
ON THE CONSTRUCTION OF CONFERENCE MATRICES OF ORDER 18 AND 26 FROM COHERENT CONFIGURATION

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Abstract

In this paper we forward methods of constructing conference matrices of order 18 and 26 by suitable combination of adjacency matrices of suitable coherent configuration.

Keywords:

Coherent Configuration;
Weighing matrix;
Conference matrix;
Symmetric
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1. Introduction:

We begin with the following definition:

1.1. WEIGHING MATRICES:

A weighing matrix M of order m and weight w is an $m \times m$ matrix with entries $(0, \pm 1)$ such that $MM^T = wI_m$, where M^T is transpose of M and I_m is identity matrix of order m . A weighing matrix of order m and weight w is denoted by $M(m, w)$. A $M(m, m)$ is a Hadamard matrix. A $M(m, m-1)$, m even with zeros on the diagonal such that $MM^T = (m-1)I_m$ is conference matrix. If $m \equiv 2 \pmod{4}$ such that $M = M^T$ is symmetric conference matrix. [6], [7] and [8].

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$$\text{Exaple: } M(6,5) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

1.2. PROPERTIES OF WEIGHING MATRICES: If M is a $M(m,w)$ then-

(i). The rows of M are pairwise orthogonal. Similarly, the columns are pairwise orthogonal.

(ii). Each row and each column of M has exactly w non-zero elements.

(iii). $MM^T = (m-1)I_m$, since the definition means that $M^{-1} = n^{-1}M^T$, where M^{-1} is the inverse of M .

(iv). If there is a $M(m,p)$ then there is asymmetric $M(m^2, p^2)$.

(v). For a weighing matrix $M(m,m-1)$, $MM^T = (m-1)I_m$ then $\det M = M(m) = (m-1)^{\frac{m}{2}}$. (Vide: [6],[7]).

1.3. CONFERENCE MATRICES:

A conference matrix of order m is an $m \times m$ matrix M with diagonal entries 0 and other entries ± 1 which satisfies $MM^T = (m-1)I_m$, where M^T is transpose of M and I_m is the identity matrix.

$$\text{Example: } M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

(Vide [6],[7])

1.4. SYMMETRIC CONFERENCE MATRIX:

A conference matrix of order m with entries 0,+1 and -1 is called symmetric conference matrix if $MM^T = M^T M = mI_m$ where M^T is transpose of M and I_m is the identity matrix.

$$\text{.Example: } M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

(Vide [2],[7])

1.5. PROPERTIES OF SYMMETRIC CONFERENCE MATRICES AND CONFERENCE MATRICES:

Some important properties of symmetric conference matrices and conference matrices are given below:

(i). The order of conference matrix is of form $4t + 2$.

(ii). $M-1$ where m is the order of a conference matrix must be the sum of two squares.

(iii). If there is a conference of order m then there is asymmetric conference matrix of order m with zero diagonal. The two forms are equivalent as one can be transformed into the other by interchanging rows (columns) or multiplying rows (columns) by -1.

(iv). A conference matrix is said to be normalized if it has first rows and columns all plus ones. [2],[7].

(v). $M^{-1} = mM^T$. (Vide[2])

1.6. SKEW – CONFERENCE MATRIX:

A conference matrix M with entries 0,+1 and -1 is called skew-symmetric matrix conference matrix if $M^T = -M$. where T denotes the transpose of matrix. Example:

$$M = \begin{bmatrix} 0 & 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & 0 & 1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 0 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 0 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 0 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 0 \end{bmatrix} \text{ (Vide [9])}$$

1.7. COHERENT CONFIGURATION:

Let X be a finite set. A coherent configuration on X is a set $C = \{C_1, \dots, C_m\}$ of binary relation on X (subset of $X \times X$) satisfying the following four conditions:

- (i) C is a prtition of $X \times X$ that is $\bigcup_{i=1}^m C_i = X \times X$;
- (ii) There exist a subset C_0 of C which is a partition of the diagonal $D = \{(x, x) : x \in X\}$
- (iii) For every relation $C_i = C_{i^*} \in C_k$
- (iv) There exist an integer P_{ij}^k for $1 \leq i, j, k \leq m$ such that for any $(\alpha, \beta) \in C_k$ the number of points $\gamma \in X$ such that $(\alpha, \gamma) \in C_i$ and $(\gamma, \beta) \in C_j$ is equal to P_{ij}^k (and in particular, is independent of the choice of $(\alpha, \beta) \in C_k$. That is we have $P_{ij}^k = |\{C_i(\alpha) \cap C_j(\beta)\}|$ for $(\alpha, \beta) \in C_k$

Coherent configuration is also defiend by adjacency matrices of classes of C. If $\{M_1, \dots, M_m\}$ are adjacency matrices of C_1, \dots, C_m respectively then the axioms takes the following from

- (i) $M_1 + \dots + M_m = J$
- (ii) There exist a sub set of $\{M_1, \dots, M_m\}$ with sum I=identity matrix;
- (iii) Each element of the set $\{M_1, \dots, M_m\}$ is closed under transposition;
- (iv) $M_i M_j = \sum_{k=1}^m P_{ij}^k M_k$ where P_{ij}^k are non-negative integers.

