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# SOME IMAGINARY STEP FUZZY SOFT SUBGROUP STRUCTURES UNDER TRIANGULAR NORMS

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**Abstract :** In this article, we combine complex soft fuzzy set and fuzzy soft subgroup. We introduce the notion of t-level complex fuzzy soft subgroup, step complex fuzzy soft subgroup and study fundamental properties. Also the basic operations like union, intersection, product, cartesian product of two complex fuzzy soft subgroups are studied. Furthermore, we define a t-level subset and t-level subgroup and then some of their properties are studied.

**Keywords :** Complex fuzzy soft group, t-level subgroup, t-norm, step complex fuzzy soft group, cartesian product, phase term.

**AMS Subject Classification [2000]:** 06F35, 03G25, 03E72, 20N25

**1.Introduction:** Soft set theory was introduced in 1999 by Molodtsov [21] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagun and Sezgin [6] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji et. al [19] presented some definitions on soft sets and based on the analysis of several operations on soft sets. Ali et. al [5] introduced several operations of soft sets and Sezgin and Atagun [26] studied on soft set operations as well. Furthermore, soft set relations and functions [10] and soft mappings [20] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies [6, 7, 23, 29]. The concept of the fuzzy set was first introduced by Zadeh in a seminal paper in 1965 [32]. This is the generalization of script set in terms of membership function. The notion of the fuzzy set A on the universe of discourse U is the set of order pair $\{(x, \mu_A(x)),$

x ∈ U} with a membership function $\mu_A(x)$, taking the value on the interval [0,1]. In [25] Rosenfeld used this concept to develop the theory of fuzzy groups. In fact, many basic properties in group theory are found to be carried over to fuzzy groups. Anthony and Sherwood [1] redefined fuzzy subgroups in terms of a t-norm which replaced the minimum operation and they characterized basic properties of t-fuzzy subgroup in [1,2]. Sherwood [27] defined products of fuzzy subgroups using t-norms and gave some properties of these products. Ramot et. al [24] extended the fuzzy set to complex fuzzy set with membership function $z = r_s\, e^{-i\omega_s(x)}$ where $i = \sqrt{-1}$ , which ranges in the interval [0,1] to a unit circle. Ramot et. al [23] also introduced different fuzzy complex operations and relations like union, intersection, complement etc. Still it is necessary to determine the membership functions correctly, which will give the appropriate or approximate result for real life applications. The membership function defined for the complex fuzzy set $z = r_s\, e^{i\omega_s(x)}$ , which compromise an amplitude term $r_s(x)$ and phase term $\omega_s$. The amplitude term retains the idea of "fuzziness" and phase term signifies declaration of complex fuzzy set, for which the second dimension of membership is required. The complex fuzzy set allows extension of fuzzy logic that is to continue with one dimension gradeness of membership. Xin Fu et. al [28] defined the fuzzy complex membership function of the form z = a+ib, where a, b are two fuzzy numbers with membership functions $\mu_A(a)$, $\mu_B(b)$ respectively. If b does not exist, z degenerates to a fuzzy number. Xin Fu et. al [28] also discussed a complex number in cartesian form where a= $r_s$ cos(x) and b= $r_s$ sin(x), which are in polar forms defined in [8]. The fuzzy number is created by interpolating complex number in the support of fuzzy set [3, 4, 5, 8].The membership functions are usually difficult to determine accurately and one may argue of accurate or precise membership functions are necessary in reality.

Following the above resent development the fuzzy set theory Zhang et. al [30] studies d-equality of complex fuzzy set. This is the logical development since a complex membership function should more difficult that a real membership function to be determine in practice. Complex fuzzy set is a unique framework over the advantage of traditional fuzzy set .The support of complex fuzzy set is unrestricted, may include any kind of object such as number, name etc., which is off course a complex number.

This paper is organized in the following order. The review of complex fuzzy soft set and operations on it which are discussed in section- 2 and section -3 discussed properties of complex fuzzy soft sets, some important results proved in section-4, t-level subgroup and its properties are investigated in section-5. The idea of step fuzzy complex soft subgroups are studied in section- 6, followed by summary and suggestion for future work are given in section -7.

## 2. COMPLEX FUZZY SETS AND OPERATORS

**2.1 Definition**[5]: A pair (F,A) is called a soft set over U, where F is a mapping given by F : A→P(U).

In other words, a soft set (F,A) over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by $F_A$. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by $F_A$, $F_B$, $F_C$ respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by $F_A$,$G_A$, $H_A$ respectively. For more details, we refer to [7,11,17,18,26,29].

**2.2 Definition**[19]: The relative complement of the soft set $F_A$ over U is denoted by $F^r{}_A$, where

$F^r{}_A : A \rightarrow P(U)$ is a mapping given as $F^r{}_A(a) = U \setminus F_A(a)$, for all $a \in A$.

**2.3 Definition**[19]: Let $F_A$ and $G_B$ be two soft sets over U such that $A \cap B \neq \emptyset$. The restricted intersection of $F_A$ and $G_B$ is denoted by $F_A \Cup G_B$, and is defined as $F_A \Cup G_B$ =(H,C), where

$C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

**2.4 Definition**[19]: Let $F_A$ and $G_B$ be two soft sets over U such that $A \cap B \neq \emptyset$. The restricted union of $F_A$ and $G_B$ is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B$ = (H,C),where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

**2.5 Definition[7]:** A complex fuzzy soft subset A, defined on a universe of discourse X, is characterized by a membership function $\tau_A(x)$ that assigns any element x∈X and a complex valued grade of membership in A. The values of $\tau_A(x)$ all lie within the unit circle in the complex plane and thus all of the form $P_A(x)\, e^{j\mu_A(x)}$ where $P_A(x)$ and $e^{j\mu_A(x)}$ are both real valued and $P_A(x) \in [0,1]$. Here $P_A(x)$ is termed as amplitude term and $e^{j\mu_A(x)}$ is termed as phase term.

