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SOFT SUBSTRUCTURES OF SENSIBLE FUZZY SOFT RIGHT R-SUBGROUPS OF NEAR- RINGS

M.Subha[1],G. Subbiah[2*] and M.Navaneethakrishnan [3]

1.Assistant Professor in Mathematics, Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India
*2 *Associate Professor in Mathematics, Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India.*
3 Associate Professor in Mathematics, Kamaraj College, Thoothukudi-628 003, Tamilnadu, India.

Abstract: In this paper , we introduce the notion of S- anti-fuzzy soft right R-subgroups of near-rings and its basic properties are investigated. We also study the homomorphic image and pre image of S- anti-fuzzy soft right R- subgroups. Using S-norm, we introduce the notion on sensible anti-fuzzy soft right R-subgroups in near- rings and some related properties of a near-rings 'R' are discussed.

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Section-1: Introduction. The concept of fuzzy subset was introduced by Zadeh [29]. Fuzzy set theory is a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situation by attributing a degree to which a certain object belong to a set. Schweizar and Sklar [26] introduce the notions of Triangular norm (t-norm) and Triangular co-norm (S-norm) are the most general families of binary operations that satisfy the requirement of the conjunction and disjunction operators respectively. First, Abuosman[6] introduced the notion of fuzzy subgroup with respect to t-norm. Zaid [1] introduced the concept of R-subgroups of a near-rings and Hokim [17] introduced the concept of fuzzy R- subgroups of a near-ring. Then Zhan [30] introduced the properties of fuzzy hyper ideals in hyper near-rings with t-norm. Recently, Cho et. al [11] introduced the notion of fuzzy subalgebras with respect to S-norm of BCK algebras and Akram [7] introduced the notion of sensible fuzzy ideal of [2] and [7]. Soft set theory was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagun and Sezgin [5] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji et. al [20] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et.al [4] introduced several operations of soft sets and Sezgin and Atagun [25] studied on soft set operations as well. Furthermore, soft set relations and functions [8] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29]. In this paper, we will redefine anti-fuzzy soft right R- subgroups of a near-ring 'R' with respect to a S-norm and investigate it is related properties. We also study the homomorphic image and pre image of S- anti-fuzzy soft right R-subgroups. Using S-norm, we introduce the notion on sensible anti-fuzzy soft right R-subgroups in near-rings and some related properties of a near-ring 'R' are discussed.

2. Preliminaries

A ring 'S' is a system consisting of a non-empty set 'S' together with two binary operations on 'S' called addition and multiplication such that

(i)'S' together with addition is a semi group.

(ii) 'S' together with multiplication is a semi group.

(iii) a (b+c) = ab + ac and (a+b)c = ac+bc for all a,b,c ∈ S. A semi ring 'S' is said to be additively commuatative if a+b = b+a for all a,b ∈ S. A zero element of a semi ring 'S' is an element 'o' such that o.x = x.o = o and o+x = x+o = x for all x∈ S. By a near-ring we mean a non-empty set 'R' with two binary operations '+' and ' · '

Satisfies the following axioms

(i) (R,+) is a group.
(ii) (R, ·) is a semi group.
(iii) (b+c)a =ba+ca for all a,b,c ∈ R.

Precisely speaking it is a right near-ring because it satisfies the right distribution law x·y. Note that xo= o and x(-y) = -(xy) but in general ox ≠ o for some x∈R. A two sided R- subgroups in a near- ring 'R' is a subset 'N'of 'R' such that

(i) (N, +) is a subgroup of (R,+).
(ii) RN ⊂ N
(iii) NR ⊂ N

If 'N'satisfies (i) and (ii) then it is called a right 'R' subgroup of 'R'. we now review some fuzzy logic concepts.A fuzzy set 'μ'in a set 'R'is a function μ: R→ [0,1]. Let Im(μ) denote the image set of μ. Let 'μ' be a fuzzy set in 'R'. For t∈ [0,1], the set L(μ: α) = { x ∈ R / μ(x) ≤ α } is called a lower level subset of 'μ'.