(Vide:Singh and Manjhi[12])

2. MAIN WORK:

In[7] and [8] methods of construction of weighing/conference matrices of order 6 and order 10 and 14 is given.

In the similar approach in this paper we forward methods of construction of conference matrices of order 18 and 26 by suitable linear combination of adjacency matrices of suitable coherent configurations:

2.1 .CONSTRUCTION OF SYMMETRIC CONFERENCE MATRIX OF ORDER 18:

Consider $X = \{i : i = 1, 2, 3, \dots, 18\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ of $X \times X$.

where:

$$C_1 = \{(i, i) : i = 1\},$$

$$C_2 = \{(1, i) : i = 2, 3, \dots, 18\},$$

$$C_3 = \{(i, 1) : i = 2, 3, \dots, 18\},$$

$$C_4 = \{(i, i) : i = 2, 3, \dots, 18\}$$

$$C_5 = \{(2, i) : i = 3, 4, 5, 6, 7, 8, 9, 10\} \cup \{(3, i) : i = 2, 5, 7, 8, 11, 12, 13, 15\} \cup \{(4, i) : i = 2, 6, 7, 10, 11, 12, 14, 16\} \\ \cup \{(5, i) : i = 2, 3, 9, 10, 11, 13, 14, 17\} \cup \{(6, i) : i = 2, 4, 8, 9, 12, 13, 14, 18\} \cup \{(7, i) : i = 2, 3, 4, 9, 11, 15, 16, 18\} \\ \cup \{(8, i) : i = 2, 3, 6, 10, 12, 15, 17, 18\} \cup \{(9, i) : i = 2, 5, 6, 7, 13, 16, 17, 18\} \cup \{(10, i) : i = 2, 4, 5, 8, 14, 15, 16, 17\} \\ \cup \{(11, i) : i = 3, 4, 5, 7, 12, 14, 17, 18\} \cup \{(12, i) : i = 3, 4, 6, 8, 11, 13, 16, 17\} \cup \{(13, i) : i = 3, 5, 6, 9, 12, 14, 15, 16\} \\ \cup \{(14, i) : i = 4, 5, 6, 10, 11, 13, 15, 18\} \cup \{(15, i) : i = 3, 7, 8, 10, 13, 14, 16, 18\} \cup \{(16, i) : i = 4, 7, 9, 10, 12, 13, 15, 17\} \\ \cup \{(17, i) : i = 5, 8, 9, 10, 11, 12, 16, 18\} \cup \{(18, i) : i = 6, 7, 8, 9, 11, 14, 15, 17\}.$$

$$C_6 = \{(2, i) : i = 11, 12, 13, 14, 15, 16, 17, 18\} \cup \{(3, i) : i = 4, 6, 9, 10, 14, 16, 17, 18\} \cup \{(4, i) : i = 3, 5, 8, 9, 13, 15, 17, 18\} \\ \cup \{(5, i) : i = 4, 6, 7, 8, 12, 15, 16, 18\} \cup \{(6, i) : i = 3, 5, 7, 10, 11, 15, 16, 17\} \cup \{(7, i) : i = 5, 6, 8, 10, 12, 13, 14, 17\} \\ \cup \{(8, i) : i = 4, 5, 7, 9, 11, 13, 14, 16\} \cup \{(9, i) : i = 3, 4, 8, 10, 11, 12, 14, 15\} \cup \{(10, i) : i = 3, 6, 7, 9, 11, 12, 13, 18\} \\ \cup \{(11, i) : i = 2, 6, 8, 9, 10, 13, 15, 16\} \cup \{(12, i) : i = 2, 5, 7, 9, 10, 14, 15, 18\} \cup \{(13, i) : i = 2, 4, 7, 8, 10, 11, 18\} \\ \cup \{(14, i) : i = 2, 3, 7, 8, 9, 12, 16, 17\} \cup \{(15, i) : i = 2, 4, 5, 6, 9, 11, 12, 17\} \cup \{(16, i) : i = 2, 3, 5, 6, 8, 11, 14, 18\} \\ \cup \{(17, i) : i = 2, 3, 4, 6, 7, 13, 14, 15\} \cup \{(18, i) : i = 2, 3, 4, 5, 10, 12, 13, 16\}.$$

