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# Analysis of Degraded Machining System with Standby Switching Failures

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**Abstract**

This paper studied the repairable system with M operating machines, S warm standby machines and R heterogeneous servers where warm standby machines have switching failure probability q. Once all spares are utilized, system starts working under degraded mode due to load sharing. It is assumed that failure and repair times of the machines follow the exponential distribution. Steady state probability is evaluated using successive over relaxation (SOR) technique. A cost model is developed in order to obtain the optimum number of repairmen and spare machines maintaining the system availability at minimum specified level. We performed sensitivity analysis under optimum operating conditions.



***Keywords:***

Degraded failure;
Switching failures;
Common cause failure;
Repairable system;
Sensitivity analysis.

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## 1. Introduction

In this paper, we deal with machine repair problem with the warm-standby and standby switching failures, where group of identical machines (operating or spare machines) are operated by one or more repairmen as availability at repair facilities. There is a possibility of failures during the switching of a machine from standby state to operating state. Therefore, we may also consider the possibility that a failure probability is associated with switching device. Chien (2010) presented a model for preventive replacement of failure. By this model, we obtain the optimal number of minimal repairs before ordering spares which is help in minimizing the system cost. The machine repair problem with spares has been studied by several researchers. Recently, Jain and Preeti (2014), Shree et al. (2015), Jharotia and Sharma (2017), Shekhar et al. (2018) etc. have studied in this area. Once all the spares are utilized, the system starts working under stress in degraded mode due to load sharing. Jain et al. (2012) provided the queueing analysis of multi-component machining system. Who have incorporated the concept of standby switching failure and used successive over relaxation (SOR) technique. Recently machine repairing systems with standby switching failure studied by Jain et al. (2014), Ke et al. (2016), Shekhar et al. (2017) etc.. Best of our knowledge, analysis of degrading machining system with standby switching failures has never been investigated. This model reduces the system cost with respect to above models. The main purpose of this paper is threefold.

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- Firstly, we solve steady state equations of our model using computer software MATLAB.
- Secondly, we have been developed a cost model to determine optimum number of repairmen and spare machines, maintaining the system availability at minimum specified level.
- Thirdly, we performed sensitivity analysis to study the effect of system parameters on the optimum cost.

Spare support is most important factor for smooth running of any machining system and avoids loss of production. The provision of spares may be helpful in improving the system reliability/availability. The machine repair problem with spares has been studied by several researchers since long including Gupta and Rao (1996), Sharma (2012), Jain and Preeti (2014), Shree et al. (2015), Jharotia and Sharma (2015, 2017) etc. Once all the spares are utilized, the system starts working under stress in degraded mode due to load sharing. Jain and Upadhyaya (2009) studied the degraded machining system with multiple vacations and multiple type spares under threshold N-policy. Kumar and Jain (2013) provided a threshold N-policy for degraded machining system with standby switching failure.

Major impact on availability and reliability of repairable system are affected by the common cause failures. Several situations like as humidity, temperature, shock voltage fluctuation etc. that prevail in many machining system. Research works in this area have been done by Hughes (1987), Kvam and Miller (2002), Jain and Mishra (2006), Dai and Wang (2007) etc. studied the common cause failures on machine repair problem. Jain et al. (2013) studied the machine repair problem with common cause failures and switching failure. Jharotia and Sharma (2016) also discussed the common cause failures in machining system.

In machine repair problems, most of researchers assumed that the switchover from standby machines to operating machines is always perfect. Although these assumptions simplify the analysis of the problem, they might not reflect certain real situations. Possibility of failures, during the switching machine is always considered. Concept of switching failure in the reliability with standby system was first introduced by Lewis (1996). Kuo and Zuo (2003) studied the various type of system reliability models such as series, parallel, standby, multistate system etc. Wang et al. (2006) extended Sivazlian and Wang (1989) model to consider standbys switching failures. Hsu et al. (2011) studied the standby system with general repair, reboot delay and switching failure using statistical inference procedures. Hsu et al. (2014) extend previous model for profit analysis. Shekhar et al. (2014) provided the fuzzy analysis of machine repair problem with switching failure and reboot.