Let 'R' be a near-ring and let 'μ' be a fuzzy set in 'R'. we say that 'μ' is a fuzzy near-ring of 'R' if , for all x,y ∈ R,

(FS1) μ (x-y) ≥ min { μ(x) , μ(y) }

(FS2) μ (xy) ≥ min { μ(x) , μ(y) }. If a fuzzy set ' μ' in a near-ring 'R' satisfies the property (FS1) then μ(0) ≥ μ(x) for all x ∈ R.

2.1 Definition[22]: A pair (F,A) is called a soft set over U, where F is a mapping given by F : A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively. For more details, we refer to [11,17,18,26,29,7].

2.2 Definition[6] :The relative complement of the soft set F_A over U is denoted by F^r_A, where $F^r_A : A \to P(U)$ is a mapping given as $F^r_A(a)$ =U \$F_A(a)$, for all a ∈ A.

2.3 Definition[6]: Let F_A and G_B be two soft sets over U such that A∩B ≠ Φ. The restricted intersection of F_A and G_B is denoted by $F_A \Cap G_B$ and is defined as $F_A \Cap G_B$ =(H,C), where C = A∩B and for all c ∈ C, H(c) = F(c)∩G(c).

2.4 Definition[6]: Let F_A and G_B be two soft sets over U such that A∩B ≠ Φ . The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$ and is defined as $F_A \cup_R G_B$ = (H,C),where C = A∩B and for all c ∈ C, H(c) = F(c)∪G(c).

2.5 Definition[12]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set ψ (F_A) over U, where ψ (F_A) : B→P(U) is a set valued function defined by ψ (F_A)(b) =$\bigcup${F(a) | a ∈ A and ψ (a) = b},

If ψ^{-1}(b) ≠ Φ, = 0 otherwise for all b ∈ B. Here, ψ (F_A) is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B)$: A → P(U) is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all a ∈ A. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

2.6 Definition[13]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A)$: B→P(U) is a set-valued function defined by $\psi^\star(F_A)(b)=\bigcap${F(a) | a ∈ A and ψ (a) = b}, if ψ^{-1}(b) ≠ Φ,

=0 otherwise for all b ∈ B. Here, $\psi^\star(F_A)$ is called the soft anti image of F_A under ψ.

2.7 Definition [8]: Let f_A be a soft set over U and α be a subset of U. Then, upper α-inclusion of a soft set f_A, denoted by $f^{\alpha}A$, is defined as $f^{\alpha}A = \{x \in A : f_A(x) \supseteq \alpha\}$

2.8 Definition : By a s- norm 'S' , we mean a function S: [0,1]→ [0,1] satisfying the following conditions ;

(S1) S(x ,0) = x

(S2) S(x,y) ≤ S(x,z) if y ≤ z

(S3) S(x,y) = S(y,x)

(S4) S(x, S(y,z)) = S(S(x,y),z), for all x,y,z ∈ [0,1].

Replacing 0 by 1 in condition 'S1' we obtain the concept of t- norm 'T'.

2.9 Proposition : For a S-norm , then the following statement holds S(x,y) ≥ max{x,y}, for all x,y ∈ [0,1].

2.10 Definition : Let 'S' be a s-norm. A fuzzy soft set 'μ' in 'R' is said to be sensible with respect to 'S' if Im(μ) ⊂ Δs, where Δs = { s(α, α) = α / α ∈ [0,1] }.

2.11 Definition : Let (R,+, ·) be a near-ring. A fuzzy soft set 'μ' in 'R' is called an anti fuzzy right (resp. left) R- subgroup of 'R' if

(AF1) μ(x-y) ≤ max { μ(x) , μ(y) }, for all x,y ∈ R.

(AF2) μ(xr) ≤ μ(x) for all r,x ∈ R.

2.12 Definition : Let (R,+, ·) be a near-ring. A fuzzy soft set 'μ' in R is called a fuzzy soft right (resp. left) R-subgroup of 'R' if

(FR1) 'μ' is a fuzzy subgroup of (R,+).

(FR2) μ(xr) ≥ μ(x) (resp. μ(rx) ≥ μ(x)) , for all r,x ∈ R.