Then adjacency matrices $M_1, M_2, M_3, M_4, M_5,$ and M_6 of $C_1, C_2, C_3, C_4, C_5,$ and C_6 respectively are given below:

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$M_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

We see that

1. $M_1 + M_4 = I_{18}$
2. $M_1 + M_2 + M_3 + M_4 + M_5 + M_6 = J_{18}$
3. $M_1' = M_1, M_2' = M_3, M_3' = M_2,$
 $M_4' = M_4, M_5' = M_5, M_6' = M_6$
4. We see the following calculations:

- (i) $M_1^2 = M_1, M_1M_2 = M_2, M_1M_3 = 0, M_1M_4 = 0, M_1M_5 = 0, M_1M_6 = 0$
- (ii) $M_2^2 = 0, M_2M_3 = 17M_1, M_2M_4 = M_2, M_2M_5 = 8M_2, M_2M_6 = 8M_2$
- (iii) $M_3^2 = 0, M_3M_4 = 0, M_3M_5 = 0, M_3M_6 = 0$
- (iv) $M_4^2 = M_4, M_4M_5 = M_5, M_4M_6 = M_6$
- (v) $M_5^2 = 8.M_1 + 3.M_5 + 4.M_6, M_5M_6 = 4.(M_5 + M_6)$
- (vi) $M_6^2 = 8.M_1 + 4.M_5 + 3.M_6$

Hence, product of any two adjacency matrices is some linear combinations of adjacency matrices.

Thus, the set $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ is a C.C.

Consider the matrix $M = 0.M_1 + 1.M_2 + 1.M_3 + 0.M_4 + 1.M_5 + (-1).M_6$

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 0 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 0 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 0 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 0 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 1 & -1 & 0 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & 0 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 0 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 0 & 1 & -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & 0 & -1 \\ 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 0 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 0 \end{bmatrix}$$

$$MM^T = \begin{bmatrix} 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 \end{bmatrix} = M^T M$$

$$= 17I_n = (18-1)I_n$$

$$\Rightarrow MM^T = M^T M = (18-1)I_n$$

Which show that M is a symmetric conference matrix of order 18.

2.2. CONSTRUCTION OF SYMETRIC CONFERENCE MATRIX OF ORDER 26:

Consider $X = \{i : i = 1, 2, 3, \dots, 26\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ of $X \times X$.

where:

$$\begin{aligned}
 C_1 &= \{(i, i) : i = 1\}, C_2 = \{(1, i) : i = 2, 3, 4, \dots, 26\}, C_3 = \{(i, 1) : i = 2, 3, 4, \dots, 26\}, C_4 = \{(i, i) : i = 2, 3, 4, \dots, 26\}. \\
 C_5 &= \{(2, i) : i = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\} \cup \{(3, i) : i = 2, 5, 6, 7, 11, 14, 17, 18, 22, 24, 25, 26\} \\
 &\cup \{(4, i) : i = 2, 6, 7, 8, 10, 13, 16, 17, 21, 23, 25, 2\} \cup \{(5, i) : i = 2, 3, 6, 7, 8, 12, 15, 16, 21, 22, 24, 26\} \\
 &\cup \{(6, i) : i = 2, 3, 4, 8, 11, 14, 15, 20, 21, 22, 23, 25\} \cup \{(7, i) : i = 2, 3, 4, 5, 10, 13, 19, 20, 22, 23, 24, 26\} \\
 &\cup \{(8, i) : i = 2, 4, 5, 6, 9, 12, 18, 19, 21, 23, 24, 25\} \cup \{(9, i) : i = 2, 5, 8, 11, 12, 13, 16, 17, 18, 20, 22, 23\} \\
 &\cup \{(10, i) : i = 2, 4, 6, 12, 13, 14, 15, 17, 18, 19, 21, 22\} \cup \{(11, i) : i = 2, 3, 6, 9, 13, 14, 16, 18, 19, 20, 21, 26\} \\
 &\cup \{(12, i) : i = 2, 5, 8, 9, 10, 14, 15, 17, 19, 20, 25, 26\} \cup \{(13, i) : i = 2, 4, 7, 9, 10, 11, 15, 16, 18, 20, 25, 26\} \\
 &\cup \{(14, i) : i = 2, 3, 6, 10, 11, 12, 15, 16, 17, 19, 23, 24\} \cup \{(15, i) : i = 5, 6, 10, 12, 13, 14, 16, 20, 21, 22, 24, 25\} \\
 &\cup \{(16, i) : i = 4, 5, 9, 11, 13, 14, 15, 17, 21, 23, 24, 26\} \cup \{(17, i) : i = 3, 4, 9, 10, 12, 14, 16, 18, 22, 23, 25, 26\} \\
 &\cup \{(18, i) : i = 3, 8, 9, 10, 11, 13, 14, 19, 21, 22, 24, 25\} \cup \{(19, i) : i = 7, 8, 10, 11, 12, 14, 18, 20, 21, 23, 24, 26\} \\
 &\cup \{(20, i) : i = 6, 7, 9, 11, 12, 13, 15, 19, 22, 23, 25, 26\} \cup \{(21, i) : i = 4, 5, 6, 8, 10, 11, 15, 16, 18, 19, 22, 26\} \\
 &\cup \{(22, i) : i = 3, 5, 6, 7, 9, 10, 15, 17, 18, 20, 21, 23\} \cup \{(23, i) : i = 4, 6, 7, 8, 9, 14, 16, 17, 19, 20, 22, 25, 26\} \\
 &\cup \{(24, i) : i = 3, 5, 7, 8, 13, 14, 15, 16, 18, 19, 25\} \cup \{(25, i) : i = 3, 4, 6, 8, 12, 13, 15, 17, 18, 20, 23, 24, 26\} \\
 &\cup \{(26, i) : i = 3, 4, 5, 7, 11, 12, 16, 17, 19, 20, 21, 23, 25\}.
 \end{aligned}$$