### 1.1. Model description

We have considered a machine repair problem having M operating machines, S standby machines and R repairmen in repair facilities. The following assumptions are taken into consideration for the mathematical formulation of the model:

- The operating machines as well as spare machines fails according to exponential distribution with rate $\lambda$ and $\alpha$ $(0 < \alpha < \lambda)$ respectively.
- The machining system may also fail due to some common cause in Poisson fashion with rate $\lambda_c$.
- When all spares are exhausted, and new machine fails, the system works under stress in degraded mode with degraded failure rate $\lambda_d$ .
- As soon as operating machine fails, it is replaced by an available standby machine and immediately sent for repair to repair facility, where it is repaired in order of breakdowns with repair rate $\mu$. The switch-over time is instantaneous.
- It is also assumed that when a spare machine move into an operating state, its failure characteristics will be same as an operating machine.
- It is also assumed that there is always a possibility of failures during switching of machine from standby state to operating state means that switching may not perfect. So that the switching device has a failure probability $q$. If a spare machine fails to switch to operating state, the next available spare attempts to switch. This process is continues till successful switching or all the spares have failed.
- Once a failed machine is repaired, it is good as new one and goes into standby state or operating state. It may goes into operating state, if system is short (less than required operating machines).

## 2. Research Method

For the machine repairable system with standby switching failures, we describe the states of the system by $\{P(i): i = 0, 1, \ldots, M + S\}$, where $i$ denotes the number of failed machines including the operating and spare in the system.



*Figure 1: State transition rate diagram*

The combination of failure rate and service rate is given as -

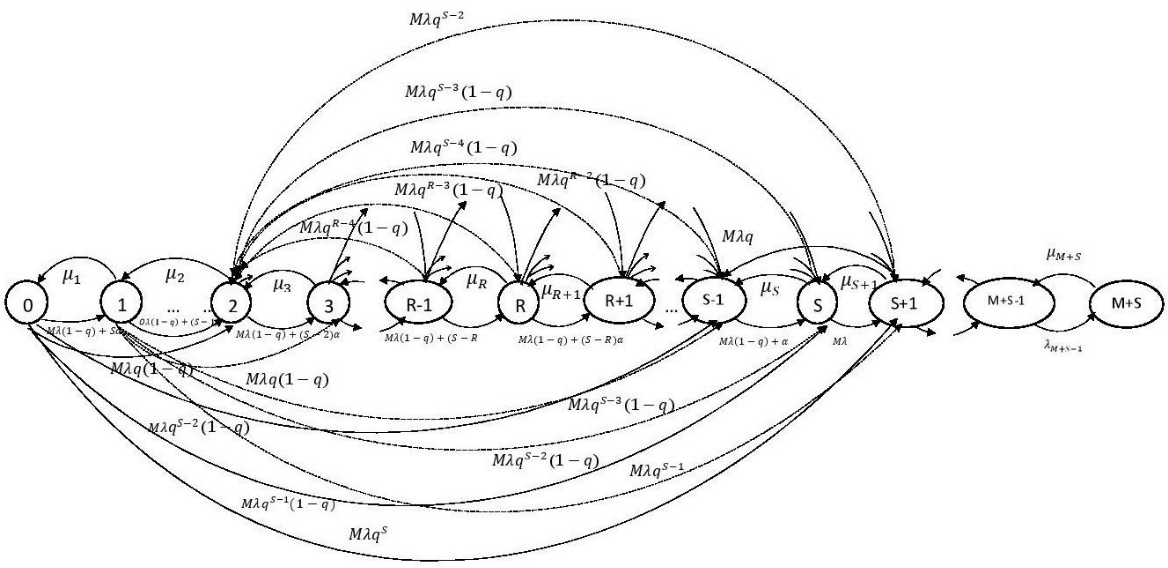
$$\lambda_i = \begin{cases} M\lambda + (S-i)\alpha + \lambda_c, & 0 \le i \le S \\ (M+S-i)\lambda_d, & S+1 \le i \le M+S-1 \\ 0, & \text{otherwise} \end{cases}$$

The mean repair rate $\mu_i$ is given as

$$\mu_i = \begin{cases} i\mu, & if\ 1 \le i < R \\ R\mu, & if\ R+1 \le i \le M+S \\ 0, & \text{otherwise} \end{cases}$$

In steady-state, let

$P(i) \equiv$ Probability that there are $i$ failed machines in the system, where $i = 0, 1, \ldots, M + S$.