2.13 Definiton : Let 'S' be a s- norm. A function μ : R→ [0,1] is called a fuzzy soft right (resp. left) R-subgroup of 'R' with respect to 'S' if

(C1) μ(x-y) ≤ S(μ(x), μ(y) }

(C2) μ(xr) ≤ μ(x) (resp. μ(rx) ≤ μ(x) for all r,x ∈ R. If a fuzzy soft R-subgroup 'μ' of R with respect to 'S' is sensible , we say that 'μ' is a sensible fuzzy soft R- subgroup of R with respect to 'S'.

2.14 Example : Let 'K' be the set of natural numbers including '0' and 'K' is a R-subgroup with usual addition and multiplication.

2.15 Proposition : Define a fuzzy subset $\mu: R \rightarrow [0,1\}$ by

$$\mu(x) = \begin{cases} 0 & \text{if x is even 0} \\ 1 & \text{otherwise.} \end{cases}$$

And let $S_m : [0,1] \rightarrow [0,1]$ by a function defined by $S_m(\alpha,\beta) = \min\{x+y , 1\}$ for all $x,y \in [0,1]$. Then S_m is a t-norm, By routine calculation, we know that ‘μ’ is sensible R-fuzzy soft subgroup of R.

SECTION-3: PROPERTIES OF ANTI-FUZZY SOFT R SUBGROUPS.

3.1 Proposition: Let ‘S’ be a s-norm. Then every sensible S-anti fuzzy soft right R- subgroups ‘μ’ of R is an anti- fuzzy soft R- subgroups of R.

Proof: Assume that ‘μ’ is a sensible S- anti fuzzy soft right R-subgroups of R, then we have (AF1) $\mu(x-y) \leq S(\mu(x), \mu(y))$ and (AF2) $\mu(xr) \leq \mu(x)$ for all $x,y \in S$.

Since ‘μ’ is sensible, we have

$$\begin{aligned} \text{Max}\{\mu(x), \mu(y)\} &= S(\min\{\mu(x), \mu(y)\}, \min\{\mu(x), \mu(y)\}) \\ &\geq S(\mu(x), \mu(y)) \\ &\geq \max\{\mu(x), \mu(y)\} \end{aligned}$$

and so $S(\mu(x), \mu(y)) = \max\{\mu(x), \mu(y)\}$. It follows that

$$\mu(x-y) \leq S(\mu(x), \mu(y)) = \max\{\mu(x), \mu(y)\} \text{ for all x,y in R.}$$

clearly $\mu(xr) \leq \mu(x)$ for all r,x in R. so ‘μ’ is an anti-fuzzy soft R- subgroups of R.

3.2 Proposition : If ‘μ’ is a S- anti fuzzy soft right R-subgroups of a near ring R and ‘θ’ is an endomorphism of R, then μ[θ] is a S- anti fuzzy soft right R- subgroups of R.

Proof: For any $x,y \in R$, we have

(i) $$\begin{aligned} \mu_{[\theta]}(x-y) &= \mu(\theta(x-y)) \\ &= \mu(\theta(x) - \theta(y)) \\ &\leq S(\mu_{[\theta]}(x), \mu_{[\theta]}(y)) \end{aligned}$$

(ii) $$\begin{aligned} \mu_{[\theta]}(xr) &= \mu(\theta(xr)) \\ &= \mu(\theta(x)r) \\ &\leq \mu(\theta(x)) \\ &\leq \mu_{[\theta]}(x) \end{aligned}$$

. Hence μ[θ] is a S-anti fuzzy soft right R-subgroups of R.

3.3 Definition : Let ‘f’ be a mapping defined on R. If ‘ψ’ is a fuzzy soft subset in f(R) , then the fuzzy soft subset $\mu = \psi$ in R (ie) $\mu(x) = \psi(f(x))$ for all x in R is called the pre -image of ‘ψ’ under ‘f’.

3.4 Proposition : An onto homomorphic pre image of a S- anti fuzzy soft right R- subgroups of a near- ring is S-anti fuzzy soft right R- subgroups of R.

Proof: Let $f: R \rightarrow R^1$ be an onto homomorphism of near- ring and let ‘ψ’ be an S- anti fuzzy soft right R-subgroups of R and ‘μ’ the pre image of ‘ψ’ under ‘f’. Then we have

(i) $\mu(x\text{-}y) = \psi(f(x\text{-}y))$

$= \psi(f(x)\text{-}f(y))$

$\leq S(\psi(f(x), \psi(f(y))$

$= S(\mu(x), \mu(y))$

(ii) $\mu(xr) = \psi(f(xr))$

$= \psi(f(x)\, r)$

$\leq \psi(f(x))$

$= \mu(x)$. Hence 'μ' is a S- anti fuzzy soft right

R-subgroups of R.

3.5 Proposition : An onto homomorphic image of a anti fuzzy soft right R- subgroups with the inf property is a anti-fuzzy soft right R- subgroups of R .

Proof: Let $f{:}R \rightarrow R^1$ be an onto homomorphism of near-ring and let 'μ' be an S-anti fuzzy soft right R-subgroup of R with inf property. Given x, y ∊ R, we let $x_o \in f^{-1}(x^1)$ and yo $\in f^{-1}(y^1)$ be such that $\mu(xo) = \inf \mu(h)$, $\mu(yo) = \inf \mu(h)$

$h \in f^{-1}(x^1) \qquad h \in f^{-1}(y^1)$

respectively. Then we can deduce that

$\mu^f(x^1\text{-}y^1) = \inf\limits_{z \in f^{-1}(x^1\text{-}y^1)} \mu(z)$

$\leq \max \{ \mu(x_o) , \mu(y_o) \}$

$= \max \{ \inf\limits_{h \in f^{-1}(x^1)} \mu(h) , \inf\limits_{h \in f^{-1}(y^1)} \mu(h) \}$

$= \max \{ \mu^f(x^1) , \mu^f(y^1) \}$

$\mu^f(xr) = \inf\limits_{z \in f^{-1}(x^1 r^1)} \mu(z) \leq \mu(yo)$

$= \inf\limits_{h \in f^{-1}(y^1)} \mu(h) = \mu^f(y^1)$

Hence μ^f is anti fuzzy soft right R- subgroups of R.

The above proposition can be further strengthened, we first give the following definition.

3.6 Definition : A s- norm S on [0,1] is called a continous function from $[0,1] \times [0,1] \rightarrow [0,1]$with respect to the usual topology. We observe that the function 'max' is always a continous S- norm .

3.7 Proposition : Let $f{:} R \rightarrow R^1$ be a homomorphism of near-rings. If 'μ' is a S- anti fuzzy soft right R-subgroups of R^1, then μ^f is S- anti fuzzy soft right R- subgroup of R.

Proof: suppose 'μ' is a S- anti fuzzy soft right R- subgroups of R^1, then

(i) for all $x,y \in R$, we have

$\mu^f(x-y) = \mu f(x-y)$

$\leq S(\mu f(x), \mu f(y))$

$\leq S(\mu^f(x), \mu^f(y))$

(ii) for all $x,y \in R$, we have

$\mu^f(xr) = \mu f(xr)$

$= \mu(f(x), r)$

$\leq \mu(f(x))$

$\leq \mu^f(x)$

Hence μ^f is a S- anti fuzzy soft right R- subgroup of R.

3.8 Proposition : Let $f : R \rightarrow R^1$ be a homomorphism of near-rings. If 'μ^f' is a S- anti fuzzy soft right R- subgroups of R, then μ is S- anti fuzzy soft right R- subgroup of R^1.

Proof : Let x^1,y^1 in R^1.There exist $x,y \in R$, such that $f(x)=x^1$ and $f(y)=y^1$.

We have (i) $\mu(x^1-y^1)=\mu(f(x)-f(y))$

$=\mu(f(x-y))$

$= \mu^f(x-y)$

$\leq S(\mu f(x), \mu f(y))$

$= S(\mu(f(x), \mu f(y))$

$= S(\mu(x^1), \mu(y^1))$

(ii) Let $x^1, r^1 \in R^1$. There exist $x,r \in R$, such that $f(x) = x^1$, $f(y) = r^1$.

we have $\mu(x^1r^1) = \mu(f(x), f(y)) = \mu(f(xr)) \leq \mu^f(x) \leq \mu(f(x)) \leq \mu(x^1)$.

Hence μ is S- anti fuzzy soft right R- subgroup of R^1.

