets have increased rapidly in many fields such as topology and algebra. The soft group and the first algebraic structure of soft sets were first defined by Aktas and Cagman [3] in 2007. In 2008, Jun [11] has defined soft BCK/BCI algebras and has applied soft sets in ideal theory of BCK/BCI algebras. Jun et.al [13] has also defined soft p-ideals of soft BCI algebras. Cagman et.al [6] has defined a new structure called soft int-group and has obtained some properties of this new structure. Acar et.al [1] has constructed a ring structure on soft sets. The concept of a fuzzy soft group structure was defined by Aygunoglu and Aygun [5] and intuitionistic fuzzy soft groups were being introduced by Karaaslan et.al [14]. The Bipolar fuzzy set was introduced by Zhang [25] as the generalization of a fuzzy set. The Bipolar fuzzy set is an extension of a fuzzy set whose membership degree interval is [−1, 1]. Naz and Shabir [20] have proposed the concept of fuzzy bipolar soft sets and have investigated algebraic structures on the fuzzy bipolar soft sets. In a soft set, the element of initial universe belongs to the image set related to parameter or not. But, in some cases, an element of universal set may not belong to the image set and to the complement of image set which is related to the parameters. In order to express such cases, Shabir and Naz [21] has proposed the concept of bipolar soft set and has defined some of their set theoretical operations such as union, the intersection and the complement. But the complement of bipolar soft sets which is defined by Shabir and Naz [20] has not allowed constructing some structures topological and algebraical, thus the notions of bipolar soft sets and their operations are defined by Karaaslan and Karatas [15]. Muhammad et.al [26] has defined the concept of bipolar fuzzy soft $\Gamma-$sub semi group and bipolar fuzzy soft $\Gamma-$ideals in a $\Gamma-$semi group. As it is mentioned above, many studies have been made on group structures and the other algebraic structures of soft sets. Since the concept of bipolar soft set is novel, there are not enough studies on algebraic structures of the bipolar soft sets.

2. PRELIMINARIES

Definition 2.1 A fuzzy subset of X is a function from X into the unit interval [0, 1]. The set of all fuzzy subset of X is called fuzzy power of x and is denoted by FP(x).

Definition 2.2 Let $\mu, v \in FP(x)$ of $\mu(x) \leq v(x)$ for all $x \in X$. Then μ is said to be contained in

v and we write $\mu \subseteq v$ (or) $v \geq \mu$. Clearly, the inclusion relation $\subseteq$ is a partial order on FP(x).

Definition 2.3 Let $\mu, v \in FP(x)$. Then $\mu \vee v$ and $\mu \wedge v$ are fuzzy subsets of X, defined as follows for all $x \in X$, $(\mu \vee v)(x) = \mu(x) \vee v(x)$ and $(\mu \wedge v)(x) = \mu(x) \wedge v(x)$. The fuzzy subsets $\mu \vee v$ and $\mu \wedge v$ are called the union and intersection of μ and v respectively.

Definition 2.4 Two fuzzy subsets ϕ and X which map every element of X onto 0 and 1 respectively. We call ϕ as the empty set or null fuzzy subset and X is the whole-fuzzy subset of X.

Definition 2.5 A bipolar fuzzy set μ in X is defined as $\mu = \{(x, \mu^P(x), \mu^N(x)) : x \in X\}$ where $\mu^P : X \to [0,1]$ and $\mu^N : X \to [-1,0]$ are mappings. The positive membership degree $\mu^P(x)$ denotes the satisfaction degree of x to the property corresponding to a bipolar fuzzy set μ and the negative membership degree $\mu^N(x)$ denotes the satisfaction degree of x to same implicit counter property of μ .

If $\mu^P(x) \neq 0$ and $\mu^N(x) = 0$, it is the situation that x is regarded as having only as possible satisfaction for μ .

If $\mu^P(x) = 0$ and $\mu^N(x) \neq 0$, it is the situation that x does not satisfy the property of μ . But some what satisfies the counter property of μ , it is possible for x to be $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of the domain.

For the sake of the simplicity, we shall write $\mu = (\mu^P, \mu^N)$ for the bipolar fuzzy set $\mu = \{(x, \mu^P(x), \mu^N(x)) : x \in X\}$.

Definition 2.6 Let U be an initial universe and E be the set of parameters such that $A \subseteq E$ and P(U) is the power set of U. Then δ_A is called a soft set, where $\delta : A \to P(U)$.

Definition 2.7 For two soft sets δ_A and Δ_B over a common universe A, we say that δ_A is a soft subset of Δ_B denoted by $\delta_A \subseteq \Delta_B$ if it satisfies

(i) $A \subset B$

(ii) $\forall a \in A, \delta(a)$ is a subset of $\Delta(a)$.

Definition 2.8 If δ_A and Δ_B are two soft sets over a common universe U. The union of δ_A and Δ_B is defined to be the soft set γ_c satisfying the following conditions:

(i) $C = A \cup B$

(ii) for all $e \in C$, $\gamma(e) = \delta(e)$ if $e \in A/B$

$= \Delta(e)$ if $e \in B/A$

$= \delta(e) \cup \Delta(e)$ if $e \in A \cup B$

This relation is denoted by $\gamma_C = \delta_A \cup \Delta_B$.

Definition 2.9 Let δ_A and Δ_B be two soft sets over a common universe U such that $A \subset E$ and P(U) is the collection of all fuzzy subsets of U. Then δ_A is called a fuzzy soft set, where $\delta : A \to P(U)$

3. BIPOLAR FUZZY SOFT SET

Definition 3.1 Let U be a universe and E be the set of parameters such that $A \subset E$. Define $\delta : A \to BF^U$, where BF^U is the combination of bipolar fuzzy subsets of U. Then δ_A is said to be bipolar fuzzy soft set over U. It is denoted by $\delta_A = \left\{ (x, \mu_e^{\ P}(x),\ \mu_e^{\ N}(x)) : x \in U \text{ and } e \in A \right\}$.

Example 1 Let $U = \{b_1, b_2, b_3, b_4\}$ be the set of four bikes under consideration and $E = \{ e_1 = \text{Steel},\ e_2 = \text{Light},\ e_3 = \text{Stylish},\ e_4 = \text{Heavy duty} \}$ be the set of parameters and $A = \{e_1, e_2\}$ be subset of E. Then

$$\delta_A = \begin{Bmatrix} \delta(e_1) = \{(b_1, 0.3, -0.4), (b_2, 0.1, -0.4), (b_3, 0.7, -0.6), (b_4, 0.3, -0.7)\} \\ \delta(e_2) = \{(b_1, 0.6, -0.4), (b_2, 0.4, -0.9),\ (b_3, 0.2, -0.5), (b_4, 0.1, -0.3)\} \end{Bmatrix}$$

Definition 3.2 Let U be a universe and E be the set of attributes. Then U_E is the collection of all bipolar fuzzy subsets on U with attributes from E and is said to be bipolar fuzzy soft class.

Definition 3.3 A bipolar fuzzy soft set δ_A is said to be a null bipolar fuzzy soft set denoted by empty set ϕ if for all $e \in A, \delta(e) =$ ϕ.

Definition 3.4 A bipolar fuzzy soft set δ_A is said to be an absolute fuzzy soft set if for all $e \in A,\ \delta(e) = BF^U$.

Definition 3.5 The complement of a bipolar fuzzy soft set δ_A is denoted by $\delta_A^{\ c}$ and is defined by $\delta_A^{\ c} = \left\{ (x, 1 - \delta_A^{\ P}(x), -1 - \delta_A^{\ N}(x)) : x \in U \right\}$.

Example 2 Let $U=\{m_1,m_2,m_3,m_4\}$ be the set of four men under consideration and $E=\{e_1=\text{Businessman},\ e_2=\text{Smart},\ e_3=\text{Educated},\ e_4=\text{Government Employee}\}$ be the set of parameters and $A=\{e_1,e_2,e_3\}$. Then

$$\delta_A=\begin{cases}\delta(e_1)=\{(c_1,0.1,-0.5),(c_2,0.3,-0.6),(c_3,0.4,-0.2),(c_4,0.7,-0.2)\}\\ \delta(e_2)=\{(c_1,0.3,-0.5),(c_2,0.4,-0.2),(c_3,0.5,-0.2),(c_4,0.4,-0.2)\}\\ \delta(e_3)=\{(c_1,0.8,-0.1),(c_2,0.3,-0.6),(c_3,0.4,-0.3),(c_4,0.6,-0.2)\}\end{cases}$$

The complement of a bipolar fuzzy soft set δ_A is

$$\delta_A{}^C=\begin{cases}\delta(e_1)=\{(c_1,0.9,-0.5),(c_1,0.7,-0.4),(c_1,0.6,-0.8),(c_1,0.3,-0.8)\}\\ \delta(e_1)=\{(c_1,0.7,-0.5),(c_1,0.6,-0.8),(c_1,0.5,-0.8),(c_1,0.6,-0.8)\}\\ \delta(e_1)=\{(c_1,0.2,-0.9),(c_1,0.7,-0.4),(c_1,0.6,-0.7),(c_1,0.4,-0.8)\}\end{cases}$$

For a bipolar fuzzy soft set $\delta=<X;\mu^P,\mu^N>$ and $(t,s)\in[0,1]\times[-1,0]$, we define $\delta_t^P=\{x\in X/\mu^P(x)\ge t\}$ and $\delta_s^N=\{x\in X/\mu^N(x)\le s\}$ which are called the +ive t-cut of ϕ and the -ive s-cut of ϕ respectively. For $(t,s)\in[0,1]\times[-1,0]$, the set $\delta_{(t,s)}=\delta_t^P\cap\phi_s^N$ is called the (t, s)-cut of ϕ. In what follows, G will denote $\Gamma-$ near ring, unless otherwise specified.

4. BASIC PROPERTIES OF BIPOLAR FUZZY SOFT SETS

Property 4.1 (Identity Laws)

Let δ_A be a bipolar fuzzy soft set over a common universe U. Then

(i) $\delta_A\cup\delta_A=\delta_A$

(ii) $\delta_A\cap\delta_A=\delta_A$

(iii) $\delta_A\cup\phi=\delta_A$, where ϕ is a null bipolar fuzzy soft set.

(iv) $\delta_A\cap\phi=\delta_A$, where ϕ is a null bipolar fuzzy soft set.

Property 4.2 (Absorption Laws)

Let δ_A and Δ_B be two bipolar fuzzy soft sets over a common universe U. Then

(i) $\delta_A\cup(\delta_A\cap\Delta_B)=\delta_A$

(ii) $\delta_A\cap(\delta_A\cup\Delta_B)=\delta_A$

Property 4.3 (Idempotent Laws)

If δ_A and Δ_B are two bipolar fuzzy soft sets over U, then

(i) $\delta_A\cap\delta_A=\delta_A$

(ii) $\delta_A\cup\delta_A=\delta_A$

Property 4.4 (Associative Laws)

Let δ_A, Δ_B and γ_C be three bipolar fuzzy soft sets over U. Then

(i) $\delta_A \cap (\Delta_B \cap \gamma_C) = (\delta_A \cap \Delta_B) \cap \gamma_C$

(ii) $\delta_A \cup (\Delta_B \cup \gamma_C) = (\delta_A \cup \Delta_B) \cup \gamma_C$

Property 4.5 (Distributive Laws)

Let δ_A, Δ_B and γ_C be three bipolar fuzzy soft sets over U. Then

(i) $\delta_A \cap (\Delta_B \cup \gamma_C) = (\delta_A \cap \Delta_B) \cup (\delta_A \cap \gamma_C)$

(ii) $\delta_A \cup (\Delta_B \cap \gamma_C) = (\delta_A \cup \Delta_B) \cap (\delta_A \cup \gamma_C)$

Result:

Let δ_A and Δ_B are two BFS sets over a common universe U. Then

(i) $\delta_A \subset \Delta_B \Rightarrow \delta_A \cap \Delta_B = \delta_A$

(ii) $\delta_A \subset \Delta_B \Rightarrow \delta_A \cup \Delta_B = \Delta_B$

Property 4.6 (Commutative Laws)

Let δ_A and Δ_B be two bipolar fuzzy soft sets. Then

(i) $\delta_A \cap \Delta_B = \Delta_B \cap \delta_A$

(ii) $\delta_A \cup \Delta_B = \Delta_B \cup \delta_A$

5. BIPOLAR FUZZY SOFT $\Gamma-$NEAR RING

Definition 5.1 Let G be a $\Gamma-$near ring and δ_G be a bipolar fuzzy soft set over U. Then δ_G is said to be a bipolar fuzzy soft $\Gamma-$ near ring shortly (BFSΓN) over U if it satisfies the following conditions hold:

$(BFS\,\Gamma N_1)$: $\delta_G{}^P(x-y) \geq \min\{\delta_G{}^P(x), \delta_G{}^P(y)\}$

$\delta_G{}^N(x-y) \leq \max\{\delta_G{}^N(x), \delta_G{}^N(y)\}$

$(BFS\,\Gamma N_2)$: $\delta_G{}^P(x\alpha y) \geq \min\{\delta_G{}^P(x), \delta_G{}^P(y)\}$

$\delta_G{}^N(x\alpha y) \leq \max\{\delta_G{}^N(x), \delta_G{}^N(y)\}$ for all $x, y \in G$ and $\alpha \in \Gamma$.

Example 3 Let $G=\{0,x,y,z\}$ and $\Gamma=\{\alpha,\beta\}$ be non empty sets. The binary operations defined as

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

α	0	x	y	z
0	0	0	0	0
x	0	x	x	x
y	0	x	y	z
z	0	0	z	y

β	0	x	y	z
0	0	0	0	0
x	0	x	0	x
y	0	y	0	z
z	0	0	0	y

Assume that G is the set of parameters and

$A=\left\{\begin{bmatrix} a & a \\ b & b \end{bmatrix} / a,b \in Z_4\right\}$, 2 x 2 matrices with Z_4 terms, in the universal set. We construct a bipolar fuzzy soft set $\delta_G{}^P$ over A by

$$\delta_G{}^P(0)=\left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix},\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix},\begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix},\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\right\}$$

$$\delta_G{}^P(x)=\left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix},\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix},\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix},\begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}\right\}$$

$$\delta_G{}^P(y)=\left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix},\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix},\begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}\right\}$$

$$\delta_G{}^P(z)=\left\{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},\begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix},\begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}\right\}$$

and

$$\delta_G{}^N(0)=\left\{\begin{bmatrix} -1 & -1 \\ -2 & -2 \end{bmatrix},\begin{bmatrix} -4 & -4 \\ -2 & -2 \end{bmatrix},\begin{bmatrix} -3 & -3 \\ -2 & -2 \end{bmatrix}\right\}$$

$$\delta_G{}^N(x)=\left\{\begin{bmatrix} -2 & -2 \\ -1 & -1 \end{bmatrix},\begin{bmatrix} -3 & -3 \\ -2 & -2 \end{bmatrix},\begin{bmatrix} -3 & -3 \\ -4 & -4 \end{bmatrix}\right\}$$

$$\delta_G{}^N(y)=\left\{\begin{bmatrix} -2 & -2 \\ -1 & -1 \end{bmatrix},\begin{bmatrix} -2 & -2 \\ -4 & -4 \end{bmatrix},\begin{bmatrix} -3 & -3 \\ -1 & -1 \end{bmatrix}\right\}$$

$$\delta_G{}^N(z)=\left\{\begin{bmatrix} -1 & -1 \\ -2 & -2 \end{bmatrix},\begin{bmatrix} -3 & -3 \\ -2 & -2 \end{bmatrix}\right\}$$

Then, one can easily show that the bipolar fuzzy soft set $\delta_G{}^P$ is a bipolar fuzzy soft $\Gamma-$near ring over A.

Note 1: If δ_G is a BFS $\Gamma-$near ring over A, then $\delta_G{}^P(0) \geq \delta_G{}^P(x)$ and $\delta_G{}^N(0) \leq \delta_G{}^N(x)$ for all $x \in G$.

Theorem 5.1

Let G be a $\Gamma-$near ring and δ_G a BFSS over U. Then δ_G is a bipolar fuzzy soft $\Gamma-$near ring over U iff

(i) $\delta_G{}^P(x-y) \geq \min\{\delta_G{}^P(x), \delta_G{}^P(y)\}$

$\delta_G{}^P(x\alpha y) \geq \min\{\delta_G{}^P(x), \delta_G{}^P(y)\}$ and

(ii) $\delta_G{}^N(x-y) \leq \max\{\delta_G{}^N(x), \delta_G{}^N(y)\}$

$\delta_G{}^N(x\alpha y) \leq \max\{\delta_G{}^N(x), \delta_G{}^N(y)\}$

for all $x, y \in G$ and $\alpha \in \Gamma$.

Note 2: Let δ_G be a BFS $\Gamma-$near ring over U.

(i) If $\delta_G{}^P(x-y)=0$ and $\delta_G{}^N(x-y)=0$ for any $x, y \in G$, then $\delta_G{}^P(x)=\delta_G{}^P(y)$ and $\delta_G{}^N(x)=\delta_G{}^N(y)$.

(ii) If $\delta_G{}^P(x-y)=\delta_G{}^P(0)$ and $\delta_G{}^N(x-y)=\delta_G{}^N(0)$ for any $x, y \in G$, then $\delta_G{}^P(x)=\delta_G{}^P(y)$ and $\delta_G{}^N(x)=\delta_G{}^N(y)$.

It is known that if $(G,+,\Gamma)$ is a $\Gamma-$near ring, then $(G,+)$ is a group but not necessarily abelian. That is, for any $x, y \in G$, $x+y$ need not be equal to $y+x$. However, we have the following theorems.

Theorem 5.2

Let δ_G be a BFS $\Gamma-$near ring over U and $x \in G$. Then

$\delta_G{}^P(x) = \delta_G{}^P(0) \Leftrightarrow \delta_G{}^P(x+y) = \delta_G{}^P(y+x)$ and

$\delta_G{}^N(x) = \delta_G{}^N(0) \Leftrightarrow \delta_G{}^N(x+y) = \delta_G{}^N(y+x)$

Proof:

It is straight forward.

Theorem 5.3

Let G be a $\Gamma-$near field and δ_G be a BFS set over U. If

$\delta_G{}^P(0) \geq \delta_G{}^P(1_G) = \delta_G{}^P(x)$ and $\delta_G{}^N(0) \leq \delta_G{}^N(1_G) = \delta_G{}^N(x)$ for all $0 \neq x \in G$, then δ_G is a BFS $\Gamma-$near ring over U.

Theorem 5.4

If δ_G and Δ_G are BFS $\Gamma-$near rings over U, then $\delta_G \cap \Delta_G$ is also BFS $\Gamma-$near ring over U.

Definition 5.2 Let S be a sub $\Gamma-$near ring of $\Gamma-$near ring G and δ_S be a bipolar fuzzy soft subset of δ_G over U. Then δ_S is called a BFS sub $\Gamma-$ near ring of δ_G over U. It is denoted by $\delta_S \leq_i \delta_G$.

Example 4 In example (3), assume that $G=\{0,x,y,z\}$ is again the set of parameters and $U=D_3=\{(a,b):a^3=b^3=(ab)^2=e, ab=ba^2\}=\{e,a,a^2,b,ba,ba^2\}$, dihedral group, the universal set. We define a BFS set δ_G over U by $\delta_G{}^P(0)=D_3$, $\delta_G{}^N(0)=\{e,a,a^2,b\}$, $\delta_G{}^P(x)=\{e,a,a^2,b,ba\}$, $\delta_G{}^N(x)=\delta_G{}^N(y)=\{e,a,a^2,ab,ba\}$, $\delta_G{}^P(y)=\{e,a,a^2,b\}$, $\delta_G{}^P(z)=\{e,a,a^2,b\}$, $\delta_G{}^N(z)=D_3$. Then δ_G is BFS $\Gamma-$ near ring over U. Now, let $S=\{0,x\}$ be a sub $\Gamma-$ near ring of G, the set of parameters and we defined a bipolar fuzzy soft subset δ_S of δ_G over U by $\delta_G{}^P(0)=\{e,a,a^2,b\}$, $\delta_G{}^N(0)=D_3$, $\delta_G{}^P(x)=\{e,a,a^2\}$, $\delta_G{}^N(x)=\{e,a,a^2,b\}$. It is clear that δ_G is called BFS sub $\Gamma-$ near ring over U.

Definition 5.3 Let G be a $\Gamma-$ near ring and δ_G be a BFS $\Gamma-$ near ring over U. Then δ_G

is

said to be a BFS $\Gamma-$ ideal of G over U if the following conditions hold;

$(BFS\,\Gamma I_1)$ $:\delta_G{}^P(x+y-x)\geq \min\{\delta_G{}^P(x),\delta_G{}^P(y)\}$, $\delta_G{}^N(x+y-x)\leq \max\{\delta_G{}^N(x),\delta_G{}^N(y)\}$

$(BFS\,\Gamma I_2)$ $:\delta_G{}^P(x\alpha y)\geq \delta_G{}^P(x)$, $\delta_G{}^N(x\alpha y)\leq \delta_G{}^N(x)$

$(BFS\,\Gamma I_3)$ $:\delta_G{}^P(x\alpha(y+z)-x\alpha y)\geq \delta_G{}^P(z)$, $\delta_G{}^N(x\alpha(y+z)-x\alpha y)\leq \delta_G{}^N(z)$

for all $x,y,z\in G$ and $\alpha\in\Gamma$.

If δ_G is a BFS $\Gamma-$ near ring over U and the conditions $(BFS\,\Gamma I_1)$ and $(BFS\,\Gamma I_2)$ hold, then δ_G is called a BFS $\Gamma-$ right ideal of G over U and if conditions $(BFS\,\Gamma I_1)$ and $(BFS\,\Gamma I_3)$ hold, then δ_G is called BFS $\Gamma-$ left ideal of G over U.

Example 5 Let $G=\{0,x,y,z\}$ and $\Gamma=\{\alpha,\beta\}$ be non empty sets. The binary operations defined as

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

α	0	x	y	z
0	0	0	0	0
x	0	x	x	x
y	0	x	y	z
z	0	0	z	y

β	0	x	y	z
0	0	0	0	0
x	0	x	0	x
y	0	y	0	z
z	0	0	0	y

Clearly, $(G,+,\Gamma)$ is a $\Gamma-$ near ring.

Assume that G is the set of parameters and

$U = D_3 = \{<x,y>: x^3 = y^3 = (xy)^2 = e, xy = yx^2\} = \{e, x, x^2, y, yx, yx^2\}$, dihedral group, the universal set .We define a BFS set δ_G over U by

$\delta_G{}^P(0) = \delta_G{}^P(y) = D_3$, $\delta_G{}^P(z) = \delta_G{}^P(x) = \{e, x\}$,

$\delta_G{}^N(0) = \{e, x^2\}$, $\delta_G{}^N(x) = \delta_G{}^N(y) = D_3$, $\delta_G{}^N(z) = \{e, x\}$.

Then, clearly δ_G is a BFS $\Gamma-$ left ideal and right ideal of G over U. Therefore, δ_G is a BFS $\Gamma-$ ideal of G over U.

Theorem 5.5

Let G be a $\Gamma-$ near field and δ_G be a BFS $\Gamma-$ ideal of G over U. Then $\delta_G{}^P(0) \geq \delta_G{}^P(1_G) = \delta_G{}^P(x)$ and $\delta_G{}^N(0) \leq \delta_G{}^N(1_G) = \delta_G{}^N(x)$ for all $0 \neq x \in G$.

Note 3: For a near ring G, the zero-symmetric part of G denoted by G_0 is defined by $G_0 = \{g \in G / g \cdot 0 = 0\}$. It is known that if G is a zero-symmetric near ring and $I \lhd_i G$, then $GI \leq G$. Here, we have an analog for this case.

Theorem 5.6

Let $G = G_0$ and δ_G be a BFS set of G over U. Then $\delta_G{}^P(x\alpha(y+z) - x\alpha y) \geq \delta_G{}^P(z)$ and $\delta_G{}^N(x\alpha(y+z) - x\alpha y) \leq \delta_G{}^N(z)$ implies that $\delta_G{}^P(xz) \geq \delta_G{}^P(z)$ and $\delta_G{}^N(xz) \leq \delta_G{}^N(z)$ for all $x, y, z \in G$.

Theorem 5.7

If δ_G and Δ_H are BFS $\Gamma-$ideals over U, then $\delta_G \wedge \Delta_H$ is also BFS $\Gamma-$ideal over U.

Theorem 5.8

If δ_G and Δ_H are BFS $\Gamma-$ideals over U_1 and U_2, then $L_{G\times H} = \delta_G \times \Delta_H$ is also BFS $\Gamma-$near ring over $U_1 \times U_2$.

Theorem 5.9

Let δ_A be bipolar fuzzy soft set in A. Then $\delta_A = (G; \delta_A{}^P, \delta_A{}^N)$ is bipolar fuzzy soft $\Gamma-$ ideal of G if and only if it satisfies the following axioms:

(i) $\delta_t{}^P \neq \phi \Rightarrow \delta_t{}^P$ is an ideal of A $\forall\ t \in [0,1]$

(ii) $\delta_s{}^N \neq \phi \Rightarrow \delta_s{}^N$ is an ideal of A $\forall\ s \in [-1,0]$

6. APPLICATIONS OF BFS $\Gamma-$ NEAR RINGS AND BFS $\Gamma-$ IDEALS

In this section, we give the applications of bipolar fuzzy soft image, BF soft pre-image and upper inclusion of sets to near ring theory with respect to BFS $\Gamma-$near rings and BFS $\Gamma-$near ideals of a near ring.

Theorem 6.1

If δ_G is a BFS $\Gamma-$ideal of $\Gamma-$near ring G over U, then

$\delta_G = \{ x \in G / \delta_G(x) = \delta_G(0) \}$ is a $\Gamma-$ideal of G over U.

Proof:

It is obvious that $0 \in G_\delta \leq G$. We need to prove that

(i) $x - y \in G_\delta$

(ii) $n + x - n \in G_\delta$

(iii) $x \alpha n \in G_\delta$ and $n\alpha(i+x) - n\alpha i \in G_\delta$ for all $x, y \in G_\delta$, $n, i \in G$ and $\alpha \in \Gamma$.

If $x, y \in G_\delta$, then $\delta_G{}^P(x) = \delta_G{}^P(y) = \delta_G{}^P(0)$ and $\delta_G{}^N(x) = \delta_G{}^N(y) = \delta_G{}^N(0)$. So, by Note (i), it follows that $\delta_G{}^P(0) \geq \delta_G{}^P(x-y)$, $\delta_G{}^P(0) \geq \delta_G{}^P(n+x-n)$, $\delta_G{}^P(0) \geq \delta_G{}^P(x\alpha n)$ and $\delta_G{}^P(0) \geq \delta_G{}^P(n\alpha(i+x) - n\alpha i)$ for all $x, y \in G_\delta$, $n, i \in G$ and $\alpha \in \Gamma$.

Also $\delta_G{}^N(0) \leq \delta_G{}^N(x-y)$, $\delta_G{}^N(0) \leq \delta_G{}^N(n+x-n)$, $\delta_G{}^N(0) \leq \delta_G{}^N(x\alpha n)$ and $\delta_G{}^N(0) \leq \delta_G{}^N(n\alpha(i+x) - n\alpha i)$ for all $x, y \in G_\delta$, $n, i \in G$ and $\alpha \in \Gamma$.

Since δ_G is a BFS $\Gamma-$ideal of G over U, so

(i) $\delta_G{}^P(x-y) \geq \min\{\delta_G{}^P(x), \delta_G{}^P(y)\}$

$= \delta_G{}^P(0)$ and

$\delta_G{}^N(x-y) \leq \max\{\delta_G{}^N(x), \delta_G{}^N(y)\}$

$= \delta_G{}^N(0)$

(ii) $\delta_G{}^P(n+x-n) \geq \delta_G{}^P(x)$

$= \delta_G{}^P(0)$ and

$\delta_G{}^N(n + x - n) \le \delta_G{}^N(x)$

$= \delta_G{}^N(0)$

(iii) $\delta_G{}^P(x\alpha n) \ge \delta_G{}^P(x)$

$= \delta_G{}^P(0)$ and

$\delta_G{}^N(x\,\alpha n) \le \delta_G{}^N(x)$

$= \delta_G{}^N(0)$,

$\delta_G{}^P(n\alpha(i + x) - n\alpha i) \ge \delta_G{}^P(x)$

$= \delta_G{}^P(0)$ and

$\delta_G{}^N(n\alpha(i + x) - n\alpha i) \le \delta_G{}^N(x)$

$= \delta_G{}^N(0)$.

This implies that

(i) $\delta_G{}^P(x - y) = \delta_G{}^P(0)$ and $\delta_G{}^N(x - y) = \delta_G{}^N(0)$.

(ii) $\delta_G{}^P(n + x - n) = \delta_G{}^P(0)$ and $\delta_G{}^N(n + x - n) = \delta_G{}^N(0)$.

(iii) $\delta_G{}^P(x\alpha n) = \delta_G{}^P(0)$ and $\delta_G{}^N(x\alpha n) = \delta_G{}^N(0)$,

$\delta_G{}^P(n\alpha(i + x) - n\alpha i) = \delta_G{}^P(0)$ and $\delta_G{}^N(n\alpha(i + x) - n\alpha i) = \delta_G{}^N(0)$

for all $x, y \in G_\delta$, $n, i \in G$ and $\alpha \in \Gamma$. Thus, $\delta_G = \{ x \in G / \delta_G(x) = \delta_G(0) \}$ is a $\Gamma-$ideal of G over U.

Theorem 6.2

Let δ_G be a BFS set over U and β be a subset of U such that $\phi \ne \beta \le \delta_G{}^P(0)$, $\phi \ne \beta \le \delta_G{}^N(0)$. If δ_G is a BFS $\Gamma-$ideal G over U, then $\delta_G{}^{\supseteq\beta} = \{ x \in G / \delta_G(x) \ge \beta,\ \delta_G{}^N(x) \le \beta \}$ is a BFS $\Gamma-$ideal of G over U.

Proof:

Since $\delta_G{}^P(0) \ge \beta$ and $\delta_G{}^N(0) \le \beta$, so $0 \in \delta_G{}^{\supseteq\beta}$ and $\phi \ne \delta_G{}^{\supseteq\beta} \le G$. Take $x, y \in \delta_G{}^{\supseteq\beta}$, $n, i \in G$ and $\alpha \in \Gamma$, which implies that $\delta_G{}^P(x) \ge \beta$ and $\delta_G{}^N(x) \le \beta$. $\delta_G{}^P(y) \ge \beta$ and $\delta_G{}^N(y) \le \beta$.

Now we need to prove that

(i) $x-y \in \delta_G^{\supseteq\beta}$

(ii) $n+x-n \in \delta_G^{\supseteq\beta}$

(iii) $x\alpha n$ and $n\alpha(i+x)-n\alpha i \in \delta_G^{\supseteq\beta}$ for all $x, y \in \delta_G^{\supseteq\beta}$, $n, i \in G$ and $\alpha \in \Gamma$.

Since δ_G is a BFS $\Gamma-$ideal of G over U, so it follows that

(i) $\delta_G^{\ P}(x-y) \geq \min\{\delta_G^{\ P}(x), \delta_G^{\ P}(y)\}$

$\geq \min\{\beta, \beta\}$

$\geq \beta$ and

$\delta_G^{\ N}(x-y) \leq \max\{\delta_G^{\ N}(x), \delta_G^{\ N}(y)\}$

$\leq \max\{\beta, \beta\}$

$\leq \beta$

(ii) $\delta_G^{\ P}(n+x-n) \geq \delta_G^{\ P}(x) \geq \beta$ and

$\delta_G^{\ N}(n+x-n) \leq \delta_G^{\ N}(x) \leq \beta$

(iii) $\delta_G^{\ P}(x\alpha n) \geq \delta_G^{\ P}(x) \geq \beta$ and

$\delta_G^{\ N}(x\alpha n) \leq \delta_G^{\ N}(x) \leq \beta$,

$\delta_G^{\ P}(n\alpha(i+x)-n\alpha i) \geq \delta_G^{\ P}(x) \geq \beta$ and

$\delta_G^{\ N}(n\alpha(i+x)-n\alpha i) \leq \delta_G^{\ N}(x) \leq \beta$

Thus, $\delta_G^{\supseteq\beta} = \{x \in G / \delta_G(x) \geq \beta,\ \delta_G^{\ N}(x) \leq \beta\}$ is a BFS $\Gamma-$ideal of G over U.

Theorem 6.3

Let δ_G and Δ_H be BFS sets over U and f be a $\Gamma-$near ring isomorphism from G to H.

(i) If δ_G is a BFS $\Gamma-$ideal of G over U, then $f(\delta_G)$ is a BFS $\Gamma-$ideal of H over U.

(ii) If Δ_H is a BFS $\Gamma-$ideal of H over U, then $f^{-1}(\Delta_H)$ is a BFS $\Gamma-$ideal of G over U.

Proof:

(i) Let $x_1, x_2 \in H$. Since f is surjective, there exist $y_1, y_2, y_3 \in G$ such that

$f(y_1) = x_1$, $f(y_2) = x_2$ and $f(y_3) = x_3$. Then

$f(\delta_G^{\ P})(x_1 - x_2) \quad = \cup\{\delta_G^{\ P}(y) : y \in G, f(y) = x_1 - x_2\}$

$= \cup\{\delta_G^{\ P}(y) : y \in G, y = f^{-1}(x_1 - x_2)\}$

$$= \bigcup \left\{ \delta_G{}^P(y) : y \in G, y = f^{-1}(f(y_1 - y_2)) = y_1 - y_2 \right\}$$
$$= \bigcup \left\{ \delta_G{}^P(y_1 - y_2) : y_i \in G, f(y_i) = x_i, i = 1, 2 \right\}$$
$$\geq \bigcup \left\{ \min\left\{ \delta_G{}^P(y_1), \delta_G{}^P(y_2) \right\} : y_i \in G, f(y_i) = x_i, i = 1, 2 \right\}$$
$$= \bigcup \left\{ \min\left\{ \delta_G{}^P(y_1) : y_1 \in G, f(y_1) = x_1, \delta_G{}^P(y_2) : y_2 \in G, f(y_2) = x_2 \right\} \right\}$$
$$= \min\left\{ f(\delta_G{}^P)(x_1), f(\delta_G{}^P)(x_2) \right\}$$

Thus, $f(\delta_G{}^P)(x_1 - x_2) \geq \min\left\{ f(\delta_G{}^P)(x_1), f(\delta_G{}^P)(x_2) \right\}$.

Similarly, we can prove that $f(\delta_G{}^N)(x_1 - x_2) \leq \max\left\{ f(\delta_G{}^N)(x_1), f(\delta_G{}^N)(x_2) \right\}$.

Also

$$f(\delta_G{}^P)(x_1 + x_2 - x_1) = \bigcup \left\{ \delta_G{}^P(y) : y \in G, f(y) = x_1 + x_2 - x_1 \right\}$$
$$= \bigcup \left\{ \delta_G{}^P(y) : y \in G, y = f^{-1}(x_1 + x_2 - x_1) \right\}$$
$$= \bigcup \left\{ \delta_G{}^P(y) : y \in G, y = f^{-1}(f(y_1 + y_2 - y_1)) = y_1 + y_2 - y_1 \right\}$$
$$= \bigcup \left\{ \delta_G{}^P(y_1 - y_2) : y_i \in G, f(y_i) = x_i, i = 1, 2 \right\}$$
$$\geq \bigcup \left\{ \delta_G{}^P(y_2) : y_2 \in G, f(y_2) = x_2 \right\}$$
$$= f(\delta_G{}^P)(x_2)$$

Thus, $f(\delta_G{}^P)(x_1 + x_2 - x_1) \geq f(\delta_G{}^P)(x_2)$.

Similarly, we can prove that $f(\delta_G{}^N)(x_1 + x_2 - x_1) \leq f(\delta_G{}^N)(x_2)$.

Now, let $x_1, x_2 \in H, y_1, y_2 \in G, \alpha \in \Gamma$ and $\alpha_1 \in \Gamma_1$. So

$$f(\delta_G{}^P)(x_1\, \alpha_1\, x_2) = \bigcup \left\{ \delta_G{}^P(y) : y \in G, f(y) = x_1\, \alpha_1\, x_2 \right\}$$
$$= \bigcup \left\{ \delta_G{}^P(y) : y \in G, y = f^{-1}(x_1\, \alpha_1\, x_2) \right\}$$
$$= \bigcup \left\{ \delta_G{}^P(y) : y \in G, y = f^{-1}(f(y_1\, \alpha\, y_2)) = y_1\, \alpha\, y_2 \right\}$$
$$= \bigcup \left\{ \delta_G{}^P(y_1\, \alpha\, y_2) : y_i \in G, f(y_i) = x_i, i = 1, 2 \right\}$$
$$\geq \bigcup \left\{ \delta_G{}^P(y_2) : y_2 \in G, f(y_2) = x_2 \right\}$$
$$= \min\left\{ f(\delta_G{}^P)(x_1), f(\delta_G{}^P)(x_2) \right\}$$

Similarly, we can prove that

$f(\delta_G{}^N)(x_1\, \alpha_1\, x_2) \leq \max\left\{ f(\delta_G{}^N)(x_1), f(\delta_G{}^N)(x_2) \right\}$.

Now, let $x_1, x_2, x_3 \in H, y_1, y_2, y_3 \in G, \alpha \in \Gamma$ and $\alpha_1 \in \Gamma_1$. Then

$$f(\delta_G{}^P)(x_1\,\alpha_1(x_2+x_3)-x_1\,\alpha_1\,x_2) = \bigcup\left\{\delta_G{}^P(y) : y\in G, f(y)=x_1\,\alpha_1(x_2+x_3)-x_1\,\alpha_1\,x_2\right\}$$

$$=\bigcup\left\{\delta_G{}^P(y) : y\in G, y=f^{-1}(x_1\,\alpha_1(x_2+x_3)-x_1\,\alpha_1\,x_2)\right\}$$

$$=\bigcup\left\{\begin{array}{l}\delta_G{}^P(y) : y\in G, y=f^{-1}(f(y_1\,\alpha(y_2+y_3)-y_1\,\alpha\,y_2)) \\ \qquad\qquad = y_1\,\alpha(y_2+y_3)-y_1\,\alpha\,y_2\end{array}\right\}$$

$$=\bigcup\left\{\begin{array}{l}\delta_G{}^P(y_1\,\alpha(y_2+y_3)-y_1\,\alpha\,y_2) : y_i\in G, \\ f(y_i)=x_i, i=1,2,3\end{array}\right\}$$

$$\geq\bigcup\left\{\delta_G{}^P(y_3) : y_3\in G,\ f(y_3)=x_3\right\}$$

$$\geq\delta_G{}^P(x_3)$$

Similarly, we can prove that

$$f(\delta_G{}^N)(x_1\,\alpha_1(x_2+x_3)-x_1\,\alpha_1\,x_2) \leq \delta_G{}^N(x_3).$$

Hence $f(\delta_G)$ is a BFS $\Gamma-$ideal of H over U.

(ii) Let $x_1, x_2 \in H$, $\alpha_1\in\Gamma$ and $\beta_1\in\Gamma_1$. Then

$$(f^{-1}(\Delta_H{}^P))(x_1\,\alpha_1\,x_2) = \Delta_H{}^P(f(x_1\,\alpha_1\,x_2))$$

$$=\Delta_H{}^P(f(x_1)\,\beta_1\,f(x_2))$$

$$\geq\Delta_H{}^P(f(x_1))\cap\Delta_H{}^P(f(x_2))$$

$$=(f^{-1}(\Delta_H{}^P))(x_1)\cap(f^{-1}(\Delta_H{}^P))(x_2)$$

Similarly, we can prove that

$$(f^{-1}(\Delta_H{}^N))(x_1\,\alpha_1\,x_2) \leq (f^{-1}(\Delta_H{}^N))(x_1)\cup(f^{-1}(\Delta_H{}^N))(x_2).$$

Also, we can prove that

$$(f^{-1}(\Delta_H{}^P))(x_1-x_2)\geq\min\left\{(f^{-1}(\Delta_H{}^P))(x_1),(f^{-1}(\Delta_H{}^P))(x_2)\right\}$$

$$(f^{-1}(\Delta_H{}^N))(x_1-x_2)\leq\max\left\{(f^{-1}(\Delta_H{}^N))(x_1),(f^{-1}(\Delta_H{}^N))(x_2)\right\}$$

Also, $(f^{-1}(\Delta_H{}^P))(x_1+x_2-x_1)=\Delta_H{}^P(f(x_1+x_2-x_1))$

$$=\Delta_H{}^P(f(x_1)+f(x_2)-f(x_1))$$

$$\geq\Delta_H{}^P(f(x_2))$$

$$=(f^{-1}(\Delta_H{}^P))(x_2).$$

Similarly, we can prove that

$$(f^{-1}(\Delta_H{}^N))(x_1+x_2-x_3) \leq (f^{-1}(\Delta_H{}^N))(x_2).$$

Now, let $x_1, x_2, x_3\in H$, $\alpha_1\in\Gamma$ and $\beta_1\in\Gamma_1$.

Then

$$(f^{-1}(\Delta_H{}^P))(x_1\alpha_1(x_2+x_3)-x_1\alpha_1\ x_2)=(\Delta_H{}^P)(f(\ x_1\alpha_1(x_2+x_3)-x_1\alpha_1\ x_2))$$
$$=(\Delta_H{}^P)(f(x_1)\beta_1 f(x_2)+f(x_3)-f(x_1)\beta_1 f(x_2))$$
$$\geq(\Delta_H{}^P)(f(x_3))$$
$$=(f^{-1}(\Delta_H{}^P))(x_3)$$

Similarly, we can prove that

$(f^{-1}(\Delta_H{}^N))(x_1\alpha_1(x_2+x_3)-x_1\alpha_1\ x_2)\leq(f^{-1}(\Delta_H{}^N))(x_3)$.

Hence $f^{-1}(\Delta_H)$ is a BFS $\Gamma-$ideal of G over U.

Definition 6.1 Let δ_G be a bipolar fuzzy soft set (BFSS) over U. Then δ_G is called bipolar fuzzy soft characteristic if $\delta_G{}^f(x)=\delta_G(f(x))\ \forall\ x\in G$.

Theorem 6.4

Let $f:\ X\rightarrow Y$ be a homomorphism and δ_G be a bipolar fuzzy soft set (BFSS) over U. Then $\delta_G{}^f$ is BFS $\Gamma-$near ring over U.

Theorem 6.5

Let G and G' be two $\Gamma-$near rings and $\theta:G\rightarrow G'$ a homomorphism. If $B=(\delta_B{}^P,\delta_B{}^N)$ is a BFS $\Gamma-$ideal of G',

then $\theta^{-1}(B)=\left(\delta^P_{\theta^{-1}(B)},\ \delta^N_{\theta^{-1}(B)}\right)$ of B under θ is BFS $\Gamma-$ideal of G.

Theorem 6.6

Let G and G' be two $\Gamma-$near rings and $\theta:G\rightarrow G'$ an epimorphism. If $B=(\delta_B{}^P,\delta_B{}^N)$ is a BFS set in G' such that $\theta^{-1}(B)=\left(\delta^P_{\theta^{-1}(B)},\ \delta^N_{\theta^{-1}(B)}\right)$ of B under θ is BFS $\Gamma-$ideal of G, then B is a BFS $\Gamma-$ideal of G'.

7. Normal BFS $\Gamma-$ideal:

In this section, we introduce Normal BFS $\Gamma-$ideal and characterize normal BFS $\Gamma-$ideal of G.

Definition 7.1 A BFS $\Gamma-$ideal $A=(\delta_A{}^P,\delta_A{}^N)$ of G is said to be normal if there exists $x\in G$ such that $A(x)=(1,-1)$. (ie) $\delta_A{}^P(x)=1$ and $\delta_A{}^N(x)=-1$.

Definition 7.2 An element $x_0\in G$ is said to be an extremal element of a BFS-set $A=(\delta_A{}^P,\delta_A{}^N)$ if $\delta_A{}^P(x_0)\geq\delta_A{}^P(x)$ and $\delta_A{}^N(x_0)\leq\delta_A{}^N(x)$ for all $x\in G$.

Preposition 7.1

A BFS set $A=(\delta_A{}^P,\delta_A{}^N)$ of G is a normal BFS $\Gamma-$ideal if and only if $A(x)=(1,-1)$.

Theorem 7.1

If x_0 is an extremal element of a BFS $\Gamma-$ideal $A=(\delta_A{}^P,\delta_A{}^N)$ of G, then a BFS set $\overline{A}=(\overline{\delta}_A{}^P,\overline{\delta}_A{}^N)$ of G, defined by $\overline{\delta}_A{}^P(x)=\delta_A{}^P(x)+1-\delta_A{}^P(x_0)$ and $\overline{\delta}_A{}^N(x)=\delta_A{}^N(x)-1-\delta_A{}^N(x_0)$ for all $x\in G$, is a normal BFS $\Gamma-$ideal of G containing A. Clearly $A\subset\overline{A}$.

Theorem 7.2

A non-constant maximal element of $(N(G),\subseteq)$ only takes a value among $(0,0)$, $(1,-1)$ and $(1,0)$.

Preposition 7.2

A maximal BFS $\Gamma-$ideal of G is normal and it takes a value among (0, 0), (1, -1) and (1, 0).

Conclusion:

The Basic properties of bipolar fuzzy soft set and its application in $\Gamma-$near rings and $\Gamma-$ideals has been discussed in this paper. Also, normal Bipolar Fuzzy soft $\Gamma-$ideal and its isomorphism discussed. Finally, the main application area of Bipolar fuzzy soft set on a group action has been investigated.

Future work:

One can obtain this similar idea in to the field of module theory, cryptotography theory and various algebraic structures.

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