Referring to the state-transition-rate diagram for the machine repair problem with standby switching failures shown in Fig. 1 lead to the following steady-state equations:

$$\lambda_0 P(0) = \mu_1 P(1) \tag{1}$$

$$(\lambda_1 + \mu_1)P(1) = [M\lambda(1-q) + S\alpha]P(0) + \mu_2 P(2) \tag{2}$$

$$(\lambda_i + \mu_i)P(i) = [M\lambda(1-q) + (S-i+1)\alpha]P(i-1) + \mu_{i+1}P(i+1) + \sum_{n=0}^{i-2} M\lambda q^{i-n-1}(1-q)P(n), \quad 2 \le i \le S \tag{3}$$

$$(\lambda_{S+1} + \mu_{S+1})P(S+1) = \lambda_S P(S) + \textstyle\sum_{n=0}^{S-1} M\lambda q^{S-n}(1-q)P(n) + \mu_{S+2}P(S+2) \qquad i = S+1 \tag{4}$$

$$(\lambda_i + \mu_i)P(i) = \lambda_{i-1}P(i-1) + \mu_{i+1}P(i+1), \qquad S+2 \le i \le M+S-1 \tag{5}$$

$$\mu_{M+S}P(M+S) = \lambda_{M+S-1}P(M+S-1) \qquad i = M+S \tag{6}$$

Eqs. (1) – (6) cannot be solved recursively to obtain closed-form expressions for the $P(i)$. The steady-state solutions

$P(i)(i = 0, 1, \ldots, M + S)$ always exist because the number of states is finite. With the normalizing condition,

$$\sum_{i=0}^{M+S} P(i) = 1 \tag{7}$$

The steady-state equations of our model can be represented in form of $Ax = b$. The linear system of equations has been solved by SOR technique with parameter value $\omega = 1.25$ in MATLAB 8.1.

### 2.1. Queuing and reliability measure

Performance measures of our model defined as follows:

- The expected number of failed machines in the system is given as
  $L_f = \sum_{i=1}^{M+S} i\, P(i)$ (8)
- The expected number of operating units in the system is given as
  $E[O] = M - \sum_{i=1}^{M} i\, P(S+i)$ (9)
- The expected number of spare units in the system is given as
  $E[S] = \sum_{i=0}^{S} (S-i)\, P(i)$ (10)
- The expected number of idle servers in the system is given as
  $E[I] = \sum_{i=0}^{R-1} (R-i)\, P(i)$ (11)
- The expected number of busy servers in the system is given as
  $E[B] = R - E[I]$ (12)
- The expected switching failure rate is given as
  $S.R. = \sum_{i=1}^{S} M\, \lambda q P(i-1)$ (13)

- The machine availability of the system is given as
  $M.A. = 1 - \frac{L_f}{M+S}$ (14)
- The fraction of busy servers (operative utilization) is given as
  $O.U. = \frac{E[B]}{R}$ (15)
- The steady-state availability of the system is given as
  $A_v = \sum_{i=0}^{S} P(i)$ (16)

### 2.2. Cost model

In this section, we construct a steady-state expected cost function and impose a constraint on the availability of system in which R and S are decision variables (discrete). Our main objective is to determine the optimum value of R and S, say R* and S*, so as minimize this function to maintain the system availability at minimum specified level. Cost parameters are defined as -

$C_O \equiv$ Cost per unit time when one of machine is in an operating condition,

$C_S \equiv$ Cost per unit time when one machine is acting as a standby,

$C_I \equiv$ Cost per unit time when one of repairman is idle,

$C_B \equiv$ Cost per unit time when one of repairman is busy,

$C_{SF} \equiv$ Cost per unit time when one of standby machine has switching failure,

The optimum value $(R^*, S^*)$ solved by cost minimization problem which can be stated as follows –

$$\text{Min}\, F(R,S) = C_O E[O] + C_S E[S] + C_I E[I] + C_B E[B] + C_{SF} S.R$$

$$s.t.\ A_v \geq A_0$$

Where, $A_v$ is the steady state availability of the system defined by Eq. (16) in performance measures and $A_0$ is the pre-specified level of system availability.

## 3. Results and Analysis

Direct search method (heuristic approach) has used to find the optimum value of repairmen and spares. Due to discrete property of R and S, we use direct substitution of successive values of R and S into the cost function until the optimum value of $F(R, S)$, say $F(R^*, S*)$ is achieved and constraint is satisfied.

### 3.1. *To find optimum value of spares and repairmen (S*,R*)*

A Numerical illustration is provided by considering the following cost parameters:

$$C_O = 200, C_S = 100, \qquad C_I = 80, C_B = 125, C_{SF} = 100.$$

For computation purpose, initially, we fixed number of operating machine M = 10 and choose parameter values $A_0 = 0.9, \lambda = 0.3, \alpha = 0.2, \mu = 1.0, \lambda_c = 0.15, \lambda_d = 0.6, q = 0.2.$

The expected profit of the system $F(R, S)$ and system availability $A_v$ are presented by table 1 for different value of R and S. This table shows that the minimum expected cost per day (426.06) and system availability $A_v = .904$ is obtained at $R^* = 5$ and $S^* = 6$.

### 3.2. *Effect of system parameters on optimum cost*

In Fig. 2, given below, we drawn the three curves corresponding to $\alpha$ = 0.1, 0.2, 0.3 respectively. This figure shows that optimum cost increase significantly as $\alpha$ increases and increasing the failure rate of operating units ($\lambda$). In Fig. 3, we drawn the three curves corresponding to $\alpha$ = 0.2, 0.3, 0.4, respectively. This figure shows that the optimum cost increase as increases $\alpha$ and common cause failure rate ($\lambda_c$). In Fig. 4, we drown the three curves corresponding to $\alpha$ = 0.2, 0.4, 0.6, respectively. This figure shows that optimum cost decrease till $\lambda_d < 0.6$ but from $\lambda_d = 0.6$ it almost constant while optimum cost decrease is decrease on increasing of ($\alpha$). In Fig. 5, we drown the three curves corresponding to $\lambda$ = 0.5, 1.0, 1.5, respectively. This figure shows that the optimum cost increase as increases failure rate of spare units ($\alpha$) whereas it increases slowly by increasing the $\lambda$. In Fig. 6, we drown the three curves corresponding $\alpha$ = 0.2, 0.4, 0.6, respectively. This figure shows that the optimum cost increase significantly as $\mu$ increases whereas it increases as increasing $\alpha$. Optimum costs are similar at $\mu = 1$ for all three curves.

***Table 1:*** The expected cost $F(R, S)$ and system availability $A_v$ under optimum operating condition

| R \ S | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 2 | 50.31<br>0.096 | 50.34<br>0.187 | 50.37<br>0.312 | 50.40<br>0.429 | 50.42<br>0.506 | 50.45<br>0.548 | 50.49<br>0.600 |
| 3 | 175.48<br>0.105 | 175.52<br>0.248 | 175.56<br>0.332 | 175.60<br>0.503 | 175.64<br>0.623 | 175.68<br>0.734 | 175.72<br>0.753 |
| 4 | 300.64<br>0.121 | 300.69<br>0.278 | 300.74<br>0.592 | 300.80<br>0.627 | 300.85<br>0.894 | 300.90<br>0.924 | 300.96<br>0.959 |
| 5 | 425.80<br>0.150 | 425.86<br>0.343 | 425.93<br>0.603 | 426.00<br>0.833 | **426.06**<br>**0.904** | 426.13<br>0.944 | 426.23<br>0.962 |
| 6 | 550.96<br>0.161 | 551.04<br>0.328 | 551.12<br>0.742 | 551.20<br>0.88 | 551.28<br>0.944 | 551.36<br>0.985 | 551.47<br>0.992 |
| 7 | 676.16<br>0.142 | 676.21<br>0.417 | 676.30<br>0.632 | 676.40<br>0.873 | 676.49<br>0.932 | 676.61<br>0.989 | 676.68<br>0.998 |

