VARIANCE OF TIME TO RECRUITMENT FOR A SINGLE GRADE MANPOWER SYSTEM WITH DIFFERENT EPOCHS FOR EXITS AND TWO TYPES OF DECISIONS WITH CORRELATED WASTAGES HAVING TWO THRESHOLDS INVOLVING TWO COMPONENTS

G. RAVICHANDRAN*, A. SRINIVASAN**

|  | Abstract: |
| :---: | :---: |
|  | In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment involving optional and mandatory thresholds for a single grade manpower system where wastages (loss in manpower) occur due to attrition generated by its policy decisions and frequent breaks taken by the personnel working in the system. Assuming that (i) the policy decisions and exits occur at different epochs (ii) the number of exits form a homogeneous Poisson process (iii) both the optional and mandatory thresholds for the cumulative loss of manpower have independently a normal component due to attrition and a second component due to frequent breaks (iv) wastage due to attrition and frequent breaks form separately a sequence of exchangeable and constantly correlated exponentially distributed random variables and <br> (v) interpolicy decision times are of two types, one with high rate of attrition and the other having low rate of attrition, a stochastic model is constructed and variance of time to recruitment is obtained when the inter-policy decision times form (i) a geometric process and (ii) an order statistics. |
| Keywords: <br> Single <br> grade <br> manpower system; different decision and exit epochs; two types of policy decisions; correlated wastages; optional and mandatory thresholds with two components; geometric process; order statistics; univariate policy of recruitment and variance of time to recruitment. |  |
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*Author correspondence:
Assistant Professor in Mathematics,
TRP Engineering College (SRM GROUP), Trichy - 621 105, Tamil Nadu, India.

## 1. INTRODUCTION

Wastages due to attrition are usual in any marketing organization. A judicious and planned recruitment has to be advocated as frequent recruitment is not advisable. In [1, 3] several stochastic models for manpower planning have been discussed. In [2] some manpower planning problems have been analyzed using statistical techniques. In [4] the author has studied the problem of time to recruitment for a single grade manpower system when the loss of manpower forms a sequence of exchangeable and constantly correlated exponentially distributed random variables. Assuming different epochs for decisions and exits and the number of exits form a homogeneous Poisson process, variance of time to recruitment is obtained in $[6,8,9]$ using univariate policy of recruitment and Laplace transform in the analysis according as the inter decision times are independent and identically distributed exponential random variables or forming a geometric process or an order statistics. Recently, in $[10,11,12]$ the research work in $[6,8,9]$ have been studied by considering
optional and mandatory thresholds as two control limits with single(normal) component, which is a variation from the work of [5] in the context of considering non-instantaneous exits at decision epochs. In [13] the research work in [10, 11, 12] have been studied when the policy decisions are classified into two types according to their intensities of attrition. In [14] the manpower planning problem in [13] is studied when the optional and mandatory thresholds have two components, the first component is the normal component for the cumulative wastage due to attrition and the second one is the component due to frequent breaks taken by the personnel working in the system. In the present paper, for a single grade manpower system, a mathematical model is constructed in which attrition due to policy decisions take place at exit points and there are optional and mandatory thresholds with two components. A univariate policy of recruitment based on shock model approach is used to determine the variance of time to recruitment when the policy decisions are classified into two types according to their intensities of attrition. The present paper extends the research work in [14] when the loss of manpower forms a sequence of exchangeable and constantly correlated exponentially distributed random variables.

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*Assistant Professor in Mathematics, TRP Engineering College (SRM GROUP), Trichy - }621\mathrm{ 105, Tamil
Nadu, India.
**Professor Emeritus, PG & Research Department of Mathematics, Bishop Heber College, Trichy -
620 017, Tamil Nadu, India.
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## 2. MODEL DESCRIPTION

Consider an organization taking decisions at random epochs in ( $0, \infty$ ) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let $X_{i}$ be the continuous random variable representing the amount of depletion of manpower (loss of man hours) caused at the $i^{\text {th }}$ exit point and $S_{k}$ be the total loss of manpower up to the first k exit points. It is assumed that $X_{i}$ 's are exchangeable and constantly correlated exponential random variables with mean $\frac{1}{\alpha}(\alpha>0)$. Let $M_{k}($.$) be the distribution of S_{k}, k=1,2,3, \ldots$. . Let $\rho$ be the correlation between $X_{i}$ and $X_{j}$ where $i \neq j$ and $\phi(n, x)=\int_{0}^{x} e^{-z} z^{n-1} d z$. Let $b=a(1-\rho)$ where $a$ is the mean of $X_{i}, i$ $=1,2,3, \ldots$ Let $U_{k}$ be the continuous random variable representing the time between the $(k-1)^{\text {th }}$ and $\mathrm{k}^{\text {th }}$ policy decisions. Let $W_{i}$ be the continuous random variable representing the time between the ( $\mathrm{i}-$ $1)^{\text {th }}$ and $\mathrm{i}^{\text {th }}$ exit times. It is assumed that $W_{i}$ 's are independent and identically distributed random variables with probability density function $\mathrm{g}($.$) , probability distribution function \mathrm{G}($.$) and mean$ $\frac{1}{\delta},(\delta>0)$. Let $N_{e}(t)$ be the number of exit points lying in $(0, t]$. Let $Y$ and $Z$ be the optional and mandatory threshold levels for the cumulative wastage in the organization. Let $Y_{1}$ be the first component of $Y$ corresponding to cumulative wastage due to attrition and $Y_{2}$ be its second component corresponding to frequent breaks taken by the personnel working in the system. Let $Z_{1}$ be the first component of $Z$ corresponding to cumulative wastage due to attrition and $Z_{2}$ be its second component corresponding to frequent breaks taken by the personnel working in the system.

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Then $Y=Y_{1}+Y_{2}$ and $Z=Z_{1}+Z_{2}$. It is assumed that (i) $Y<Z$ (ii) $Y_{1}$ and $Y_{2}$ are independent and (iii) $Z_{1}$ and $Z_{2}$ are independent. Let $H_{1}(),. H_{2}(),. H_{11}(),. H_{12}(),. H_{21}($.$) and H_{22}($.$) be the distribution of Y, Z, Y_{1}$, $Y_{2}, Z_{1}$ and $Z_{2}$ respectively. It is assumed that $H_{11}(y)=1-e^{-\theta_{1} y}, H_{12}(y)=1-e^{-\theta_{2} y}$. It is also assumed that $H_{21}(z)=1-e^{-\gamma_{1} z}, H_{22}(z)=1-e^{-\gamma_{2} z}$. Let $p$ be the probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower. Let $q$ be the probability that every policy decision has exit of personnel. As $q=0$ corresponds to the case where exits are impossible, it is assumed that $q \neq 0$. Let T be the random variable denoting the time to recruitment with probability distribution function $\mathrm{L}($.$) , density function$ $l($.$) , mean \mathrm{E}(\mathrm{T})$ and variance $\mathrm{V}(\mathrm{T})$. Let $\bar{a}($.$) be the Laplace transform of \mathrm{a}($.$) . The univariate CUM$ policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative wastage in the organization exceeds the mandatory threshold. The organization may or may not go for recruitment if the cumulative wastage exceeds the optional threshold.

## 3. MAIN RESULT

From the recruitment policy, we note that
$P(T>t)=\sum_{k=0}^{\infty} P\left[N_{e}(t)=k\right] P\left(S_{k} \leq Y\right)+p \sum_{k=0}^{\infty} P\left[N_{e}(t)=k\right] P\left(S_{k}>Y\right) P\left(S_{k} \leq Z\right)$
(1)

As in Gurland [4],
$M_{k}(y)=(1-\rho) \sum_{i=0}^{\infty} \frac{(k \rho)^{i}}{(1-\rho+k \rho)^{i+1}} \frac{\phi\left(k+i, \frac{y}{b}\right)}{(k+i-1)!}$, where $\phi\left(k+i, \frac{y}{b}\right)=\int_{0}^{\frac{y}{b}} e^{-z} z^{\mathrm{k}+\mathrm{i}-1} d z$
(2)