$$\begin{aligned}
 C_6 &= \{(2, i) : i = 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\} \cup \{(3, i) : i = 4, 8, 9, 10, 12, 13, 15, 16, 19, 20, 21, 23\} \\
 &\cup \{(4, i) : i = 3, 5, 9, 11, 12, 14, 15, 18, 19, 20, 22, 24\} \cup \{(5, i) : i = 4, 6, 10, 11, 12, 14, 17, 18, 19, 20, 23, 25\} \\
 &\cup \{(6, i) : i = 5, 7, 9, 10, 12, 13, 16, 17, 18, 19, 24, 26\} \cup \{(7, i) : i = 6, 8, 9, 11, 12, 14, 15, 16, 17, 18, 21, 25\} \\
 &\cup \{(8, i) : i = 3, 7, 10, 11, 13, 14, 15, 16, 17, 20, 22, 26\} \cup \{(9, i) : i = 3, 4, 6, 7, 10, 14, 15, 19, 21, 24, 25, 26\} \\
 &\cup \{(10, i) : i = 3, 5, 6, 8, 9, 11, 16, 20, 23, 24, 25, 26\} \cup \{(11, i) : i = 4, 5, 7, 8, 10, 12, 15, 19, 22, 23, 24, 25\} \\
 &\cup \{(12, i) : i = 3, 4, 6, 7, 11, 13, 16, 18, 21, 22, 23, 24\} \cup \{(13, i) : i = 3, 5, 6, 7, 12, 14, 17, 19, 21, 22, 23, 26\} \\
 &\cup \{(14, i) : i = 4, 5, 7, 8, 9, 13, 18, 20, 21, 22, 25, 26\} \cup \{(15, i) : i = 2, 3, 4, 7, 8, 9, 11, 17, 18, 19, 23, 26\} \\
 &\cup \{(16, i) : i = 2, 3, 6, 7, 8, 10, 12, 18, 19, 20, 22, 25\} \cup \{(17, i) : i = 2, 5, 6, 7, 8, 11, 13, 15, 19, 20, 21, 24\} \\
 &\cup \{(18, i) : i = 2, 4, 5, 6, 7, 12, 14, 15, 16, 20, 23, 26\} \cup \{(19, i) : i = 2, 3, 4, 5, 6, 9, 13, 15, 16, 17, 22, 25\} \\
 &\cup \{(20, i) : i = 2, 3, 4, 5, 8, 10, 14, 16, 17, 18, 21, 24\} \cup \{(21, i) : i = 2, 3, 7, 9, 12, 13, 14, 17, 20, 23, 24, 25\} \\
 &\cup \{(22, i) : i = 2, 4, 8, 11, 12, 13, 14, 16, 19, 24, 25, 26\} \cup \{(23, i) : i = 2, 3, 5, 10, 11, 12, 13, 15, 18, 21, 24\} \\
 &\cup \{(24, i) : i = 2, 4, 6, 9, 10, 11, 12, 17, 20, 21, 23, 23, 26\} \cup \{(25, i) : i = 2, 5, 7, 9, 10, 11, 14, 16, 19, 21, 22\} \\
 &\cup \{(26, i) : i = 2, 6, 8, 9, 10, 13, 14, 15, 18, 22, 24\}.
 \end{aligned}$$

Then adjacency matrices M_1, M_2, M_3, M_4, M_5 , and, M_6 of C_1, C_2, C_3, C_4, C_5 , and, C_6 respectively are given below:

