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Dynamical system in semigroups

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Abstract

A dynamical system is an order pair (S, f) whereS is a semi group and f is a homomorphism fromS to S which is an endomorphism. In thisdynamical system (S, f) we introduced fixedpoints and periodic points and related resultsare obtained. Regular dynamical system andinversedynamical system is also defined andrelatedresults are obtained. In this paper it isimportant observe that regular element in asemi group is also a regular element in thedynamical system. Inverse dynamical system isintroduced and important results are obtained.

Keywords:

Discrete dynamical system;
Homomorphism
periodic point;
fixed point;
strongly invariant,
regular dynamical system,
inverse dynamical system

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Introduction

Discrete dynamical system is an exciting and very active field in pure and Applied Mathematics that involves tools and techniques from many areas such as analysis, geometry and number theory. Dynamics in semigroups is associated with the dynamical evaluation of quantum systems of the simplest kind that involves mathematical structure associated with infinitely many degrees of freedom. The dynamics of such a system is represented by a one parameter semi group of homomorphism from semi group to itself. If we introduce a natural casual structure into a dynamical system then a pair of one parameter semi groups of endomorphism emerge. It is useful to think of this pair as representing the past and future with respect to the given causality. In this paper mainly we have introduced dynamical system, inverse dynamical system and the related results are obtained. Mainly it is observed that an element which is regular in a semi group is also a regular element in the dynamical system (S,f).

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Preliminary definitions:

1 **Dynamical system:** Let S be a semi group and f:S→S is a homomorphism then (S,f) is called a dynamical system.
2 **Fixed point** : Let (S,f) be a dynamical system. A point a ε S is called a fixed point if f(a)=a.
3 **Periodic point** : Let (S,f) be a dynamical system. A point a ε S is called a Periodic point if there exists a positive integer n
such that $f^n(a)=a$ where $f^n(a)$=fofo…fof(a).
4 **Invariant set** : In the dynamical system (S,f) a subset A of S said to be invariant if f(A) ⊂ A. (A⊆S)
5 **Strongly invariant set:** In the dynamical system (S,f) a subset A of S said to be invariant if f(A)=A.
6 **Regular Dynamical system:** Let (S,f) be dynamical system if each aε(S,f) there exists xε(S,f) such that f(axa)=f(a) then (S,f) is called regular dynamical system.
7 **Inverse dynamical system** : Let (S,f) be dynamical system if each aε(S,f) There exists xε(S,f) such that f(axa)=f(a) and
f(xax)=f(x) is called regular dynamical system.i.e. to each aεS there exist xεS such that f(axa)=f(a) & f(xax) =f(x), then S is called Inverse dynamical system.

Dynamical system is a system which is autonomous.

Dynamical system in ordinary differential equations of order 1 is as follows.

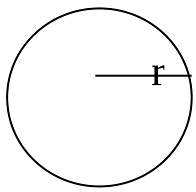
Result 1: A dynamical system is a system of ordinary differential equations.
Consider the simple dynamical system $x' = y$

$$y' = -x$$
$$\Rightarrow x'' = -x$$
$$\Rightarrow x'' + x = 0$$
$$\Rightarrow x(t) = A\cos t + B\sin t$$
$$y(t) = B\cos t - A\sin t$$

$x' = y \Rightarrow y(t) = B\cos t - A\sin t \quad y' = -x$

$$\Rightarrow x(t) = A\cos t + B\sin t$$

For the polar form Put $A = r\cos\theta, B = r\sin\theta$
$x(t) = A\cos t + B\sin t$
$= r\cos\theta\cos t + r\sin\theta\sin t$
$= r(\cos\theta\cos t + \sin\theta\sin t)$r



$$= r\cos(\theta - t)$$
$$y(t) = r\sin(\theta - t)$$

The graph of above solution is a circle with centre (0,0) radius r.

From the above it is observed that an ordinary differential equation which is of order 1 is a dynamical system which contains a single parameter t.

An algebraic system which contains a single binary operation under homomorphism is a dynamical system in semigroups. In this dynamical system the following results are obtained.

Result 2: In the dynamical system (S,f) if a and b are fixed elements then aob is also a fixed element.

Proof: Since a and b are fixed elements so that f(a)=a & f(b)=b ⇒f(aob)=f(a)o f(b) =aob. Hence aob is also a fixed element.

Remark : If $a_1,a_2...a_n$ are fixed elements of a dynamical system (S,f), $a_1 o a_2 o ... o\ a_n$ is also a fixed element.

Result 3:In the dynamical system (S,f), if a is a fixed element and b is a periodic point then aob is also a periodic point .

Proof: Since a is a fixed point f(a)=a. Let b be a periodic point with period k then $f^k(b)$ =b. Since $f^2(a)$=f0f(a)=f(f(a))=f(a)=a and also $f^k(a)$ =a. Hence by the homomorphism property of $f^{k,}f^k(a0b)= f^k(a)0\ f^k(b)$ =a0b which implies that aob is also a periodic point .

Result 4: If a is a regular element in a semigroup S then f(a) is a regular element in (S,f).
Proof: Since a is a regular element in a semigroup S⇒ to each aεS there exists xεs such that axa=a.Now, we have f(axa)= f(a) f(x) f(a) ⇒ f(a)= f(a) f(x) f(a).

Remark5:It is first observed that in the regular dynamical system idempotent elements are fixed elements.
From the following result it is observed that f(ax) and f(xa) are fixed elements in the dynamical system.

Result 6: In the dynamical system (S,f) , f(ax) and f(xa) are fixed elements.

From the following result it is observed that f(a) f(x) and

f(x) f(a) Idempotent elements in the dynamical system.
Result 7 : In the regular dynamical system (S,f); f(a) f(x) and f(x) f(a) are Idempotent elements .

Proof: We have $[f(a)\ f(x)]^2$ = [f(a) f(x)]. [f(a) f(x)]=[f(a) f(x) f(a)]. f(x) = f(axa). f(x) = f(a). f(x)

Similarly $[\ f(x)\ f(a)]^2$ = f(x). f(a).

In general f(a′) need not be equal to [f(a)]′.From the following result it is observed that [f(a)]′=f(a′) on an inverse dynamical system.

Result 8: In the inverse dynamical system (S,f), [f(a)]′=f(a′) where a′ is the inverse of a .

Result 9: In the inverse dynamical system (S,f), $f^n(a')= [f^n(a)]'$ for any aεS.
Proof: From the above result [f(a)]′=f(a′) by using mathematical induction $f^n(a')= [f^n(a)]'$.
The following is an example for a dynamical system.
Let S= {0,a,b,e,f} Define . on S by the composition table as follows:
Example1:

.	0	a	b	E	f
0	0	0	0	0	0
a	0	0	e	0	a
b	0	f	0	B	0
e	0	a	0	E	0
f	0	0	b	0	f

Clearly (S,.) is a semi group.

Define Φ: S→S by Φ(0)=0; Φ(a)=a; Φ(b)=b ; Φ(e)=e; Φ(f)=f.

Then (S, Φ) is a dynamical system. It is observed that 0, b, e, f are periodic points with period 1.

Example 2:

The following is an example for a dynamical system.

Let S= {e,a,x,y} Define . On S by the composition table as follows:

.	e	a	x	Y
e	e	a	x	Y
a	a	e	x	Y
x	x	y	x	Y
y	y	x	x	Y

Define Φ on S

$$e\Phi = \begin{pmatrix} e & a & x & y \\ e & a & x & y \end{pmatrix}$$

$$a\Phi = \begin{pmatrix} e & a & x & y \\ a & e & x & y \end{pmatrix}$$

$$x\Phi = \begin{pmatrix} e & a & x & y \\ x & x & x & x \end{pmatrix}$$

$$y\Phi = \begin{pmatrix} e & a & x & y \\ y & y & y & y \end{pmatrix}$$

Clearly (S, Φ) is a dynamical system and e, x, y are idempotent elements in semi group S.

From the above it is observed that a Dynamical system corresponding to first order ordinary differential equations are obtained and an algebraic structure under a homomorphism is obtained.

Conclusion: From the above it is observed that Dynamical system is obtained for first order ordinary differential equations. Dynamical system is also obtained for semigroups, a non empty set together with a binary operation form a semi group under the homomorphism is obtained. Dynamical system is also extendable for regular semi groups, idempotent semi groups and inverse semigroups.

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