The complex fuzzy soft set may be represented in the set form as A = {(x, $\tau_A$(x)) / x∈X }. It is denoted by CFSS.

The phase term of complex membership function belongs to (0,2$\pi$). Now we take those forms which Ramot et.al presented in [23] to define the game of winner, neutral and loser.

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & \text{if } p_A > p_B \\ \mu_B(x) & \text{if } p_A < p_B \end{cases}.$$

This is a novel concept and it is the generalization of the concept " winner take all" introduced by Ramot et.al [23] for the union of phase terms.

**2.6 Example:** Let X = {$x_1$, $x_2$, $x_3$} be a universe of discourse. Let A and B be complex fuzzy soft sets in X as shown below.

$$A = \{0.6\, e^{i(0.8)},\ 0.3\, e^{i\frac{3\pi}{4}},\ 0.5\, e^{i(0.3)}\}$$

$$B = \{0.8\, e^{i(0.9)},\ 0.1\, e^{i\frac{\pi}{4}},\ 0.4\, e^{i(0.5)}\}$$

$$A\cup B = \{0.8\, e^{i(0.9)},\ 0.3\, e^{i\frac{3\pi}{4}},\ 0.5\, e^{i(0.3)}\}$$

We can easily calculate the phase terms $e^{i\mu_{A\cap B}(x)}$ on the same line by winner, neutral and loser game.

**2.7 Definition:** Here a complex fuzzy soft subset A of a group G is said to be a complex fuzzy soft subgroup of G if for all x ,y ∈ G

(CFSSG1) A(xy) ≥ min { A(x), A(y) },

(CFSSG2) A($x^{-1}$) ≥ A(x) where the product x and y is denoted by xy and the inverse of x by $x^{-1}$. It is well known and easy to see that a complex fuzzy soft subgroup G satisfies A(x)≤ A(e) and A($x^{-1}$) = A(x) for all x ∈ G, where 'e' is the identity of G.

**2.8 Definition[11]:** A triangular norm (briefly a t-norm) is a function T:[0,1] x [0,1] → [0,1] satisfying, for each p, q, r, s ∈ [0,1]

(1) T(p,1) = p

(2) T(p, q) ≤ T(r, s) if p≤r and q≤s,

(3) T(p, q) = T(q, p),

(4) T( p, T( q, r)) = T( T( p, q),r).

## 3.PROPERTIES OF OPERATIONS ON COMPLEX FUZZY SOFT SETS.

**3.1 Proposition:** Let A, B, C be three complex fuzzy soft sets on X. Then

(i) (A∪B) ∩ C = (A∩ C) ∪(B∩C)

(ii) (A∩B) ∪ C = (A∪ C) ∩(B∪C)

**Proof:** Let A, B, C be three complex fuzzy sets on X and $\mu_A$ (x), $\mu_B$ (x), $\mu_C$ (x) are membership functions. Then we have

$$\tau_{(A\cup B)\cap C}(x) = P_{(A\cup B)\cap C}(x)\, e^{i\mu_{(A\cup B)\cap C}(x)}$$
$$= \min\{P_{A\cup B}(x), P_C(x)\}\, e^{i\min\{\mu_{A\cup B}(x),\mu_C(x)\}}$$
$$= \min\{\max\{P_A(x), P_B(x)\}, P_C(x)\}\, e^{i\min\{\max\{\mu_A(x),\mu_B(x)\},\mu_C(x)\}}$$
$$= \max\{\min\{P_A(x), P_C(x)\}, \min\{P_B(x), P_C(x)\}\}\, e^{i\max\{\min\{\mu_A(x),\mu_C(x)\},\min\{\mu_B(x),\mu_C(x)\}\}}$$
$$= \max\{P_{A\cap C}(x), P_{B\cap C}(x)\}\, e^{i\max\{\mu_{A\cap C}(x),\mu_{B\cap C}(x)\}}$$
$$= P_{(A\cap C)\cup(B\cap C)}(x)\, e^{i\mu_{(A\cap C)\cup(B\cap C)}(x)}$$
$$= \tau_{(A\cap C)\cup(B\cap C)}(x)$$

Similarly, we can show (ii).

**3.2 Proposition:** Let A and B be two complex fuzzy soft sets in X. Then

(i) (A∪B) ∩ A = A

(ii) (A∩B) ∪ A = A

**Proof:** Let A and B be two complex fuzzy sets in X and $\mu_A$ (x), $\mu_B$ (x)are membership functions. Then we have

$$\tau_{(A\cup B)\cap A}(x) = P_{(A\cup B)\cap A}(x)\, e^{i\mu_{(A\cup B)\cap A}(x)}$$
$$= \min\{P_{A\cup B}(x), P_A(x)\}\, e^{i\min\{\mu_{A\cup B}(x),\mu_A(x)\}}$$
$$= \min\{\max\{P_A(x), P_B(x)\}, P_A(x)\}\, e^{i\min\{\max\{\mu_A(x),\mu_B(x)\},\mu_A(x)\}}$$
$$= P_A(x)\, e^{i\mu_A(x)}$$
$$= \tau_A(x)$$

Similarly, we can show (ii).