Since $Y$ is the sum of two exponential random variables and is independent of $S_{k}$, using law of total probability and on simplification, we get

$$
\begin{equation*}
P\left(S_{k} \leq Y\right)=(1-\rho) N_{1}\left\{\frac{\left(b \theta_{2}+1\right)}{\theta_{2}} A_{k}-\frac{\left(b \theta_{1}+1\right)}{\theta_{1}} B_{k}\right\} \tag{3}
\end{equation*}
$$

where $A_{k}=\frac{1}{\left(b \theta_{2}+1\right)^{k}\left[(1-\rho+k \rho)\left(b \theta_{2}+1\right)-k \rho\right]}$,

$$
B_{k}=\frac{1}{\left(b \theta_{1}+1\right)^{k}\left[(1-\rho+k \rho)\left(b \theta_{1}+1\right)-k \rho\right]} \quad \text { and } \quad N_{1}=\frac{\theta_{1} \theta_{2}}{\theta_{1}-\theta_{2}}
$$

(4)

Similarly

$$
\begin{equation*}
P\left(S_{k} \leq Z\right)=(1-\rho) N_{2}\left\{\frac{\left(b \gamma_{2}+1\right)}{\gamma_{2}} C_{k}-\frac{\left(b \gamma_{1}+1\right)}{\gamma_{1}} D_{k}\right\} \tag{5}
\end{equation*}
$$

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where $C_{k}=\frac{1}{\left(b \gamma_{2}+1\right)^{k}\left[(1-\rho+k \rho)\left(b \gamma_{2}+1\right)-k \rho\right]}$,

$$
\begin{equation*}
B_{k}=\frac{1}{\left(b \gamma_{1}+1\right)^{k}\left[(1-\rho+k \rho)\left(b \gamma_{1}+1\right)-k \rho\right]} \tag{6}
\end{equation*}
$$

and $N_{2}=\frac{\gamma_{1} \gamma_{2}}{\gamma_{1}-\gamma_{2}}$
Substituting (3), (4), (5) and (6) in (1) and on simplification, we have

$$
\begin{aligned}
& \mathrm{P}(\mathrm{~T}>\mathrm{t})=(1-\rho) N_{1}\left\{\frac{\left(b \theta_{2}+1\right)}{\theta_{2}}\left[A_{0}+\sum_{k=1}^{\infty} G_{k}(t)\left(A_{k}-A_{k-1}\right)\right]-\frac{\left(b \theta_{1}+1\right)}{\theta_{1}}\left[B_{0}+\sum_{k=1}^{\infty} G_{k}(t)\left(B_{k}-B_{k-1}\right)\right]\right\} \\
& +p\left\{(1-\rho) N_{2}\left[\frac{\left(b \gamma_{2}+1\right)}{\gamma_{2}}\left[C_{0}+\sum_{k=1}^{\infty} G_{k}(t)\left(C_{k}-C_{k-1}\right)\right]-\frac{\left(b \gamma_{1}+1\right)}{\gamma_{1}}\left[D_{0}+\sum_{k=1}^{\infty} G_{k}(t)\left(D_{k}-D_{k-1}\right)\right]\right]\right\} \\
& -p(1-\rho)^{2} N_{1} N_{2}\left\{\left[\frac{\left(b \theta_{2}+1\right)\left(b \gamma_{2}+1\right)}{\theta_{2} \gamma_{2}}\left[A_{0} C_{0}+\sum_{k=1}^{\infty} G_{k}(t)\left(A_{k} C_{k}-A_{k-1} C_{k-1}\right)\right]-\frac{\left(b \theta_{2}+1\right)\left(b \gamma_{1}+1\right)}{\theta_{2} \gamma_{1}}\left[A_{0} D_{0}+\sum_{k=1}^{\infty} G_{k}(t)\left(A_{k} D_{k}-A_{k-1} D_{k-1}\right)\right]\right]\right\} \\
& +p(1-\rho)^{2} N_{1} N_{2}\left\{\left[\frac{\left(b \theta_{1}+1\right)\left(b \gamma_{2}+1\right)}{\theta_{1} \gamma_{2}}\left[B_{0} C_{0}+\sum_{k=1}^{\infty} G_{k}(t)\left(B_{k} C_{k}-B_{k-1} C_{k-1}\right)\right]+\frac{\left(b \theta_{1}+1\right)\left(b \gamma_{1}+1\right)}{\theta_{1} \gamma_{1}}\left[B_{0} D_{0}+\sum_{k=1}^{\infty} G_{k}(t)\left(B_{k} D_{k}-B_{k-1} D_{k-1}\right)\right]\right]\right\}
\end{aligned}
$$

From (7), we get

$$
\begin{align*}
& \bar{l}(\mathrm{~s})=1-(1-\rho) N_{1}\left\{\frac{\left(b \theta_{2}+1\right)}{\theta_{2}}\left[A_{0}+\sum_{k=1}^{\infty} \bar{g}(s)^{k}\left(A_{k}-A_{k-1}\right)\right]-\frac{\left(b \theta_{1}+1\right)}{\theta_{1}}\left[B_{0}+\sum_{k=1}^{\infty} \bar{g}(s)^{k}\left(B_{k}-B_{k-1}\right)\right]\right\} \\
& -p\left\{(1-\rho) N_{2}\left[\frac{\left(b \gamma_{2}+1\right)}{\gamma_{2}}\left[C_{0}+\sum_{k=1}^{\infty} \bar{g}(s)^{k}\left(C_{k}-C_{k-1}\right)\right]-\frac{\left(b \gamma_{1}+1\right)}{\gamma_{1}}\left[D_{0}+\sum_{k=1}^{\infty} \bar{g}(s)^{k}\left(D_{k}-D_{k-1}\right)\right]\right]\right\} \\
& +p(1-\rho)^{2} N_{1} N_{2}\left\{\left[\frac{\left(b \theta_{2}+1\right)\left(b \gamma_{2}+1\right)}{\theta_{2} \gamma_{2}}\left[A_{0} C_{0}+\sum_{k=1}^{\infty} \bar{g}(s)^{k}\left(A_{k} C_{k}-A_{k-1} C_{k-1}\right)\right]-\frac{\left(b \theta_{2}+1\right)\left(b \gamma_{1}+1\right)}{\theta_{2} \gamma_{1}}\left[A_{0} D_{0}+\sum_{k=1}^{\infty} \bar{g}(s)^{k}\left(A_{k} D_{k}-A_{k-1} D_{k-1}\right)\right]\right]\right\} \\
& -p(1-\rho)^{2} N_{1} N_{2}\left\{\left[\frac{\left(b \theta_{1}+1\right)\left(b \gamma_{2}+1\right)}{\theta_{1} \gamma_{2}}\left[B_{0} C_{0}+\sum_{k=1}^{\infty} \bar{g}(s)^{k}\left(B_{k} C_{k}-B_{k-1} C_{k-1}\right)\right]+\frac{\left(b \theta_{1}+1\right)\left(b \gamma_{1}+1\right)}{\theta_{1} \gamma_{1}}\left[B_{0} D_{0}+\sum_{k=1}^{\infty} \bar{g}(s)^{k}\left(B_{k} D_{k}-B_{k-1} D_{k-1}\right)\right]\right]\right\} \tag{8}
\end{align*}
$$

It is known that $\mathrm{E}\left(\mathrm{T}^{\mathrm{r}}\right)=(-1)^{r}\left[\frac{d^{r}}{d s^{r}} \bar{l}(s)\right]_{s=0}, r=1,2,3, \ldots$
From (8) and (9), it can be shown that

$$
\begin{aligned}
E(\mathrm{~T})= & \bar{g}^{\prime}(0)\left\{(1-\rho) N_{1}\left[\frac{\left(b \theta_{2}+1\right)}{\theta_{2}} \sum_{k=1}^{\infty} k\left(A_{k}-A_{k-1}\right)-\frac{\left(b \theta_{1}+1\right)}{\theta_{1}} \sum_{k=1}^{\infty} k\left(B_{k}-B_{k-1}\right)\right]\right. \\
& -p(1-\rho) N_{2}\left[\frac{\left(b \gamma_{2}+1\right)}{\gamma_{2}} \sum_{k=1}^{\infty} k\left(C_{k}-C_{k-1}\right)-\frac{\left(b \gamma_{1}+1\right)}{\gamma_{1}} \sum_{k=1}^{\infty} k\left(D_{k}-D_{k-1}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& +p(1-\rho)^{2} N_{1} N_{2}\left[\frac{\left(b \theta_{2}+1\right)\left(b \gamma_{2}+1\right)}{\theta_{2} \gamma_{2}} \sum_{k=1}^{\infty} k\left(A_{k} C_{k}-A_{k-1} C_{k-1}\right)-\frac{\left(b \theta_{2}+1\right)\left(b \gamma_{1}+1\right)}{\theta_{2} \gamma_{1}} \sum_{k=1}^{\infty} k\left(A_{k} D_{k}-A_{k-1} D_{k-1}\right)\right] \\
& \left.-p(1-\rho)^{2} N_{1} N_{2}\left[\left[\frac{\left(b \theta_{1}+1\right)\left(b \gamma_{2}+1\right)}{\theta_{1} \gamma_{2}} \sum_{k=1}^{\infty} k\left(B_{k} C_{k}-B_{k-1} C_{k-1}\right)+\frac{\left(b \theta_{1}+1\right)\left(b \gamma_{1}+1\right)}{\theta_{1} \gamma_{1}} \sum_{k=1}^{\infty} k\left(B_{k} D_{k}-B_{k-1} D_{k-1}\right)\right]\right]\right\}
\end{aligned}
$$

(10)
and

$$
\begin{aligned}
& E\left(\mathrm{~T}^{2}\right)=-(1-\rho) N_{1}\left[\bar{g}^{\prime \prime}(0)+(k-1) \bar{g}^{\prime}(0)^{2}\right]\left[\frac{\left(b \theta_{2}+1\right)}{\theta_{2}} \sum_{k=1}^{\infty} k\left(A_{k}-A_{k-1}\right)-\frac{\left(b \theta_{1}+1\right)}{\theta_{1}} \sum_{k=1}^{\infty} k\left(B_{k}-B_{k-1}\right)\right] \\
& -p\left\{\left[\bar{g}^{\prime \prime}(0)+(k-1) \bar{g}^{\prime}(0)^{2}\right]\right\}\left\{(1-\rho) N_{2}\left[\frac{\left(b \gamma_{2}+1\right)}{\gamma_{2}} \sum_{k=1}^{\infty} k\left(C_{k}-C_{k-1}\right)-\frac{\left(b \gamma_{1}+1\right)}{\gamma_{1}} \sum_{k=1}^{\infty} k\left(D_{k}-D_{k-1}\right)\right]\right. \\
& -(1-\rho)^{2} N_{1} N_{2}\left[\frac{\left(b \theta_{2}+1\right)\left(b \gamma_{2}+1\right)}{\theta_{2} \gamma_{2}} \sum_{k=1}^{\infty} k\left(A_{k} C_{k}-A_{k-1} C_{k-1}\right)-\frac{\left(b \theta_{2}+1\right)\left(b \gamma_{1}+1\right)}{\theta_{2} \gamma_{1}} \sum_{k=1}^{\infty} k\left(A_{k} D_{k}-A_{k-1} D_{k-1}\right)\right]
\end{aligned}
$$

$$
\begin{equation*}
\left.-(1-\rho)^{2} N_{1} N_{2}\left[\left[\frac{\left(b \theta_{1}+1\right)\left(b \gamma_{2}+1\right)}{\theta_{1} \gamma_{2}} \sum_{k=1}^{\infty} k\left(B_{k} C_{k}-B_{k-1} C_{k-1}\right)+\frac{\left(b \theta_{1}+1\right)\left(b \gamma_{1}+1\right)}{\theta_{1} \gamma_{1}} \sum_{k=1}^{\infty} k\left(B_{k} D_{k}-B_{k-1} D_{k-1}\right)\right]\right]\right\} \tag{11}
\end{equation*}
$$

Variance of time to recruitment can be computed from (10) and (11).
We now determine variance of time to recruitment for two different cases on inter-policy decision times.
Case(i): $\left\{U_{k}\right\}_{k=1}^{\infty}$ form a geometric process with rate $c,(c>0)$. The distribution $F($.$) of U_{1}$ is $F(t)=1-\left[p_{1} e^{-\lambda_{1} t}+\left(1-p_{1}\right) e^{-\lambda_{2} t}\right], \lambda_{1}, \lambda_{2}>0$, where $p_{1}$ and $\left(1-p_{1}\right)$ are proportions of policy decisions with high and low rates of attritions $\lambda_{1}$ and $\lambda_{2}$ respectively.
It can be shown that the distribution function $\mathrm{G}($.$) of the inter-exit times \mathrm{W}$ satisfy the relation
$G(x)=q \sum_{n=1}^{\infty}(1-q)^{n-1} F_{n}(\mathrm{x})$.
Therefore $\bar{g}(s)=q \sum_{n=1}^{\infty}(1-q)^{n-1} \overline{f_{n}}(s)$, where $\overline{f_{n}}(s)=\prod_{k=1}^{n} \bar{f}\left(\frac{s}{c^{k-1}}\right)$
From (13), we get

$$
\begin{equation*}
\bar{g}^{\prime}(0)=\frac{c}{(c-1+q)} \bar{f}^{\prime}(0), \text { where } \bar{f}^{\prime}(0)=-\left(\frac{p_{1}}{\lambda_{1}}+\frac{1-p_{1}}{\lambda_{2}}\right) \tag{14}
\end{equation*}
$$

From (13) and on simplification, we get

$$
\begin{equation*}
\bar{g}^{\prime \prime}(0)=\frac{c^{2}}{\left(c^{2}-1+q\right)} \bar{f}^{\prime \prime}(0)+\frac{2 c^{2}(1-q)}{\left(c^{2}-1+q\right)(c-1+q)}\left(\bar{f}^{\prime}(0)\right)^{2}, \text { where } \bar{f}^{\prime \prime}(0)=2\left(\frac{p_{1}}{\lambda_{1}^{2}}+\frac{1-p_{1}}{\lambda_{2}^{2}}\right) \tag{15}
\end{equation*}
$$

Equations (10), (11) together with (14) and (15) give the mean and variance of the time to recruitment for case (i).

Case (ii): $\left\{U_{k}\right\}_{k=1}^{\infty}$ form an order statistics where the sample of size $r$ associated with this order statistics is selected from a hyper-exponential population of independent and identically distributed inter-policy decision times, where the common distribution $F($.$) is given as in case(i).$
Let $F_{u(j)}($.$) and f_{u(j)}$ (.) be the distribution and the probability density function of the $j^{\text {th }}$ order statistic selected from the sample of size $r$ from the exponential population $\left\{\boldsymbol{U}_{k}\right\}_{k=1}^{\infty}$. From the theory of order statistics [12], it is known that
$f_{u(j)}(t)=j\binom{r}{j}[F(t)]^{j-1} f(t)[1-F(t)]^{r-j}, j=1,2, \ldots, r$.
Suppose $f(t)=f_{u(1)}(t)$
From (12) and (13), we get
$\bar{g}^{\prime}(0)=\frac{1}{q} \overline{f_{u(1)}}(0)$
$\bar{g}^{\prime \prime}(0)=\frac{1}{q^{2}}\left[q \overline{f_{u(1)}}{ }^{\prime}(0)+2(1-q)\left(\overline{f_{u(1)}}(0)\right)^{2}\right]$
${\overline{f_{u(1)}}}^{\prime}(0)=-A_{r}$ where $A_{r}=\sum_{n=0}^{r} \frac{{ }^{r} C_{n} p_{1}{ }^{n}\left(1-p_{1}\right)^{r-n}}{\left[\lambda_{1} n+(r-n) \lambda_{2}\right]}$
$\overline{f_{u(1)}}{ }^{\prime \prime}(0)=2 B_{r}$, where $B_{r}=\sum_{n=0}^{r} \frac{{ }^{r} C_{n} p_{1}{ }^{n}\left(1-p_{1}\right)^{r-n}}{\left[\lambda_{1} n+(r-n) \lambda_{2}\right]^{2}}$
Equations (10), (11) together with (17), (18),(19) and (20) give the mean and variance of the time to recruitment when $f(t)=f_{u(1)}(t)$.

## Suppose $f(t)=f_{u(r)}(t)$.

From (12) and (13), we get

$$
\begin{equation*}
\bar{g}^{\prime}(0)=\frac{1}{q} \overline{f_{u(r)}}(0) \tag{21}
\end{equation*}
$$

$$
\begin{equation*}
\bar{g}^{\prime \prime}(0)=\frac{1}{q^{2}}\left[q \overline{f_{u(r)}}{ }^{\prime \prime}(0)+2(1-q)\left(\overline{f_{u(r)}}(0)\right)^{2}\right] \tag{22}
\end{equation*}
$$

$$
\begin{equation*}
\overline{f_{u(r)}}(0)=C_{r} \text {, where } C_{r}=\sum_{n=0}^{r} \sum_{n_{1}=0}^{r-n} \frac{{ }^{r} C_{n}(-1)^{r-n(r-n)} C_{n_{1}} p_{1}^{n_{1}}\left(1-p_{1}\right)^{r-n-n_{1}}}{\left[\lambda_{1} n_{1}+\left(r-n-n_{1}\right) \lambda_{2}\right]} \tag{23}
\end{equation*}
$$

$$
\begin{equation*}
\overline{f_{u(r)}}{ }^{\prime \prime}(0)=-2 D_{r} \text {, where } D_{r}=\sum_{n=0}^{r} \sum_{n_{1}=0}^{r-n} \frac{{ }^{r} C_{n}(-1)^{r-n(r-n)} C_{n_{1}} p_{1}^{n_{1}}\left(1-p_{1}\right)^{r-n-n_{1}}}{\left[\lambda_{1} n_{1}+\left(r-n-n_{1}\right) \lambda_{2}\right]^{2}} \tag{24}
\end{equation*}
$$

Equations (10), (11) together with (21), (22), (23) and (24) give the mean and variance of the time to recruitment when $f(t)=f_{u(r)}(t)$.

## Remark:

Variance of time to recruitment for the case when $\left\{U_{k}\right\}_{k=1}^{\infty}$ is a sequence of independent and identically distributed hyper exponential random variables can be obtained by taking $c=1$ in case (i).

## 4. FINDINGS

From the above results, the following observations are presented which agree with reality:

1. When $\alpha$ increases and keeping all the other parameters fixed, the average wastage increases. Therefore the mean and variance of time to recruitment increase.
2. As $\lambda$ increases, on the average, the inter-decision time decreases and consequently the mean and variance of time to recruitment decrease when the other parameters are fixed.
3. The mean and variance of the time to recruitment decrease or increase according as $c>1$ or $c<1$, since the geometric process of inter-policy decision times is stochastically decreasing when $c>1$ and increasing when $c<1$.

## 5. CONCLUSION

The models discussed in this paper are new in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points (iii) considering two types of policy decisions, one with high rate of attrition and the other having low rate of attrition and (iv) provision of optional and mandatory thresholds with two components, a normal component due to attrition and a second component due to frequent breaks. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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