International Journal of Engineering, Science and Mathematics
Vol. 6 Issue 4, August 2017,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

EFFECTS OF THERMOPHORESIS AND VARIABLE SUCTION ON MIXED CONVECTIVE HEAT AND MASS TRANSFER FLOW PAST A VERTICAL PERMEABLE PLATE IN THE PRESENCE OF THERMAL RADIATION AND UNIFORM MAGNETIC FIELD

Dr.Prabal Pratap Singh
Department of Mathematics,C.L.Jain (P.G.) College,Firozabad-283203,India
Dr.Seema Raghav
Department of Mathematics,V.S.S.D. College,Kanpur-208002,India

Abstract

The present paper deals with the study of two-dimensional steady MHD mixed convection and mass transfer flow of an incompressible, viscous, electrically conducting fluid past an impulsively started infinite vertical porous plate in the presence of thermal radiation, large variable suction and thermophoresis. Usual similarity transformations are introduced to make non-dimensional form of the equations of momentum, energy and concentration. To obtain similarity solution of the problem, the similarity equations are solved using regular perturbation technique. Numerical results for the dimensionless velocity, temperature and concentration fields as well as for the skin-friction coefficient, wall heat transfer and particle deposition rate are obtained and displayed graphically for pertinent parameters to show physical insight and interesting aspects of the problem. Finally, a through discussion of different results is presented.

Keywords: Variable Suction, Mixed Convection, Thermophoresis, Thermal radiation

Nomenclature

B_0	uniform magnetic field
C	species concentration
C_p	specific heat at constant pressure
C_∞	species concentration of the ambient fluid
D	chemical molecular diffusivity
f	dimensionless stream function
f_w	dimensionless wall suction / injection
g	acceleration due to gravity
k	thermophoretic coefficient
k_1	mean absorption coefficient
K_n	Knudsen number
M	magnetic parameter
N	radiation parameter
Pr	Prandtl number
q_r	radiative heat flux
Re_x	local Reynolds number

Sc	Schmidt number
T	temperature of the fluid in the boundary layer
T_w	temperature at the surface
T_∞	temperature of the ambient fluid
T_p	thermophoretic parameter
U_0	uniform velocity of the plate
u, v	velocity component in *x*-and *y*-directions
$v_w(x)$	permeability of the porous surface
V_T	thermophoretic velocity

Greek Symbols

β	volumetric coefficient of thermal expansion
γ	local buoyancy parameter
ρ	density of the fluid
σ	electrical conductivity
θ	dimensionless temperature
ϕ	dimensionless concentration
σ_1	Stefan-Boltzmann constant
ψ	stream function
η	similarity variable
ϑ	kinematic viscosity
μ	viscosity of the fluid
λ_g	thermal conductivity of the fluid
λ_p	thermal conductivity of the diffused particles
τ	skin-friction

1. Introduction

In the context of space technology and in processes involving high temperatures, the effects of radiation are of vital importance. Recent developments in hypersonic flights, missile reentry, rocket combustion chambers , power plants for interplanetary flight and gas cooled nuclear reactors have focused attention on thermal radiation as a mode of energy transfer and emphasize the need for improved understanding of radiative heat transfer in these processes. In addition, radiative heat and mass transfer flow plays an important role in manufacturing industries for design of reliable equipment, nuclear power plants, gas turbines and various propulsion devices for aircraft, satellites and space vehicles as well as many other astrophysical and cosmic studies.

The flow of an incompressible, electrically conducting Boussinesq fluid past an impulsively started infinite porous plate in the presence of magnetic field has applications in space science and engineering fluid dynamics. Stewartson [1] studied flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate in its own plane. In this study buoyancy effects were considered to be negligible. Soundalgekar [2] presented an analysis on free convection and mass transfer flow past an impulsively started vertical plate using regular perturbation technique. Raptis and Singh [3] investigated free convection flow along an impulsively started vertical plate embedded in a highly porous medium using finite difference method. In all these studies the presence of magnetic field is not considered. Sattar and Alam [4] studied free convective and mass transfer flow past an impulsively started vertical porous plate. In this study time dependent length scale of similarity technique was introduced. Alam and Sattar [5] extended the problem [4] to study magnetohydrodynamic free convection and mass transfer flow with thermal diffusion using similarity transformation technique. In another study, Alam and Sattar [6] presented MHD free convection and mass transfer flow in rotating system. In this study the effect of Hall currents, viscous dissipation and Joule heating were discussed. Singh [7] investigated unsteady free convection and mass transfer flow in a vertical channel consisting of semi-infinite parallel plates under the influence of magnetic field wherein a plate of the channel suddenly starts to move with a constant velocity in its own plane. In this study, the solution was obtained by the use of Laplace transform technique. In another study, Singh [8] presented a detailed analysis on hydromagnetic free convection and mass transfer flow with heat source and thermal diffusion in the presence of large suction. In this study, the similarity equations are solved using perturbation technique to obtain local similarity solutions of the problem. Recently, Ali [9] has investigated the effect of temperature dependent viscosity on laminar mixed convection boundary layer flow and heat transfer on a continuously moving vertical surface. In this study local similarity solutions are obtained for the boundary layer equations subject to isothermally moving vertical surface with uniform speed.

Cess [10] investigated the interaction of radiation with laminar free convection heat transfer from a vertical plate for an absorbing, emitting fluid in the optically thick region. In this study singular perturbation technique is used. Arpaci [11] considered a similar problem for both, the optically thin and optically thick regions. In this study approximate integral technique is used. England and Emery [12] studied the thermal radiation effect of an optically thin gray gas bounded by a stationary vertical plate, while Raptis [13] studied radiation effect on mixed convection heat transfer flow past a continuously moving plate. In these studies, the presence of electromagnetic field is not taken into account. Bestman and Adjepong [14] presented unsteady hydromagnetic free convection flow with radiative heat transfer in a rotating fluid. Azzam [15] studied radiation effects on the hydromagnetic convective heat transfer past a semi-infinite vertical porous moving plate for high temperature differences, while Chamkha [16] investigated thermal radiation effects on MHD flow over an accelerated permeable surface with heat source (sink). Cookey et. al. [17] presented an analysis on unsteady hydromagnetic free convective flow past an infinite heated vertical plate embedded in porous medium considering time dependent suction, viscous dissipation and radiation. Duwairi and Damseh [18] investigate natural convection and heat transfer from a radiative vertical porous surface in the presence of uniform magnetic field. In another study, Duwairi and Damseh [19] presented MHD buoyancy aiding and opposing flows with viscous dissipation from radiative vertical surfaces. In these studies the vertical surfaces are considered to be stationary. Recently, Mbeledogu and Ogulu [20] have presented analytical closed form solution of an unsteady hydromagnetic natural convection heat and mass transfer flow of a rotating viscous, incompressible, Boussinesq fluid in the presence of radiative heat transfer. In this analysis, the Rosseland approximation for an optically thick fluid is invoked to describe the ratiative flux. More recently, Prasad et. al. [21] have studied the interaction of free convection with thermal radiation of a viscous incompressible unsteady flow past an impulsively started vertical plate with heat and mass transfer. In this study, the dimensionless governing equations are solved using implicit finite difference method of Crank-Nicolson type.

The phenomenon by which small micron sized particles suspended in a non-isothermal gas acquire a velocity relative to the gas in the direction of decreasing temperature is termed as 'thermophorisis'. The velocity acquired by the particles is termed as 'thermophorisis velocity' and the force experienced by the suspended particles due to the temperature gradient is termed as 'thermophoritic force'. The magnitudes of thermophoretic velocity and thermophoritic force are proportional to the temperature gradient. Besides, thermophoritic velocity and force also depend on thermal conductivity of aerosol particles, the carrier gas, heat capacity of the gas, thermophoritic coefficient and Knudsen number. Due to thermophoresis small micron sized particles are deposited on cold surfaces. In this process the repulsion of particles from hot objects also takes place and a particle-free layer is observed around hot bodies. Thermophoresis plays a vital role in mass transfer mechanism of several devices involving submicron sized particles and large temperature gradients. Thermophoresis principle is utilized in the fouling of gas turbines equipment, the coagulation of condensing / evaporating aerosoles, modified chemical deposition process, the corrosion of heat exchangers with attendant reduction of the heat transfer coefficient, in determining particles trajectories in the exhaust gas from combustion devices, transpiration cooling of gas turbine blades, manufacturing of graded index silicon dioxide and germanium dioxide optical fiber used in the field of communications. Thermophoretic deposition of radioactive particles is considered to be an important factor causing accident in nuclear reactors.

Goldsmith and May [22] presented initial study for the thermophoritic transport involved simple one-dimensional flow for the measurement ofthermophoritic velocity. Hales et. al. [23] introduced first analysis on thermophoritic deposition in a geometry of engineering interest considering aerosol transport in a naturally converted boundary layer. Goren [24] presented a theoretical analysis on thermophorisis in laminar flow over a horizontal flat plate. Talbot et. al. [25] a detailed analysis on thermophorisis numerically solved a problem for the velocity and temperature fields in a laminar boundary layer adjacent to a heated plate. Batchelor and Shen [26] studied deposition of particles in gas flowing over cold surfaces due to thermophorisis. Garg and Jayaraj [27] studied thermophoritic deposition of the laminar slot jet on an inclined plate for hot, cold and adiabatic plate conditions with viscous dissipation. Chiou [28] analyzed the effect of thermophorisis on submicron particle deposition from a forced laminar boundary layer flow on to an isothermal moving plate. In this analysis the governing fundamental equations are approximated introducing the usual similarity transformation. Natural convection laminar flow over a cold vertical flat plate in the presence of thermophorisis was studied by Jayaraj [29] and Jayaraj et. al. [30] for constant and variable properties respectively. Recently, Selim et. al. [31] have investigated the effect of surface mass flux on mixed convective flow past a heated vertical flat permeable plate with detail description about thermophorisis. In this analysis a non-uniform surface mass flux through the permeable surface has been considered and the governing equations are reduced to local non-similarity boundary layer conditions using suitable transformations. More recently, Alam et. al. [32] have presented a two-dimensional steady hydromagnetic mixed convection and mass transfer flow over a semi-infinite porous inclined plate in the presence of thermal radiation, variable suction and thermophorisis.

In the present paper we consider the effects of thermophoresis and variable suction on two-dimensional steady mixed convection flow past a heated vertical flat permeable surface, moving continuously with uniform velocity in its own plane in the presence of large variable suction, thermal radiation and magnetic field. The results of the study are discussed with help of graphs and tables. The results of Alam et. al. [32] have been deduced as particular case of the present study.

2. BASIC EQUATIONS AND DESCRIPTION OF THE PROBLEM

We consider a two-dimensional, steady, MHD, laminar, mixed convection flow of a electrically conducting, incompressible, viscous fluid along a semi-infinite vertical permeable flat plate. In Cartesian coordinate system, with x-axis measured along the plate and y-axis normal to the plate. A constant magnetic field of uniform strength B_0 is applied in y-direction, which is normal to the flow. Large suction is imposed at the surface of the plate. The plates moves continuously in its own plane with uniform velocity U_0. The temperature of the surface is held uniform at T_w, which is higher than the ambient temperature T_∞. The species concentration at the surface is maintained uniform equal to that of the ambient fluid and is assumed to be C_∞. The permeability is considered to be variable depending on the distance measured from the leading edge. The effects of thermophoresis are being taken into account to help in the understanding of the mass deposition variation on the surface. The flow configuration and coordinate system are as shown in the figure (see Physical model of the Problem). In addition the present analysis based on the following assumptions :

1. The induced magnetic field is negligible compared to the applied magnetic field
2. The equation of conservation of electric charge is $\vec{\nabla} \cdot \vec{J} = 0$.
3. The Joule heating and viscous dissipation terms are negligible.
4. The term due to electrical dissipation is neglected in energy equation.
5. The Boussinesq approximation is taken into account.
6. The fluid has constant kinematic viscosity and thermal diffusivity.
7. The particle diffusivity is assumed to be constant and the concentration of particles is sufficiently dilute to assume that particles coagulation in the boundary layer is negligible.
8. The fluid is considered to be gray; absorbing-emitting radiation but not scattering medium.
9. The Rosseland approximation is used to describe the radiative heat flux in the *x*-direction is considered negligible in comparison to the *y*-direction.
10. The temperature gradient in the *y*-direction is much larger than in the *x*-direction due to boundary layer behaviour so that only the thermophoretic velocity component which is normal to the surface is of importance
11. The mass flux of the particles is sufficiently small so that the main stream velocity and temperature fields are not affected by the thermo physical processes experienced by the relatively small number of particles.

Under the above stated assumptions, the equations of mass, momentum, energy and species concentration (following Alam et. al. [32]) for this present model can be written as :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\lambda_g}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\frac{\partial}{\partial y}(V_T C) \tag{4}$$

The appropriate boundary conditions for the above model (following Singh [8]) are as follows :

$$\begin{aligned} &u=U_0, \quad v=v_w(x), \quad T=T_w, \quad C=C_\infty \quad \text{at} \quad y=0 \\ &u\to 0, \quad T\to T_\infty, \quad C\to 0 \quad \text{as} \quad y\to\infty \end{aligned} \tag{5}$$

The radiative heat flux q_r under Rosseland approximation has the form

$$\partial q_r=-\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y} \tag{6}$$

The temperature difference within the flow is assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature by expanding T^4 in Taylor series about T_∞ and neglecting higher order terms, thus

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{7}$$

Unsing (6) and (7) in Eq. (3), we have

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\lambda_g}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma_1 T_\infty^3}{3\rho C_p k_1}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The thermophoretic velocity V_T, which appears in Eq. (4), can be written as (following Talbot et. al. [25]) :

$$V_T=-k\vartheta\frac{\nabla T}{T_r}=-\frac{k\vartheta}{T_r}\frac{\partial T}{\partial y} \tag{9}$$

where T_r is some reference temperature and k is the thermophoretic coefficient which ranges in value from 0.2 to 1.2 (see Batchelor and Shen [26]). The thermophoretic coefficient (k) is defined as (see Talbot et. al. [25]) follows :

$$k=\frac{2C_s\left[(\lambda_g/\lambda_p)+C_t Kn\right]\left[1+Kn\left(C_1+C_2 e^{-C_3/Kn}\right)\right]}{(1+3C_m Kn)\left[1+(\lambda_g/\lambda_p)+2C_t Kn\right]} \tag{10}$$

where C_1, C_2, C_3, C_s, C_m, C_t are constants, λ_g is the thermal conductivity of the fluid, λ_p is the thermal conductivity of diffused particles and Kn is the Knudsen number.

In order to obtain similarity solution of the problem, we introduce the following non-dimensional variables :

$$\eta = y\sqrt{\frac{U_0}{2\vartheta x}}, \quad \psi = \sqrt{2\vartheta x U_0}\, f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C}{C_\infty}$$

where ψ is the stream function that satisfies the continuity equation (1).

Since $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, then we have :

$$u = U_0 f' \quad \text{and} \quad v = -\sqrt{\frac{\vartheta U_0}{2x}}(f - \eta f') \tag{11}$$

Here prime denotes the ordinary differentiation with respect to the similarity variables η.

Now substituting Eq. (11) in Eqs. (2), (8) and (4), we obtain the following ordinary differential equations which are locally similar :

$$f''' + ff'' + \gamma\theta - Mf' = 0 \tag{12}$$

$$(3N + 4)\theta'' + 3N\, Pr\, f\, \theta' = 0 \tag{13}$$

$$\phi'' + Sc\left(f - T_p\theta'\right)\phi' - Sc\, T_p\, \phi\theta'' = 0 \tag{14}$$

The boundary conditions (5) then turn into :

$$f = f_w, \quad f' = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0$$

$$f' \to 0, \quad \theta \to 0, \phi \to 0 \quad \text{as} \quad \eta \to \infty \tag{15}$$

where $f_w = -v_w(x)\sqrt{\frac{2x}{\vartheta U_0}}$ is the wall suction velocity at the permeable plate. Here $f_w > 0$ denotes the suction and $f_w < 0$ the injection.

The dimensionless parameters introduced in the Eqs. (12)-(14) are defined as follows :

$N = \frac{\lambda_g k_1}{4\sigma_1 T_\infty^3}$ (radiation parameter), $Re_x = \frac{U_0 2x}{\vartheta}$ (local Reynolds number)

$Pr = \frac{\vartheta \rho C_p}{\lambda_g}$ (Prandtl number), $M = \frac{\sigma B_0^2 2x}{\rho U_0}$ (local magnetic field parameter),

$Sc = \frac{\vartheta}{D}$ (Schmidt number), $T_p = \frac{-k\left(T_w - T_\infty\right) f_w^2}{T_r}$ (thermophoretic parameter),

$\gamma = \frac{Gr_x}{Re_x^2}$ (local buoyancy parameter), $Gr_x = \frac{8g\beta x^3\left(T_w - T_\infty\right)}{\vartheta^2}$ (local Grashof number)

3. METHOD OF SOLUTION

To obtain the velocity, temperature and concentration field of the equations, we assume:

$$\xi = \eta f_w, \quad f(\eta) = f_w X(\xi), \quad \theta(\eta) = f_w^2 Y(\xi), \quad \phi(\eta) = f_w^2 Z(\xi) \tag{16}$$

Substituting (16) into Eqs. (12)-(14), we obtain :

$$X''' + XX'' = \varepsilon\left(MX' - \gamma Y\right) \tag{17}$$

$$Y'' + NPr\,XY' = 0 \tag{18}$$

$$Sc.T_p\left(Y''Z + Y'Z'\right) = \varepsilon\left(Z'' + ScXZ'\right) \tag{19}$$

The boundary conditions (15) then turn to :

$$X = 1, \quad X' = \varepsilon, \quad Y = \varepsilon, \quad Z = \varepsilon \quad \text{at} \quad \xi = 0$$

$$X' \to 0, \quad Y \to 0, \quad Z \to 0 \quad \text{as} \quad \xi \to \infty \tag{20}$$

where $\varepsilon\left(= 1/f_w^2\right)$ is very small so for large suction $f_w > 1$. Hence X, Y, Z can be expressed in terms of ε as follows :

$$X(\xi) = 1 + \varepsilon X_1(\xi) + \varepsilon^2 X_2(\xi) + \varepsilon^3 X_3(\xi) + \ldots\ldots \tag{21}$$

$$Y(\xi) = \varepsilon Y_1(\xi) + \varepsilon^2 Y_2(\xi) + \varepsilon^3 Y_3(\xi) + \ldots\ldots \tag{22}$$

$$Z(\xi) = \varepsilon Z_1(\xi) + \varepsilon^2 Z_2(\xi) + \varepsilon^3 Z_3(\xi) + \ldots\ldots \tag{23}$$

Introducing $X(\xi)$, $Y(\xi)$ and $Z(\xi)$ into Eqs. (17)-(19) and considering up to $o\left(\varepsilon^3\right)$, following three sets of ordinary differential equations and corresponding boundary conditions are obtained :

First order $o(\varepsilon)$:

$$X_1''' + X_1'' = 0 \tag{24}$$

$$Y_1'' + N\,Pr\,Y_1' = 0 \tag{25}$$

$$Z_1'' + ScZ_1' = 0 \tag{26}$$

with the following boundary conditions :

$$X_1 = 0, \quad X_1' = 1, \quad Y_1 = 1, \quad Z_1 = 1 \quad \text{at} \quad \xi = 0$$

$$X_1' \to 0, \; Y_1 \to 0, \quad Z_1 \to 0 \quad \text{as} \quad \xi \to \infty \tag{27}$$

Second order $o\left(\varepsilon^2\right)$:

$$X_2''' + X_2'' + X_1 X_1'' = MX_1' - \gamma Y_1 \tag{28}$$

$$Y_2'' + N\,Pr\,Y_2' = -NPr\,X_1 Y_1' \tag{29}$$

$$Z_2'' + ScZ_2' = -ScX_1Z_1' + ScT_pY_1''Z_1 + ScT_pY_1'Z_1' \tag{30}$$

with the following boundary conditions :

$$X_2 = 0, \quad X_2' = 0, \quad Y_2 = 0, \quad Z_2 = 0 \quad \text{at} \quad \xi = 0$$

$$X_2' \to 0, \; Y_2 \to 0, \quad Z_2 \to 0 \quad \text{as} \quad \xi \to \infty \tag{31}$$

Third order $o\left(\varepsilon^3\right)$:

$$X_3''' + X_3'' = MX_2' - \gamma Y_2 - X_1 X_2'' - X_1'' X_2 \tag{32}$$

$$Y_3'' + N\,Pr\,Y_3' = -NPr\,X_1 Y_2' - NPr\,X_2 Y_1' \tag{33}$$

$$Z_3'' + ScZ_3' = ScT_p\left(Y_1''Z_2 + Y_2''Z_1 + Y_1'Z_2' + Y_2'Z_1'\right) - Sc\left(X_1Z_2' + X_2Z_1'\right) \tag{34}$$

with the following boundary conditions :

$$X_3 = 0, \quad X_3' = 0, \quad Y_3 = 0, \quad Z_3 = 0 \quad \text{at} \quad \xi = 0$$

$$X_3' \to 0, \; Y_3 \to 0, \quad Z_3 \to 0 \quad \text{as} \quad \xi \to \infty \tag{35}$$

The solution of these coupled equations, satisfying the corresponding boundary conditions are :

$$X_1 = 1 - e^{-\xi} \tag{36}$$

$$Y_1 = e^{-N\,Pr\,\xi} \tag{37}$$

$$Z_1 = e^{-Sc\,\xi} \tag{38}$$

$$X_2 = A_1 e^{-N\,Pr\,\xi} + A_2 e^{-\xi} + \frac{1}{4}e^{-2\xi} + (1+M)\xi e^{-\xi} + A_3 \tag{39}$$

$$Y_2 = A_4 e^{-N\,Pr\,\xi} - N\,Pr\,\xi e^{-N\,Pr\,\xi} - A_4 e^{-(1+N\,Pr)\xi} \tag{40}$$

$$Z_2 = -Sc\,\xi e^{-Sc\,\xi} + ScT_p e^{-(Sc+N\,Pr)\xi} - A_5 e^{-(1+Sc)\xi} + A_6 e^{-Sc\,\xi} \tag{41}$$

$$X_3 = A_{13} + A_{12}e^{-\xi} + A_7\xi e^{-\xi} - \frac{A_8}{4}e^{-2\xi} + A_9 e^{-N\,Pr\,\xi} - A_{10}e^{-(1+N\,Pr)\xi}$$
$$-A_{11}e^{-N\,Pr\,\xi}(\xi + K_1) - \frac{5}{12}e^{-3\xi} - (M+1)^2 e^{-\xi}\left(2\xi + \frac{\xi^2}{2}\right)$$
$$-\frac{1}{2}(M+1)e^{-2\xi}(\xi+2) \tag{42}$$

$$Y_3 = A_{18}e^{-N\,Pr\,\xi} - A_{14}\xi e^{-N\,Pr\,\xi} + A_{15}e^{-(1+N\,Pr)\xi} + A_{16}e^{-(2+N\,Pr)\xi} + \frac{A_1}{2}e^{-2N\,Pr\,\xi}$$
$$+A_{17}e^{-(1+N\,Pr)\xi}(\xi + K_2) + N^2Pr^2 e^{-NPr\,\xi}\left(\frac{\xi^2}{2} + \frac{\xi}{N\,Pr}\right) \tag{43}$$

$$Z_3 = A_{28}e^{-Sc\,\xi} + A_{19}e^{-(N\,Pr+Sc)\xi} - A_{20}e^{-(1+N\,Pr+Sc)\xi} - A_{21}e^{-(1+Sc)\xi} + A_{22}e^{-(2N\,Pr+Sc)\xi}$$
$$+A_{23}e^{-\xi}\left(\frac{\xi^2}{2} + \frac{\xi}{Sc}\right) + A_{24}^{-(2+Sc)\xi} - A_{25}\xi e^{-Sc\,\xi} + A_{26}e^{-(1+Sc)\xi}(\xi + K_{10})$$
$$+A_{27}e^{-(Sc+N\,Pr)\xi}\left(\frac{\xi^2}{2} + \frac{\xi}{Sc}\right) \tag{44}$$

Introducing (21)-(23) in (16), the velocity, temperature and concentration fields are obtained as :

$$u = U_0 f'(\eta) = U_0\left[X_1'(\xi) + \varepsilon X_2'(\xi) + \varepsilon^2 X_3'(\xi)\right] \tag{45}$$

$$\theta(\eta) = Y_1(\xi) + \varepsilon Y_2(\xi) + \varepsilon^2 Y_3(\xi) \tag{46}$$

$$\phi(\eta) = Z_1(\xi) + \varepsilon Z_2(\xi) + \varepsilon^2 Z_3(\xi) \tag{47}$$

The constants are given in the appendix.

4. SKIN-FRICTION, NUSSELT NUMBER AND SHERWOOD NUMBER

The quantities of chief interest are the local skin-friction, coefficient of local Nusselt number and the local Sherwood number.

The equation defining the wall skin-friction (τ) is :

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{48}$$

Hence, from equation (45), we have :

$$\tau \propto [f''(\eta)]_{\eta=0} = -1 + \varepsilon\left(N^2Pr^2A_1 + A_2 - 2M - 1\right) + \varepsilon^2(S_1 + S_2) \tag{49}$$

The local Nusselt number denoted by *Nu* is :

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{50}$$

Hence from (46), we have :

$$Nu \propto [\theta'(\eta)]_{\eta=0} = NPr + \varepsilon(NPr - A_4) + \varepsilon^2 (H_1 + H_2) \tag{51}$$

The local Sherwood number denoted by *Sh* is :

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{52}$$

Hence from (47), we have :

$$Sh \propto [\phi'(\eta)]_{\eta=0} = Sc + \varepsilon\left[ScA_6 - (1+Sc)A_5 - ScT_p(Sc + NPr) - Sc\right] + \varepsilon^2 (M_1 + M_2 + M_3) \tag{53}$$

Table-1

Variations in skin-friction (τ) at the plate

M	*N*	*Pr*	f_w	τ
0.2 0.4 0.8	0.8	0.71	0.5	5.24225 4.44225 3.64225
0.2	0.8 1.0 1.2	0.71	0.5	5.24225 3.83381 2.89483
0.2	0.8	0.71 0.90 1.00	0.5	5.24225 3.75556 3.24837
0.2	0.8	0.71	0.5 1.0 2.0	5.24225 2.06056 2.16514

Table-2

Variations in wall heat transfer or local Nusselt number (Nu) at the plate

N	Pr	f_w	Nu
0.8 1.0 1.2	0.71	0.5	2.01698 2.37081 2.69217
0.8	0.71 0.90 1.00	0.5	2.01698 2.39442 2.57778
0.8	0.71	0.5 1.0 2.0	2.01698 0.93025 0.65856

Table-3

Variations in wall deposition parameter (rate of mass transfer) (Sh) at the plate

N	Pr	f_w	T_p	Sc	Sh
0.8 1.0 1.2	0.71	0.5	0.5	0.6	-3.3816 -3.5527 -3.7224
0.8	0.71 0.90 1.00	0.5	0.5	0.6	-3.38162 -3.56474 -3.66276
0.8	0.71	0.5 1.0 2.0	0.5	0.6	-3.38162 -0.39548 0.35115
0.8	0.71	0.5	0.0 0.5 1.5	0.6	-2.74584 -3.38162 -4.74486
0.8	0.71	0.5	0.5	0.22 0.60 0.78	-1.17854 -3.38162 -4.59327

5. DISCUSSION AND CONCLUSIONS

For the present problem numerical computations have been carried out by employing the similarity solution for all ξ. Numerical calculations have been carried out for different values of suction parameter (f_w), radiation parameter (N), Schmidt number (Sc), thermophoretic parameter (T_p), Prandtl number (Pr) and magnetic parameter (M). The case of $\gamma = 1.0$ corresponds to mixed (combined free and forced) convection. Throughout this calculation we have considered $\gamma = 1.0$ unless otherwise specified.

The values of Prandtl number are chosen for air (*Pr* = 0.71), Ammonia (*Pr* = 0.90)20^{0}C and one atmospheric pressure, electrolyte solution (*Pr* = 1.0), while that of Schmidt number are chosen for hydrogen (*Sc* = 0.22), helium (*Sc* = 0.30), water vapor (*Sc* = 0.60), oxygen (*Sc* = 0.66), ammonia (*Sc* = 0.78) and methanol (*Sc* = 1.0), which represents diffusing chemical species of most common interest in air at 20^{0}C and one atmospheric pressure. The values of the remaining parameters are chosen arbitrarily, but do retain physical significance in real energy system applications.

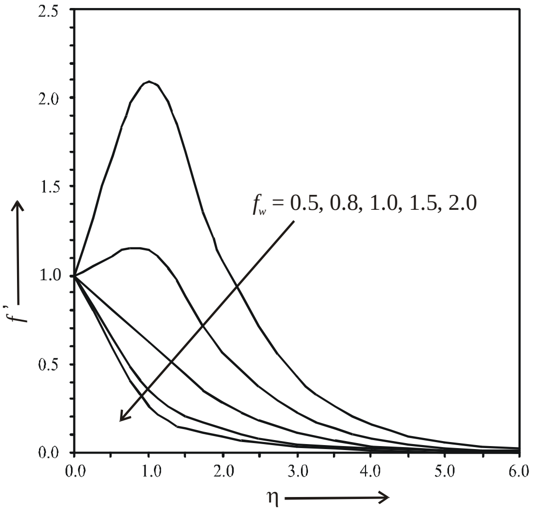

Fig. 1. Dimensionless velocity profiles for different values of f_w and for γ = 1.0, *Pr* = 0.71, *N* = 0.8 and *M* = 0.2

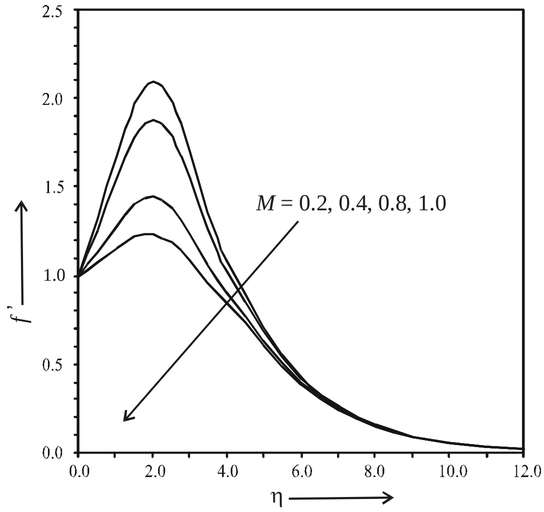

Fig. 2. Dimensionless velocity profiles for different values of *M* and for γ = 1.0, *Pr* = 0.71, *N* = 0.8 and f_w = 0.5

Fig.-1 illustrates the influence of the suction parameter (f_w) on the velocity profiles (u, η) with fixed values of $\gamma = 1.0$, $M = 0.2$, $N = 0.8$ and $Pr = 0.71$. It is observed that an increase in suction parameter decreases the hydromagnetic boundary layer causing a decrease in the fluid velocity.

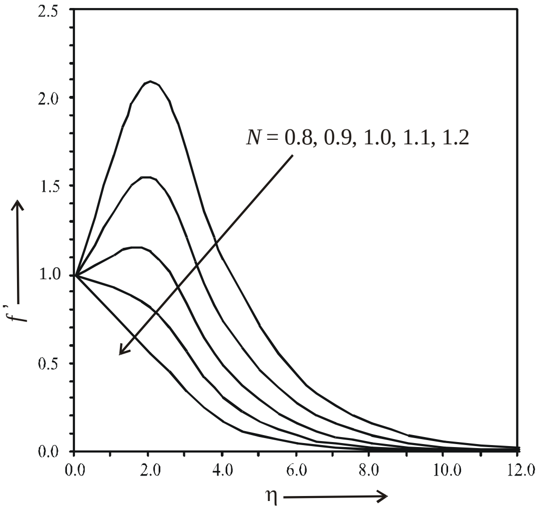

Fig. 3. Dimensionless velocity profiles for different values of *N* and for γ = 1.0, *Pr* = 0.71, f_w = 0.5 and *M* = 0.2

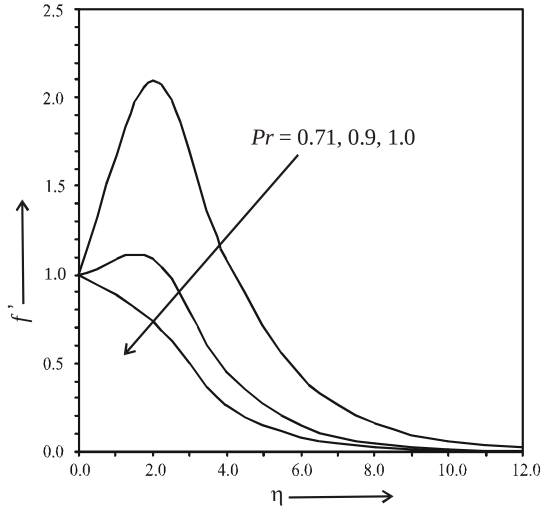

Fig. 4. Dimensionless velocity profiles for different values of *Pr* and for γ = 1.0, M = 0.2, *N* = 0.8 and f_w = 0.5

Fig.-2 demonstrates the effect of the magnetic parameter (*M*) on the velocity profiles (u,η) with fixed values of $\gamma=1.0$, $f_w=0.5$, $N=0.8$ and $Pr=0.71$. It is observed that an increase in magnetic parameter decreases the hydromagnetic boundary layer causing the fluid velocity.

Fig.-3 shows the variations in the velocity profiles (u,η) due to change in radiation parameter (*N*) with fixed values of $\gamma=1.0$, $f_w=0.5$, $M=0.2$ and $Pr=0.71$. It is observed that an increase in radiation parameter decreases the velocity.

Fig.-4 shows the variations in the velocity profiles (u,η) due to change in Prandtl number (Pr) with fixed values of $\gamma=1.0$, $f_w=0.5$, $M=0.2$ and $N=0.8$. It is observed that an increase in Prandtl number decreases the velocity.

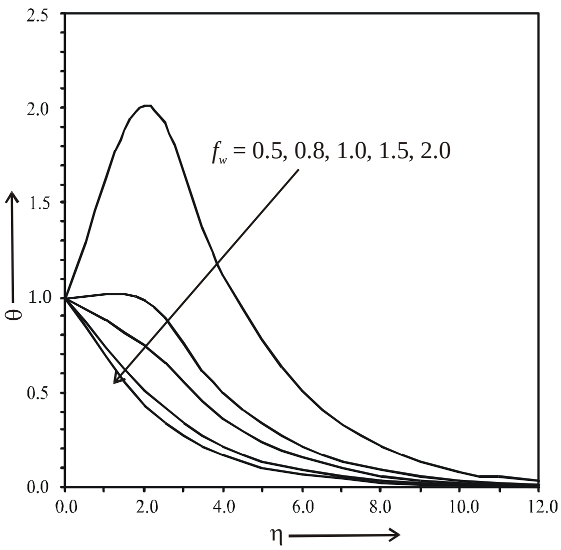

Fig. 5. Dimensionless temperature profiles for different values of f_w and for γ = 1.0, Pr = 0.71 and N = 0.8

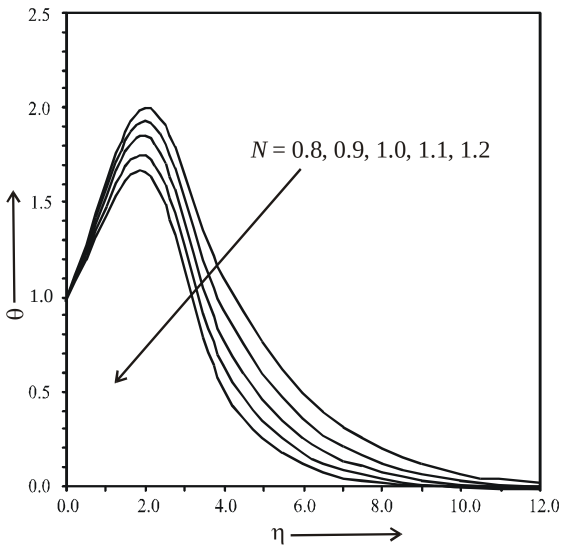

Fig. 6. Dimensionless temperature profiles for different values of N and for γ = 1.0, Pr = 0.71 and f_w = 0.5

Fig.-5 depicts the influence of the suction parameter (f_w) on the temperature profiles (θ,η) with fixed values of $\gamma=1.0$, $N=0.8$ and $Pr=0.71$. It is observed that an increase in suction parameter decreases the thermal boundary layer causing a decrease in the temperature.

Fig.-6 shows the effects of radiation parameter (*N*) on the temperature profiles (θ,η) with fixed values of $\gamma=1.0$, $f_w=0.5$ and $Pr=0.71$. It is observed that an increase in radiation parameter decreases the temperature field.

Fig.-7 shows the variations in Prandtl number (Pr) on the temperature profiles (θ,η) with fixed values of $\gamma=1.0$, $f_w=0.5$ and $N=0.8$. It is observed that an increase in Prandtl number decreases the temperature.

International Journal of Engineering, Science and Mathematics
Vol. 6 Issue 4, August 2017,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
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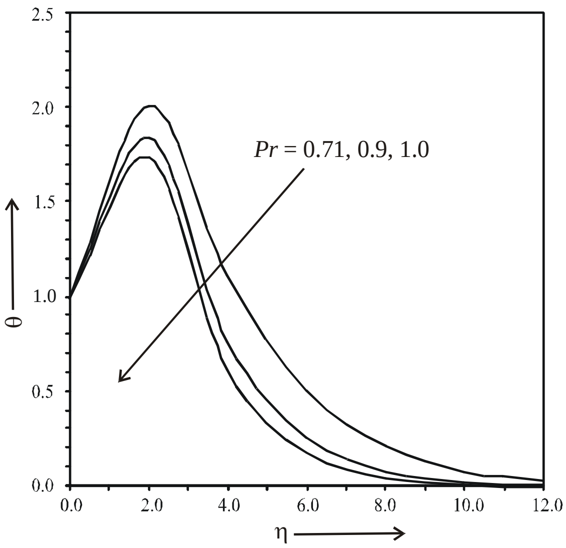

Fig. 7. Dimensionless temperature profiles for different values of *Pr* and for $\gamma = 1.0$, $N = 0.8$ and $f_w = 0.5$

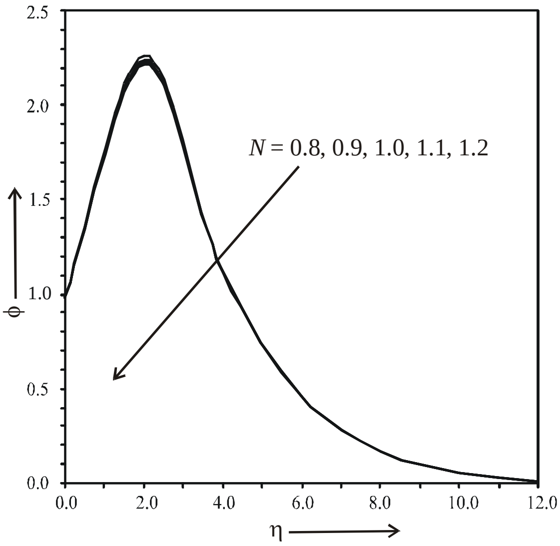

Fig. 8. Dimensionless concentration profiles for different values of *N* and for $\gamma = 1.0$, $Pr = 0.71$, $Sc = 0.60$, $T_p = 0.5$ and $f_w = 0.5$

Fig.-8 depicts the influence of the radiation parameter (*N*) on the concentration profiles (ϕ, η) with fixed values of $\gamma = 1.0$, $Sc = 0.60$, $T_p = 0.5$, $N = 0.8$ and $Pr = 0.71$. It is observed that an increase in suction parameter decreases the concentration field.

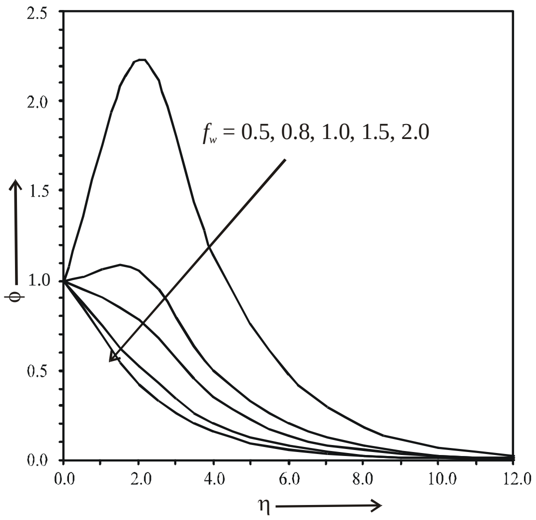

Fig. 9. Dimensionless concentration profiles for different values of f_w and for $\gamma = 1.0$, $Pr = 0.71$, $N = 0.8$, $Sc = 0.60$ and $T_p = 0.5$

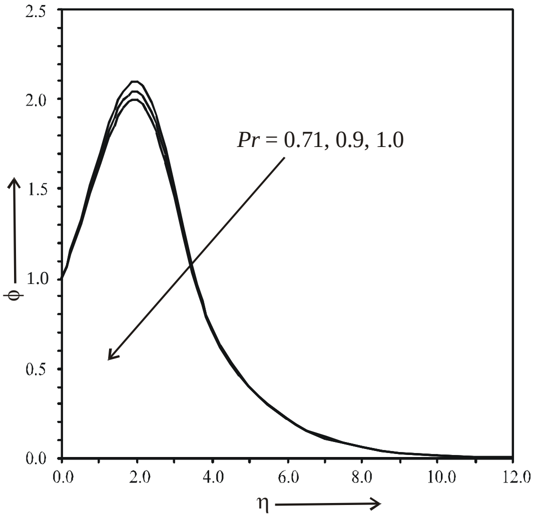

Fig. 10. Dimensionless concentration profiles for different values of *Pr* and for $\gamma = 1.0$, $T_p = 0.05$, $N = 0.8$, $Sc = 0.60$ and $f_w = 0.5$

Fig.-9 represents the influence of the suction parameter (f_w) on the concentration profiles (ϕ, η) with fixed values of $\gamma = 1.0$, $Sc = 0.60$, $T_p = 0.5$, $N = 0.8$ and $Pr = 0.71$. It is observed that an increase in suction parameter decreases the concentration boundary layer. The decreasing thickness of the concentration layer is caused by two effects first the direct action of the suction and other the indirect action of suction causing a thinner boundary layer, which corresponds to higher temperature gradient. This causes a consequent increase in the thermophoretic force and in turn higher concentration gradient.

Fig.-10 represents the influence of the Prandtl number (Pr) on the concentration profiles for fixed values of $\gamma = 1.0$, $Sc = 0.60$, $T_p = 0.5$, $N = 0.8$ and $f_w = 0.5$. It is observed that an increase in Prandtl number decreases the concentration field.

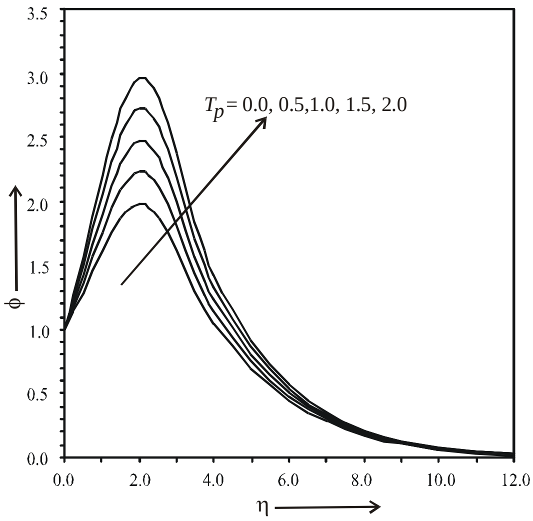

Fig. 11. Dimensionless concentration profiles for different values of T_p and for γ = 1.0, Pr = 0.71, N = 0.8, Sc = 0.60 and f_w = 0.5

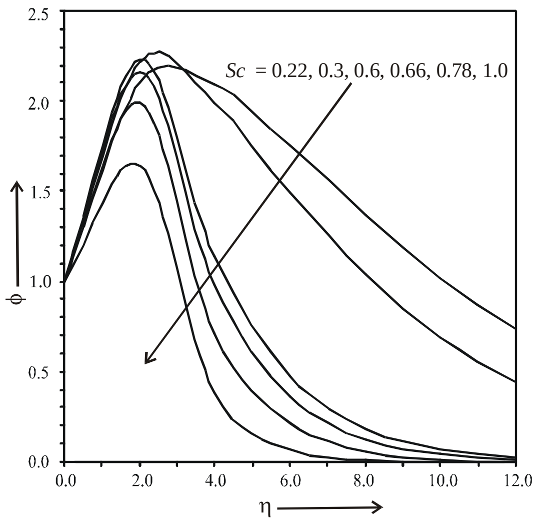

Fig. 12. Dimensionless concentration profiles for different values of Sc and for γ = 1.0, Pr = 0.71, N = 0.8, T_p = 0.5 and f_w = 0.5

Fig.-11 illustrates the influence of the thermophoretic parameter (T_p) on the concentration profiles for fixed values of $\gamma = 1.0$, $Sc = 0.60$, $f_w = 0.5$, $N = 0.8$ and $Pr = 0.71$. It is observed that an increase in thermophoretic parameter increases the concentration field.

Fig.-12 illustrates the influence of the Schmidt number (Sc) on the concentration profiles for fixed values of $\gamma = 1.0$, $f_w = 0.5$, $T_p = 0.5$, $N = 0.8$ and $Pr = 0.71$. It is observed that an increase in Schmidt number decreases the concentration boundary layer and this is analogous to the effect of increasing the Prandtl number on the thickness of a thermal boundary layer.

Table-1 indicates the effects of the parameters M, N, Pr and f_w on the skin-friction at the wall. It is observed that an increase in M, N, Pr or f_w decreases the skin-friction. Table-2 represents the variations in wall heat transfer due to change in N, Pr and f_w. Obviously, an increase in *N* or Pr increases the heat transfer but an increase in f_w decreases the heat transfer. on the skin-friction at the wall. It is observed that an increase in M, N, Pr or f_w decreases the skin-friction.

Table-3 indicates the variations in the rate of mass transfer, i.e., wall deposition parameter due to change in the numerical values of *N*, Pr, f_w, Sc and T_p. It is observed that an increase in *N*, Pr, Sc and T_p decreases the wall deposition while an increase in f_w increases the wall deposition.

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APPENDIX

$$A_1 = \frac{\gamma}{N^2 Pr^2 (NPr - 1)}, \qquad A_2 = M + \frac{1}{2} - NPr\, A_1, \qquad A_3 = A_1 (NPr - 1) - M - \frac{3}{4},$$

$$A_4 = \frac{N^2 Pr^2}{1 + NPr}, \qquad A_5 = \frac{Sc^2}{1 + Sc}, \qquad A_6 = ScT_p + A_5,$$

$$A_7 = A_3 - A_2 (1 + M) + M (1 + M)(2 + M), \qquad A_8 = 2 (A_2 - M - 1) - \frac{M}{2} - 1,$$

$$A_9 = \frac{\gamma A_4 + MNPrA_1 + N^2 Pr^2 A_1}{N^2 Pr^2 (NPr - 1)}, \qquad A_{10} = \frac{\gamma A_4 + \left(1 + NPr^2\right) A_1}{NPr (NPr + 1)^2}, \quad A_{11} = \frac{\gamma}{NPr (NPr - 1)},$$

$$A_{12} = A_7 + \frac{A_8}{2} - NPrA_9 + (1 + NPr) A_{10} + \frac{5}{24} - 2M (M + 1) - \frac{M + 1}{2} + A_{11} (NPrK_1 - 1),$$

$$A_{13} = \frac{A_8}{4} - A_9 + A_{10} + \frac{5}{72} + M + 1 + A_{11} K_1 - A_{12}, \qquad A_{14} = NPr (A_4 + A_3 + 1),$$

$$A_{15} = \frac{NPr}{1 + NPr} [NPr (A_2 - A_4 - 1) - (1 + NPr) A_4], \quad A_{16} = \frac{NPr}{2 (2 + NPr)} \left[\frac{NPr}{4} + A_4 (1 + NPr) \right],$$

$$A_{17} = \frac{N^2 Pr^2 (M + NPr + 1)}{1 + NPr}, \quad A_{18} = -A_{17} K_2 - \frac{A_1}{2} - A_{16} - A_{15}, \quad A_{19} = \frac{K_3 + K_4 + K_5 + K_6}{NPr (NPr + Sc)},$$

$$A_{20} = \frac{K_7 + K_8 + K_9}{(1 + NPr)(1 + NPr + Sc)}, \qquad A_{21} = \frac{Sc [Sc (A_2 + A_6 + 1) + A_5 (1 + Sc)]}{1 + Sc},$$

$$A_{22} = \frac{Sc^2 T_p}{2 (Sc + 2N\,Pr)}, \qquad A_{23} = Sc^2 [A_6 + A_2 - 1], \qquad A_{24} = \frac{Sc [-Sc + 4A_5 (1 + Sc)]}{2 (2 + Sc)},$$

$$A_{25} = Sc [A_6 - A_3 + 1], \qquad A_{26} = \frac{Sc^2 (Sc - 1 - M)}{1 + Sc},$$

$$A_{27} = \frac{-ScT_p \left(2NPrSc + Sc^2 + N^2 Pr^2\right)}{NPr + Sc},$$

$$A_{28} = -(A_{19} - A_{20} - A_{21} + A_{22} + A_{24} + A_{26} K_{10} + A_{27} K_{11}), \qquad ,$$

$$K_1 = \frac{3NPr - 2}{N\,Pr (N\,Pr + 1)}, \quad K_2 = \frac{2 + NPr}{1 + NPr}, \qquad K_3 = T_p N^2 Pr^2 Sc (A_6 + A_4 + 2),$$

$$K_4 = T_p NPrSc^2 (A_6 + 1), \qquad K_5 = T_p NPrSc^2 (A_4 + 1),$$

$$K_6 = Sc^2 \left[-A_1 + T_p (NPr + Sc) \right], \qquad K_7 = T_p NPrSc (NPrA_5 + ScA_5 + A_5),$$

$$K_8 = T_p Sc \left[A_4 (1 + NPr)^2 + (NPr + Sc) Sc \right], \qquad K_9 = Sc^2 T_p A_4 (1 + NPr),$$

$$K_{10} = \frac{2+Sc}{1+Sc}, \qquad K_{11} = \frac{2NPr+Sc}{NPr(NPr+Sc)}.$$

$$S_1 = A_{12} - 2A_7 - A_8 + N^2Pr^2A_9 - (1+NPr)^2 A_{10}, \quad S_1 = A_{12} - 2A_7 - A_8 + N^2Pr^2A_9 - (1+NPr)^2 A_{10},$$

$$H_1 = A_{14} + (1+NPr)A_{15} + (2+NPr)A_{16} - NPr, \quad H_2 = A_1 + A_{17}(1+NPr)K_2 - A_{17} + NPrA_{18},$$

$$M_1 = Sc(A_{19} - A_{20} - A_{21} + A_{22} + A_{24} + A_{28}) + NPr(A_{19} - A_{20} + 2A_{22}),$$

$$M_2 = A_{26}[(1+Sc)K_{10} - 1] + A_{27}[(Sc+NPr)K_{11} - 1],$$

$$M_3 = A_{25} + 2A_{24} - A_{21} - A_{20} - \frac{A_{23}}{Sc}.$$

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