**3.3 Proposition:** Let A and B be two complex fuzzy soft sets in X. Then

(i) A ∪ (A ∩ B) = A

(ii) A ∩ (A∪ B) = A (Absorption laws)

**Proof:**

$\tau_{A\cup(A\cap B)}(x) = P_{A\cup(A\cap B)}(x)\, e^{i\mu_{A\cup(A\cap B)}(x)}$

$= \max\{P_A(x), P_{A\cap B}(x)\}\, e^{i\max\{\mu_A(x),\mu_{A\cap B}(x)\}}$

$= \max\{P_A(x), \min\{P_A(x), P_B(x)\}\}\, e^{i\max\{\mu_A(x),\min\{\mu_A(x),\mu_B(x)\}\}}$

$=\min\{\max\{P_A(x), P_A(x)\},$

$\max\{P_A(x), P_B(x)\}\}\, e^{i\min\{\max\{\mu_A(x),\mu_A(x)\},\max\{\mu_A(x),\mu_B(x)\}\}}$

$= P_A(x)\, e^{i\mu_A(x)}$

$= \tau_A(x)$

Similarly, we can show (ii).

**3.4 Proposition:** Let A and B be two complex fuzzy soft sets in X. Then

(i) A ∪ B = B ∪ A

(ii) A ∩ B = B ∩ A (Commutative laws)

**Proof:** Let A and B be two complex fuzzy soft sets in X and $\mu_A(x), \mu_B(x)$ are membership functions. Then we have

$\tau_{A\cup B}(x) = P_{A\cup B}(x)\, e^{i\mu_{A\cup B}(x)}$

$= \max\{P_A(x), P_B(x)\}\, e^{i\max\{\mu_A(x),\mu_B(x)\}}$

$= \max\{P_B(x), P_A(x)\}\, e^{i\max\{\mu_B(x),\mu_A(x)\}}$

$= P_{B\cup A}(x)\, e^{i\mu_{B\cup A}(x)}$

$= \tau_{B\cup A}(x)$

Similarly, we can show (ii).

**3.5 Definition:** Let S be a groupoid and T is a t-norm. A function B: S→[0,1] is a subgroupoid of S iff for every x,y in S, B(xy) ≥ T{B(x),B(y)}. If S is group, a t-complex fuzzy soft subgroupoid B is a t- complex fuzzy soft subgroup of S iff for each x ∈ S, $B(x^{-1}) \geq B(x)$.

**3.6 Definition:** For each i = 1,2,….,n, let $A_i$ be a t- complex fuzzy soft subgroup in a group $X_i$. Let T be a t-norm. The T-product of $A_i$ (i = 1,2,….,n) is the function,

$A_1 \times A_2 \times \dots \times A_n : X_1 \times X_2 \times \dots \times X_n \longrightarrow [0,1]$ defined by

$(A_1 \times A_2 \times \dots \times A_n)(x_1, x_2, \dots, x_n) = T(A_1(x_1), A_2(x_2), \dots, A_n(x_n))$.

**3.7 Definition:** For each i = 1,2,….,n, let $A_i$ be a complex fuzzy soft subgroup under a minimum operation in a group $X_i$, the membership function of the product
$A = A_1 \times A_2 \times \dots \times A_n$ in $X = X_1 \times X_2 \times \dots \times X_n$ is defined by
$(A_1 \times A_2 \times \dots \times A_n)(x_1, x_2, \dots, x_n) = \min\{(A_1(x_1), A_2(x_2), \dots, A_n(x_n)\}$.

**3.8 Definition:** Let A be a complex fuzzy soft subset of a set X and let t ∈ [0,1]. The set $A_t = \{ x \in X : A(x) \geq t\}$ is called a level subset of A.

**3.9 Definition:** A complex fuzzy soft subgroup A of a group X is called complex fuzzy soft normal if for all x,y in G, it fulfills the following condition A(xy) = A(yx).

**3.10 Definition:** A complex fuzzy soft subgroup A of a group X is said to be conjugate to a complex fuzzy soft subgroup B of G if there exists x in G such that for all g in G, $A(g) = B(x^{-1}gx)$.

**4.RESULTS**

**4.1 Theorem:** Every complex fuzzy soft subset A of the universe of discourse X is a complex fuzzy soft subgroup of X.

**Proof:** Let A be a complex fuzzy soft subset of X. Then for all x,y ∈ X.

(CFSSG-1) $A(xy) = p(xy)\, e^{j\mu(xy)}$

$\geq \min\{p(x).p(y)\} . e^{j \min\{\mu(x) \wedge \mu(y)\}}$

$\geq \min\{p(x).p(y)\}. e^{j\mu(x)}. e^{j\mu(y)}$

$\geq \min\{p(x)\, e^{j\mu(x)}, p(y)\, e^{j\mu(y)}\}$

$\geq \min\{A(x), A(y)\}$

(CFSSG-2) $A(x^{-1}) = p(x^{-1})\, e^{j\mu(x^{-1})} \geq p(x)\, e^{j\mu(x^{-1})} \geq p(x)\, e^{j\mu(x)} \geq A(x)$.

Complex fuzzy soft subgroup conditions satisfied.

Therefore A form a complex fuzzy soft subgroup of X.

**4.2 Theorem :** Intersection of non-empty collection of complex fuzzy soft subgroups is a complex fuzzy soft subgroup.

**Proof:**

(CFSSG-1) $(\bigcap_{i\in I} A_i)(xy) = \mathrm{Inf}_{i\in I} A_i(xy)$

$\geq \mathrm{Inf}_{i\in I}\{\min\{A_i(x), A_i(y)\}\}$

$\geq \min\{\mathrm{Inf}_{i\in I} A_i(x), \mathrm{Inf}_{i\in I} A_i(y)\}$

$\geq \min\{\bigcap_{i\in I} A_i(x), \bigcap_{i\in I} A_i(y)\}$

(CFSSG-2) $(\bigcap_{i\in I} A_i)(x^{-1}) = \mathrm{Inf}_{i\in I} A_i(x^{-1}) \geq \bigcap_{i\in I} A_i(x)$

**4.3 Definition:** Let A and B be two complex fuzzy soft sets on X and $\tau_A(x) = r_A(x)\, e^{j\mu_A(x)}$ and $\tau_B(x) = r_B(x)\, e^{j\mu_B(x)}$ be their membership functions, respectively. The union and intersection

of A and B are denoted as A∪B and A∩B, which is specified by a functions

$\tau_{A\cup B}(x) = r_{A\cup B}(x)\, e^{j\mu_{A\cup B}(x)} = (r_A(x) \vee r_B(x))\, e^{j(\mu_A(x)\vee\mu_B(x))}$ , where ∨ denotes the max operator.

$\tau_{A\cap B}(x) = r_{A\cap B}(x)\, e^{j\mu_{A\cap B}(x)} = (r_A(x) \wedge r_B(x))\, e^{j(\mu_A(x)\wedge\mu_B(x))}$ , where ∧ denotes the min operator.

**4.4 Theorem:** Union of two complex fuzzy soft subgroups is also a complex fuzzy soft subgroup.

**Proof:** Let A and B be two complex fuzzy soft subgroups.

Now let x,y ∈ X,

(CFSSG-1) $\tau_{A\cup B}(xy) = r_{A\cup B}(xy)\, e^{j\mu_{A\cup B}(xy)}$

$= (r_A(xy) \vee r_B(xy))\, e^{j(\mu_A(xy)\vee\mu_B(xy))}$

$\geq ((r_A(x) \wedge r_A(y)) \vee ((r_B(x) \wedge r_B(y))\, e^{j((\mu_A(x)\wedge\mu_A(y))\vee(\mu_B(x)\wedge\mu_B(y)))}$

$\geq ((r_A(x) \vee r_A(y)) \wedge ((r_B(x) \vee r_B(y))\, e^{j((\mu_A(x)\vee\mu_A(y))\wedge(\mu_B(x)\vee\mu_B(y)))}$

$\geq (r_A(x) \vee r_B(x))\, e^{j\mu_{A\cup B}(x)} \wedge (r_A(y) \vee r_B(y))\, e^{j\mu_{A\cup B}(y)}$

$\geq \min\{\tau_{A\cup B}(x), \tau_{A\cup B}(y)\}$

(CFSSG-2) $\tau_{A\cup B}(x^{-1}) = r_{A\cup B}(x^{-1})\, e^{j\mu_{A\cup B}(x^{-1})}$

$= (r_A(x^{-1}) \vee r_B(x^{-1}))\, e^{j(\mu_A(x^{-1})\vee\mu_B(x^{-1}))}$

$\geq (r_A(x) \vee r_B(x))\, e^{j(\mu_A(x)\vee\mu_B(x))} = r_{A\cup B}(x)\, e^{j\mu_{A\cup B}(x)} \geq \tau_{A\cup B}(x)$

**4.5 Theorem:** Intersection of two complex fuzzy soft subgroups is also a complex fuzzy soft subgroup.

**Proof:** The result is obvious.

**4.6 Theorem:** The product of two complex fuzzy soft subgroups is also a complex fuzzy soft subgroup.

**Proof:** Let A and B be two complex fuzzy soft subgroups of X.

Now let x,y ∈ X,

(CFSSG-1) $\tau_{A\cup B}(xy) = r_{A\cup B}(xy)\, e^{j\mu_{A\cup B}(xy)}$

$= (r_A(xy)\,.\,r_B(xy))\, e^{j2\pi\left(\frac{\mu_A(xy)}{2\pi}\,.\,\frac{\mu_B(xy)}{2\pi}\right)}$

$\geq (\min\{r_A(x), r_A(y)\}.\ \min\{r_B(x), r_B(y)\})\ e^{j2\pi\left(\frac{\min\{\mu_A(x),\mu_A(y)\}}{2\pi}\,.\,\frac{\min\{\mu_B(x),\mu_B(y)\}}{2\pi}\right)}$

$\geq \min\{r_A(x).r_B(x), r_A(y).r_B(y)\}\ e^{j2\pi\left(\frac{\mu_{A\cup B}(x)}{2\pi}\,.\,\frac{\mu_{A\cup B}(y)}{2\pi}\right)}$

$\geq \min\{r_{A\cup B}(x), r_{A\cup B}(y)\}\ e^{j\mu_{A\cup B}(x)}\,.\,e^{j\mu_{A\cup B}(y)}$

$\geq \min\{r_{A\cup B}(x)\, e^{j\mu_{A\cup B}(x)}, r_{A\cup B}(y)\, e^{j\mu_{A\cup B}(y)}\}$

$\geq \min\{\tau_{A\cup B}(x), \tau_{A\cup B}(y)\}$.

CFSSG-1 is satisfied.

(CFSSG-2) $\tau_{A\cup B}(x^{-1}) = r_{A\cup B}(x^{-1})\ e^{j\mu_{A\cup B}(x^{-1})} \geq r_{A\cup B}(x)\ e^{j\mu_{A\cup B}(x)} \geq \tau_{A\cup B}(x)$ .

CFSSG-2 is satisfied.

**4.7 Definition:** Let A and B be two complex fuzzy soft sets on X and $\tau_A(x) = r_A(x)\, e^{j\mu_A(x)}$ and $\tau_B(x) = r_B(x)\ e^{j\mu_B(x)}$ be their membership functions, respectively. The complex product of A and B , denoted as AoB and is specified by a function

$\tau_{AoB}(x) = r_{AoB}(x)\, e^{j\mu_{AoB}(x)} = (r_A(x)\,.\,r_B(x))\, e^{j2\pi\left(\frac{\mu_A(x)}{2\pi}\,.\,\frac{\mu_B(x)}{2\pi}\right)}$

**4.8 Definition:** Let A and B be two complex fuzzy soft sets on X. Then the cartesian product of A and B is defined as

$(AxB)(x,y) = r_{AxB}(x,y)\, e^{j\mu_{AxB}(x,y)}$

$= \min\{r_A(x), r_B(y)\}.\ e^{j(\mu_A(x)\vee\mu_B(y))}$

$= \min\{r_A(x), r_B(y)\}.\ e^{j\mu_A(x)}.\, e^{j\mu_B(y)}$

$= \min\{r_A(x)\, e^{j\mu_A(x)}, r_B(y)\, e^{j\mu_B(y)}\}$

= min {A(x) ,B(x)} where A(x) = $r_A$(x) $e^{j\mu_A(x)}$ and B(x) = $r_B$(y) $e^{j\mu_B(y)}$

**4.9 Note :** Complement of complex fuzzy soft subgroup is not a complex fuzzy soft subgroup.

**Proof:** By definition, $\tau_{C(S)}(x) = P_{C(S)}(x)\, e^{j\mu_{C(S)}(x)}$

$= (1-P_S(x))\, e^{j(2\pi-\mu_S(x))}$

Now, $\tau_{C(S)}(xy) = (1-P_S(xy))\, e^{j(2\pi-\mu_S(xy))} = 1 - \tau_S(xy)$

$\leq 1-\min\{\tau_S(x), \tau_S(y)\}$

$\leq \max\{1-\tau_S(x), 1-\tau_S(y)\}$

$\leq \max\{\tau_{C(S)}(x), \tau_{C(S)}(y)\}$

∴ Complement of complex fuzzy soft subgroup is not a complex fuzzy soft subgroup.

**4.10 Proposition:** For all a,b ∈ [0,1] and 'm' is any positive integer, verify that

(i) If $a < b$, then $a^m < b^m$ and

(ii) $\min\{a,b\} = \min\{a^m, b^m\}$.

**Proof:** It is obvious.

**4.11 Proposition:** If X is a group, then $X^m = \{x, (A_X(x))^m / x \in X\}$ is a complex fuzzy soft subgroup.

**Proof:** Let X is a complex fuzzy soft subgroup, where ( X , .) is a group. Thus ($X^m$, .) is a group for all positive integer m. Let 'm' be a positive integer and x,y ∈ $X^m$.

(CFSSG-1) $(A_m(xy)) = (A(xy))^m$

$\geq \min\{A(x), A(y)\}^m$

$= \min\{(A(x))^m, (A(y))^m\}$

$= \min\{A_m(x), A_m(y)\}$

(CFSSG-2)$(A_m(x^{-1})) = (A(x^{-1}))^m \geq (A(x))^m \geq (A_m(x))$ .

Therefore ($X^m$, .) is a complex fuzzy soft subgroup.

**4.12 Corollary:** The complex fuzzy soft subgroup of $X^n$ is a complex fuzzy soft subgroup of $X^m$ if $m \leq n$.

**Proof:** Clearly $X^n$ and $X^m$ are complex fuzzy soft subgroups by above theorem(4.11), for all x ∈ X.

Also $X^m \geq X^n$ implies that $X^n \subset X^m$ (since $A_n(x) \leq A_m(x)$ for all $x \in X$).

## 5. t-level subgroup

In this section, we introduce a definition of a t-level subset of a complex fuzzy soft subset and then we give some of the important algebraic results.

**5.1 Definition:** Let 'A' be a complex fuzzy soft subset of a set X, T a t-norm and $r \in [0,1]$. Then we define a t-level subset of a complex fuzzy soft subset A as

$$A_r^T = \{ x \in G / T(A(x),r) \geq r \}.$$

The following are the main results based on the above definition 5.1

**5.2 Theorem:** Let X be a group and A be a t-complex fuzzy soft subgroup of X, then the t-level subset $A_r^T$, $r \in [0,1]$, is a subgroup of X.

**Proof:** $A_r^T = \{ x \in G / T(A(x),r) \geq r \}$is clearly non-empty.

Let $x, y \in A_r^T$, then $T(A(x),r) \geq r$ and $T(A(y),r) \geq r$ .

Since 'A' is t-complex fuzzy soft subgroup of X, $A(xy) = p(xy)\ e^{j\mu(xy)} \geq T\ (A(x),A(y))$ is satisfied.

This means $T(A(xy), r) \geq T(T(A(x),A(y)),r) = T((\ A(x), T(A(y))),r) \geq T(A(x),r) \geq r$.

Hence $xy \in A_r^T$ .

Since 'A' is t-complex fuzzy soft subgroup, $A\ (x^{-1}) = A(x)$ and hence $T(A(x^{-1}), r) = T(A(x), r) \geq r$.

This means that $x^{-1} \in A_r^T$ . Therefore $A_r^T$ is a subgroup of X.