3.9 Proposition : Let 'S' be a continuous S- norm and let 'f' be a homomorphism on a near-ring R. If 'μ' is a S- anti fuzzy soft right R- subgroup of R, then μf is a S- anti fuzzy soft right R- subgroups of f(R).

Proof: Let $A_1 = f^{-1}(y_1)$, $A_2 = f^{-1}(y_2)$ and $A_{12} = f^{-1}(y_1-y_2)$, where $y_1-y_2 \in f(R)$. Consider the set

$A_1-A_2 = \{ x \in R / x = a_1-a_2$ for some $a_1 \in A_1, a_2 \in A_2\}$. If $x \in A_1-A_2$, then $x = x_1-x_2$ for some $x_1 \in A_1$ and $x_2 \in A_2$. so that we have $f(x) = f(x_1-x_2) = f(x_1)-f(x_2) = y_1-y_2$. (ie) $x \in f^{-1}(y_1-y_2) = A_{12}$. We have $A_1-A_2 \subset A_{12}$.

It follows that

$\mu f(y_1-y_2) = \inf \{ \mu(x) / x \in f^{-1}(x_1-x_2) \}$

$= \inf \{ \mu(x) / x \in A_{12}\}$

$\leq \inf \{ \mu(x) / x \in A_1-A_2\}$

$\leq \inf \{ \mu(x_1-x_2) / x_1 \in A_1, x_2 \in A_2\}$

$\leq \inf \{ S(\mu(x_1), \mu(x_2)) / x_1 \in A_1, x_2 \in A_2\}$

Since 'S' is continous for every $\varepsilon > 0$, we see that if inf ($\mu(x_1)/ x_1 \in A_1$)-$x_1^* \leq \delta$ and

inf ($\mu(x_2)/ x_2 \in A_2$)- $x_2^* \leq \delta$, then $S(\inf\{\mu(x_1)/ x_1 \in A_1\}, \inf\{ \mu(x_2)/ x_2 \in A_2\})-S(x_1^*,x_2^*) \leq \varepsilon$. Choose $a_1 \in A_1$,and $a_2 \in A_2$ such that

$\inf\{\mu(x_1)/ x_1 \in A_1 \} - \mu(a_1) \leq \delta$ and

$\inf \{\mu(x_2)/ x_2 \in A_2\} - \mu(a_2) \leq \delta$ then

$S(\inf \{\mu(x_1)/x_1 \in A_1\} , \inf \{ \mu(x_2)/x_2 \in A_2\}) - S(\mu(a_1), \mu(a_2)) \leq \varepsilon$.

Thus we have

(i) $\mu^f(y_1-y_2) \leq \inf \{ S(\mu(x_1), \mu(x_2))/ x_1 \in A_1,\ x_2 \in A_2\}$
$= S(\inf \{ \mu(x_1)/x_1 \in A_1\} , \inf \{ \mu(x_2)/ x_2 \in A_2\})$

$= S(\mu^f(y_1), \mu^f(y_2))$.

(ii) similarly , we can prove that
$\mu^f(xr) \leq \mu^f(x)$. Hence μ^f is a S- anti fuzzy right R- subgroups of f(R) .

3.10 Lemma:Let 'T' be a t-norm. Then t- co-norm 'S' can be defined as $S(x,y) = 1- T(1-x, 1-y)$.

Proof: straight forward.

3.11 Proposition : If a fuzzy soft subset 'μ' of R is a T- anti fuzzy soft right R- subgroup of R ,then 'μ^c' is 'S' anti fuzzy soft right R- subgroup of R.

Proof: Let 'μ' be a 'T' –anti fuzzy soft right R- subgroup of R. for all x,y ∈ R, we have

(i) $\mu^c(x-y) = 1- \mu(x-y)$

$\leq 1- T(\mu(x) , \mu(y))$

$= 1- T(1- \mu^c(x) , 1- \mu^c(y))$

$= S(\mu^c(x) , \mu^c(y))$

(ii) $\mu^c(xr) = 1- \mu(xr)$

$\leq 1- \mu(x) = \mu^c(x)$

Hence μ^c is 'S' anti fuzzy soft right R- subgroup of R.

SECTION-4 SOFT STRUCTURES OF ANTI FUZZY RIGHT R- SUBGROUPS.

4.1 Definition : A fuzzy soft relation on any set 'X' is a fuzzy soft set $\mu: X\times X \rightarrow [0,1]$.

4.2 Definition : Let 'S' be a s- norm. If 'μ' is a fuzzy soft relation on a set 'R' and 'χ' be fuzzy soft set in R, Then 'μ' is a S- fuzzy soft relation on 'χ' if $\mu_\chi(x,y) \geq S(\chi(x), \chi(y))$ for all x,y in R.

4.3 Definition: Let 'S' be a s- norm. let 'μ' and 'χ' be a fuzzy soft subset of R. Then direct S- product of μ and χ is defined as

$(\mu\times \chi) (x,y) = S(\mu(x) , \chi(y))$, for all x,y ∈ R.

4.4 Lemma : Let 'S' be a s- norm. let 'μ' and 'χ' be a fuzzy soft set of R, then

(i) $\mu\times\chi$ is a S-fuzzy soft relation on S.

(ii) $L(\mu\times\chi; t) = L(\mu; t) \times L(\chi; t)$ for all $t \in [0,1]$.

Proof: obivious.

4.5 Definition: Let 'S' be a s- norm . let 'μ' be a fuzzy soft subset of R , then μ is called strongest S- fuzzy soft relation on R if

$$\mu_\chi(x,y) \geq S(\chi(x) , \chi(y)) \text{ for all x,y in R.}$$

4.6 Proposition : Let 'S' be a s- norm and let 'μ'and 'χ' be a S- anti fuzzy soft right R- subgroup of R.Then $\mu\times\chi$ is also anti fuzzy soft right R- subgroup of R.

Proof:

(i) $(\mu\times\chi)(x-y) = (\mu\times\chi)((x_1,x_2) - (y_1,y_2))$

$= (\mu\times\chi)((x_1-y_1) , (x_2-y_2))$

$= S(\mu(x_1-y_1) , \chi(x_2-y_2))$

$\leq S (S(\mu(x_1), \mu(y_1) , S(\chi(x_2) , \chi(y_2))$

$=S (S(\mu(x_1) , \chi(x_2)) , S(\mu(y_1) , \chi(y_2))$

$= S ((\mu\times\chi)(x_1,x_2) , (\mu\times\chi)(y_1,y_2))$

$= S ((\mu\times\chi)(x) , (\mu\times\chi)(y))$

(ii) $(\mu\times\chi) (xr) = (\mu\times\chi) ((x_1,x_2)(r_1,r_2))$

$= (\mu\times\chi) (x_1r_1, x_2r_2)$

$= S(\mu(x_1) , \chi(x_2))$

$= (\mu\times\chi)(x_1,x_2)$

$= (\mu\times\chi) (x).$

Hence $\mu\times\chi$ is also anti fuzzy soft right R- subgroup of R.

4.7 Proposition : Let ' μ' and 'χ' be sensible S- anti fuzzy soft right R- subgroup of a near- ring R. Then $\mu\times\chi$ is a sensible S- anti fuzzy soft right R- subgroup of R×R.

Proof: By proposition 4.6, we have $\mu\times\chi$ is S- anti fuzzy soft right R- subgroup of R.

let $x = (x_1,x_2)$ be any element of S×S, then

$S((\mu \times \chi)(x) , (\mu \times \chi)(x)) = S((\mu \times \chi)(x_1,x_2) , (\mu \times \chi)(y_1,y_2))$

$= S(S(\mu(x_1) , \chi(x_2)) , S(\mu(x_1) , \chi(x_2))$

$= S(S(\mu(x_1), \mu(x_1)), S(\chi(x_2) , \chi(x_2)))$

$= S(\mu(x_1) , \chi(x_2))$

$= (\mu \times \chi) (x_1,x_2) = (\mu\times \chi)(x).$

Hence $\mu \times \chi$ is a sensible S- anti fuzzy soft right R- subgroup of R×R

4.8 Remark : If $\mu \times \chi$ is a sensible S- anti fuzzy soft right R- subgroup of R×R , Then $\mu \times \chi$ need not be sensible S- anti fuzzy soft right R- subgroup of R.

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