**5.3 Theorem:** Let X be a group and A be a complex fuzzy soft subgroup of X. Then the t-level subset $A_r^T$, $r \in [0,1]$, $T(A(e),r) \geq r$ , is a subgroup of X, where 'e' is the identity of X.

**Proof:** $A_r^T = \{ x \in G / T(A(x),r) \geq r \}$is clearly non-empty.

Let $x, y \in A_r^T$ . Then $T(A(x),r) \geq r$ and $T(A(y),r) \geq r$ .

Since 'A' is a subgroup of X, $A(xy) = p(xy)\ e^{j\mu(xy)} \geq \min \{A(x), A(y)\}$ is satisfied.

This means that $T(\ A(xy), r) \geq T\ (\min\{A(x),A(y)\},r)$, where there are two cases:

$\min \{A(x), A(y)\}= A(x)$ or $\min \{A(x), A(y)\}= A(y)$ .

Since $x, y \in A_r^T$, also in two cases $T((\min \{A(x), A(y)\}),r) \geq r$.

Therefore $T(A(xy),r) \geq r$ . Thus we get $xy \in A_r^T$ . It is easily seen that, as above $x^{-1} \in A_r^T$ . Hence

$A_r^T$ is a subgroup of X.

**5.4 Theorem:** Let X be a group and A be a complex fuzzy soft subset of X such that $A_r^T$ is a subgroup of X for all r ∈ [0,1], T(A(x),r) ≥ r.Then A is a t-fuzzy complex soft subgroup of X.

**Proof:** Let x, y ∈ X and let $T(A(x),r_1) = r_1$ and $T(A(y),r_2) = r_2$.

Then x ∈ $A_{r_1}^T$ , y ∈ $A_{r_2}^T$ . Let us assume that $r_1 < r_2$. Then these follows $T(A(x),r_1) < T(A(y),r_2)$

and $A_{r_2}^T \subseteq A_{r_1}^T$ . So y∈ $A_{r_1}^T$ . Thus x, y ∈ $A_{r_1}^T$ and since $A_{r_1}^T$ is a subgroup of X, by hypothesis,

xy ∈ $A_{r_1}^T$ . Therefore $T(A(xy),r_1) \geq r_1 = (T(A(x),r_1) , T(A(y),r_1)) = T(T(A(x),A(y)), r_1)$.

Thus we get $T(A(xy),r_1) \geq T(T(A(x),A(y)), r_1)$ . As a t-norm is monotone with respect to each variable and symmetric, we have $A(xy) = p(xy)\, e^{j\mu(xy)} \geq T(A(x),A(y))$ .

Next, let x ∈ X and T(A(x),r)= r . Then x∈ $A_r^T$ . Since $A_r^T$ is a subgroup, $x^{-1} \in A_r^T$ .

Therefore $T(A(x^{-1}),r) \geq r$ and hence $T(A(x^{-1}),r) \geq T(A(x),r)$. So we have $A(x^{-1}) \geq A(x)$.

Thus A is a t-complex fuzzy soft subgroup of X.

**5.5 Theorem:** Let A and B be t-level subsets of the sets $X_1$ and $X_2$, respectively and let r ∈ [0,1]. Then AxB is also a t-level subset of $X_1 x X_2$.

**Proof:** Since any t-norm T is associative, using definition-5.1, we can write the following statement

$$T((AxB)(a,b),r) = T( T(A(a),B(b)),r) = T(( A(a),T(B(b))),r) \geq T(A(a), r) \geq r.$$

This completes the proof.

**5.6 Definition:** Let X be a group and A be a t-complex fuzzy soft subgroup of X. The subgroups $A_r^T$ , r ∈ [0,1] and T(A(x),r) ≥ r are called t-level subgroups of A.

The following theorems are proved based on the definition 5.6

**5.7 Theorem:** Let $X_1$ and $X_2$ be two groups. A and B be t-complex fuzzy soft subgroups of $X_1$ and $X_2$ respectively. Then the t-level subset$(AxB)_r^T$ , for r ∈ [0,1] is a subgroup of $X_1 x X_2$ , where $e_{X_1}$ and $e_{X_2}$ are identities of $X_1$ and $X_2$ respectively.

**Proof:** $(A \text{ x } B)_r^T = \{(x, y) \in X_1 x X_2 : T((AxB)(x, y), r) \geq r \}$.

Since $T((A \text{ x } B)(e_{X_1}, e_{X_2}), r) = T( T(A(e_{X_1}),B(e_{X_2})),r) \geq T(A(e_{X_1}),r) \geq r$

$\therefore$ $(A \times B)_r^T$ is non empty. Let $(x_1, y_1)$, $(x_2, y_2) \in (A \times B)_r^T$ . Then $T((A \times B)(x_1, y_1), r) \geq r$ and

$T((A \times B)(x_2, y_2), r) \geq r$. Since A x B is a t-complex fuzzy soft subgroup of $X_1 x X_2$ ,

we get $(A \times B)\ ((x_1, y_1)\ (x_2, y_2)) = T(\ A(x_1x_2), B(y_1y_2))$.

Using A and B are t-complex fuzzy soft subgroups , we get

$T\big((A \times B)(x_1x_2, y_1y_2)\big) \geq T(T(\ A(x_1x_2), B(y_1y_2)),r)$

$= T((A(x_1x_2),T(B(y_1y_2))),r) \geq T(A(x_1x_2),r) \geq r$

Hence $(x_1, y_1)$, $(x_2, y_2) \in (A \times B)_r^T$ . Again $(x,y) \in (AxB)_r^T$ implies

$T((A \times B)(x, y)^{-1}, r) = T((A \times B)(x^{-1}, y^{-1}), r)$

$= T(T(A(x^{-1}), B(y^{-1})),r) = T((A(x^{-1}), T(B(y^{-1}))),r) \geq T(A(x^{-1}),r) \geq r$

This means that $(x, y)^{-1} \in (A \times B)_r^T$ . Therefore $(A \times B)_r^T$ is a subgroup of $X_1 x\ X_2$.

**5.8 Theorem :** Let X be a group and $A_r^T$ a t-level subgroup of X . If A is a normal t-complex fuzzy soft subgroup, then $A_r^T$ is a normal soft subgroup of X.

**Proof:** By theorem-5.4, $A_r^T$ is a t-level subgroup of X. Now let us show that $A_r^T$ is normal. For all

$a \in X$ and $x \in A_r^T$ , $T(A(axa^{-1}),r) = T(A(aa^{-1}x),r) = T(A(x),r) \geq r$.

Thus $axa^{-1} \in A_r^T$ . Hence $A_r^T$ is a normal soft subgroup of X .

**5.9 Theorem:** Let A,B be complex fuzzy soft subsets of the sets $X_1$ and $X_2$ respectively, T be a

t-norm and $r \in [0,1]$. Then $A_r^T \times B_r^T = (AxB)_r^T$ .

**Proof:** Let (a,b) be an element of $A_r^T \times B_r^T$ .Then $a \in A_r^T$ and $b \in B_r^T$ . By definition-5.1, we can write $T(A(a),r) \geq r$ and $T(B(b),r) \geq r$. Using definition-5.6 ,we get

$T((A \times B)(a, b)\ , r) = T(T(A(a), B(b)),r) = T((A(a), T(B(b))),r) \geq T(A(a),r) \geq r$.

Thus we have $(a, b) \in (AxB)_r^T$. Now let $(a, b) \in (AxB)_r^T$ , this is required following statement

$T((A \times B)(a, b)\ , r) = T(T(A(a), B(b)),r) = T((A(a), T(B(b))),r) \geq r = T(1,r)$.

Thus the inequalities

$T(A(a),r) \geq r$ and $T(B(b),r) \geq r$ are satisfied. Hence (a,b) is in $A_r^T \times B_r^T$ .

This completes the proof.

**5.10 Theorem:** Let $A_1, A_2, \ldots, A_n$ be complex fuzzy soft subgroups under a minimum operation in groups $X_1, X_2, \ldots, X_n$ respectively, r ∈ [0,1]. Then

$(A_1 \times A_2 \times \ldots \times A_n)_r^T = A_{1r}^T \times A_{2r}^T \times \ldots \times A_{nr}^T$ .

**Proof:** Let $(a_1, a_2, \ldots, a_n)$ be an element of $(A_1 \times A_2 \times \ldots \times A_n)_r^T$ .

Using definition-5.6, we can write

$T(\min((A_1 \times A_2 \times \ldots \times A_n)(a_1, a_2, \ldots, a_n)), r) = T(\min(A_1(a_1), A_2(a_2), \ldots, A_n(a_n)), r)$ .

For all i=1,2,…,n, $\min(A_1(a_1), A_2(a_2), \ldots, A_n(a_n)) = A_i(a_i)$.

This gives us $T(\min(A_1(a_1), A_2(a_2), \ldots, A_i(a_i), \ldots A_n(a_n)), r) = T(A_i(a_i), r) \geq r$.

Thus we have $a_i \in A_{ir}^T$ . That is $(a_1, a_2, \ldots, a_n) \in A_{1r}^T \times A_{2r}^T \times \ldots \times A_{nr}^T$ .

Similarly, let $(a_1, a_2, \ldots, a_n)$ be an element of $A_{1r}^T \times A_{2r}^T \times \ldots \times A_{nr}^T$ . Then for all i=1,2,…,n ,

$a_i \in A_{ir}^T$ . That is, $T(A_i(a_i), r) \geq r$ .

Since $\min(A_1(a_1), A_2(a_2), \ldots, A_i(a_i), \ldots A_n(a_n)) = A_i(a_i)$ and $T(A_i(a_i), r) \geq r$, we get

$T((A_1 \times A_2 \times \ldots \times A_n)(a_1, a_2, \ldots, a_n)), r) = T(\min(A_1(a_1), A_2(a_2), \ldots, A_i(a_i), \ldots A_n(a_n)), r)$

$= T(A_i(a_i), r) \geq r$.

Thus, $(a_1, a_2, \ldots, a_n) \in (A_1 \times A_2 \times \ldots \times A_n)_r^T$ .

The proof is completed.

## 6. STEP COMPLEX FUZZY SOFT SUBGROUP

Let X be a group and T be a t-norm. We have the following.

**6.1 Definition:** Let 'A' be a complex fuzzy soft subgroup of X with respect to T. Then 'A' is called step complex fuzzy soft subgroup of X with respect to T if for every $\lambda \in [0,1]$ , $A_\lambda$ is a subgroup of X when $A_\lambda \neq \emptyset$.

**6.2 Theorem:** Let 'A' be a complex fuzzy soft subgroup of X with respect to T. Then A is a step complex fuzzy soft subgroup of X with respect to T if and only if A(xy) ≥ min{ A(x),A(y) }

for all x,y ∈ X.

**Proof:** Suppose A(xy) ≥ min{ A(x),A(y) } for all x,y ∈ X.

When $A_\lambda \neq \emptyset$, $\lambda \in [0,1]$. Let x,y ∈ $A_\lambda$. Then

$A(xy^{-1}) \geq \min\{ A(x), A(y^{-1}) \} = \min\{ A(x), A(y) \} \geq \lambda$.

Hence $xy^{-1} \in A_\lambda$ is a subgroup of X.

Conversely, suppose there are x,y ∈ X such that A(xy) ≥ min{ A(x),A(y) }.

Put $\lambda$= min{ A(x),A(y) }, then x,y ∈ $A_\lambda$.Because A(xy) ≤ min{ A(x),A(y) }= $\lambda$ .

∴ xy ∉ $A_\lambda$ . $A_\lambda$ is not a subgroup of X.

Consider the t-norm T, $T_0$(a,b) = min{a,b}, for all (a,b) ∈ [0,1]x[0,1]. We get the following corollary.

**6.3 Corollary**: Suppose A is a complex fuzzy soft subgroup of X with respect to T. Then

(i) A is a step complex fuzzy soft subgroup of X with respect to $T_0$.

(ii) Furthermore, A is a step complex fuzzy soft subgroup of X with respect to T, where T is a arbitrary t-norm.

**Proof:** (i) Because A(xy) ≥ $T_0$(A(x),A(y)) = min{ A(x),A(y) } for all x,y ∈ X, it follows from the above theorem that A is a step complex fuzzy soft subgroup of X with respect to $T_0$.

(ii) This conclusion follows from (i) and that fact $T_0 \geq$ T.

**6.4 Definition:** Let 'A' be a step complex fuzzy soft subgroup of X with respect to T. Suppose A(X) is the range of A. A(X) = { A(x) / x ∈ X }.

If the A(X) is finite; A(X) = { $a_0, a_1, \ldots, a_m$ }, with $1 \geq a_0 \geq a_1 \geq \ldots \geq a_m \geq 0$, the integer m is called the length of A.

If the A(X) is countable; A(X) = { $a_0, a_1, \ldots, a_m, \ldots$ }, with $1 \geq a_0 \geq a_1 \geq \ldots \geq a_m \geq \ldots \geq 0$, the length of A is defined by +∞.

**6.5 Theorem:** Let 'A' be a step complex fuzzy soft subgroup of X with respect to T. The length of A is $\ell$. A(X) = { $a_0, a_1, \ldots, \frac{a}{2}$ }, with $1 \geq a_0 \geq a_1 \geq \ldots \geq \frac{a}{2} \geq 0$. Suppose A= {$A_\lambda$ / $\lambda$ ∈ [0,1], $A_\lambda \neq \emptyset$ }. Then B = { $A_{C_1}, A_{C_2}, \ldots, A_{C_\ell}$ } and we can get a subgroup series without repetition in X: {e} ≤ $A_{a_0} \leq A_{a_1} \leq \ldots \leq A_{a_\ell}$ = X.

**Proof:** Let i be an integer.

Case-(i) : If $0 \leq i \leq \ell$ , then there exists $x_i$ ∈ X such that $A(x_i) = a_i$ , hence $A_{a_i} \neq \emptyset$. By definition 3.6,

$A_{a_i}$ (i=0,1,2,…, $\ell$) is a subgroup of X.

Case-(ii) : If $1 \leq i < i+1 \leq \ell$ , then clearly $A_{a_i} \subseteq A_{a_{i+1}}$. Because there is $x_{i+1}$ ∈ X such that

$A(x_{i+1}) = a_{i+1}$ and $a_{i+1} < a_i$ , then $x_{i+1} \in A_{a_{i+1}}, x_{i+1} \notin A_{a_i}$ .

Hence $A_{a_i} \neq A_{a_{i+1}}$ , $A_{a_i} < A_{a_{i+1}}$ .

Because $e = \min A(X)$ , then for all $x \in X$, $A(x) \geq a_\ell$ , thus $A_{i_\ell} \supseteq X$ . Hence $A_{a_\ell} = X$ .

Therefore, we get sequence $\{e\} \leq A_{a_0} \leq A_{a_1} \leq \ldots \leq A_{a_\ell} = X$ .

For all $A_\lambda \in B$, if $a_{i+1} \leq \lambda \leq a_i$ , $1 \leq i+1 \leq \ell$ , then

$A_\lambda = \{x / x \in X, A(x) \geq \lambda\} = \{x / x \in X, A(x) \geq a_i\} \cup \{x / x \in X, \lambda \leq A(x) \leq A_\lambda\} = A_{a_i} \cup \emptyset = A_{a_i}$ .

If $\lambda > a_0$ , then $A_\lambda = \emptyset$ , $A_\lambda \notin B$ .

If $\lambda < a_\ell$ , then $A_\lambda \supseteq A_{a_\ell} = X$ , hence $A_\lambda = A_{a_\ell}$ .

$\therefore B \subseteq \{A_{a_0}, A_{a_1}, A_{a_2}, \ldots, A_{a_\ell}\}$.

**Applications**

1.Many physical quantities are complex-valued wave function in quantum mechanics .

2.Complex amplitude and impedance in electrical engineering, etc.

3.In all these problems, expert uncertainty means that we do not know the exact value of the corresponding complex number instead, we have a fuzzy knowledge about this number.

**7. Summary and future work:** The work presented in this paper is the novel frame work of step complex fuzzy soft subgroup. The various properties and operations of complex fuzzy soft set are investigated. We also presented the properties of t-level complex fuzzy soft subgroup. A further study is required to implement these notions in real life applications such as Intuitionistic fuzzy set ,Bipolar fuzzy complex set, vague set and rough set etc.

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