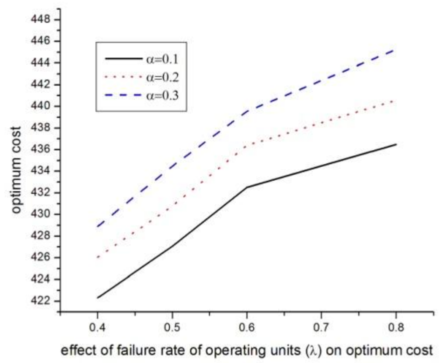



*Figure 2: Effect of failure rate of operating units on optimum cost*

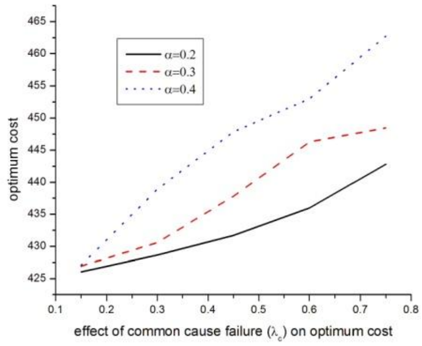



*Figure 3: Effect of common cause failure rate on optimum cost*

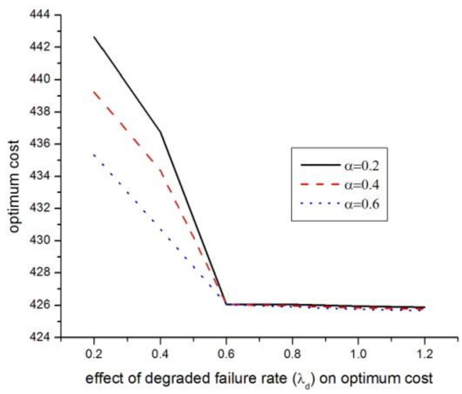



*Figure 4: Effect of degraded failure rate on optimum cost*

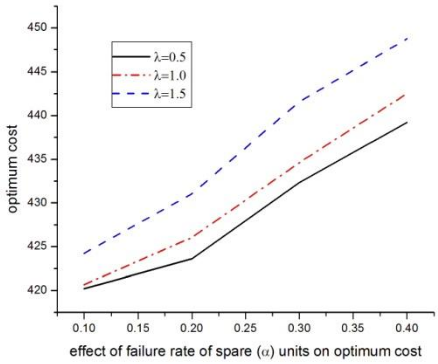



*Figure 5: Effect of failure rate of spare on optimum cost*

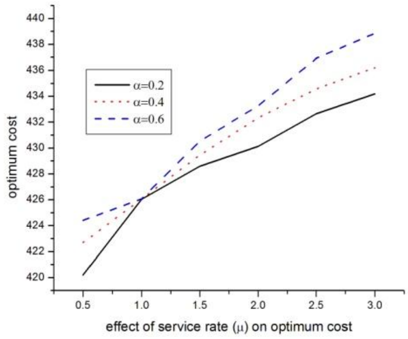



*Figure 6: Effect of service rate on optimum cost*

## 4. Conclusion

In this paper, we studied the machine repair problem to analysis the performance and also perform sensitivity analysis. The concept of common cause and degraded failure rates leads our model to deal with embedded engineering systems including electronics/ electrical and computer and communication systems etc. The optimum number of repairmen and spare machine are determined with the help of a cost model, to ensure the system availability its minimum level ($A_0 = 0.9$).
Numerical experiment may helpful to explore the effects of parameters on performance measures. This model may be applied in production and manufacturing industries and helps to decision makers to design an effective system.

### References


[1] Chien, Y. H., "Optimal number of minimal repairs before ordering spares for preventive replacement," *Applied Mathematical Modelling*, Vol. 34, No. 11, pp. 3439-3450, 2010.

[2] Dai, Y. S., Levitin, G. and Wang, X., "Optimal task partition and distribution in grid service system with common cause failure," *Future Generation Computer Systems*, Vol. 23, No. 2, pp. 209-218, 2007.

[3] Gupta, U. C. and Srinivasa Rao, T. S. S., "On the M/G/1 machine interference model with spares," *European Journal of Operational Research*, Vol. 89, No. 1, pp. 164-171, 1996.

[4] Hughes, R. P., "A new approach to common cause failure," *Reliability engineering & System Safety*, Vol. 17, No. 1, pp. 211-236, 1987.

[5] Hsu, Y. L., Ke, J. C. and Liu, T. H., "Standby system with general repair, reboot delay, switching failure and unreliable repair facility – A statistical standpoint," *Mathematics and Computers in Simulation*, Vol. 81, No. 11, pp. 2400-2413, 2011.

[6] Hsu, Y. L., Ke, J. C., Liu, T. H. and Wu, C. H., "Modelling of multi server repair problem with switching failure and reboot delay and related profit analysis", *Computer & Industrial Engineering*, Vol. 69, No. 3, pp. 21-28, 2014.

[7] Jain, M. and Mishra, A., "Multistage degraded machining system with common cause shock failure and state dependent rates," *Journal of Rajasthan Academy of Physical Sciences*, Vol. 5, No. 3, pp. 251-262, 2006.

[8] Jain, M. and Upadhyaya, S., "Threshold N-policy for degraded machining system with multiple types of spares and multiple vacations," *Quality Technology Quantitative Management*, Vol. 6, No. 2, pp. 185-203, 2009.

[9] Jain, M., Shekhar, C. and Shukla, S., "Queueing analysis of multi component machining system having heterogeneous server and impatient customers," *American Journal of Operation Research*, Vol. 2, No. 3, pp. 16-26, 2012.

[10] Jain, M., Shekhar, C. and Shukla, S., "Queueing analysis of two unreliable servers machining system with switching and common cause failure," *International Journal of Mathematics in Operational Research* , Vol. 5, No. 4, pp. 508-536, 2013.

[11] Jain M. and Preeti, "Cost Analysis of a Machine Repair Problem with Standby, Working Vacation and Server breakdown," *Int. J. of Mathematics in Operation Research*, Vol. 6, No. 4, pp. 437-451, 2014.

[12] Jain, M., Shekhar, C. and Shukla, S., "Markov model for switching failure of warm spares in machine repair system," *Journal of Reliability and Statistical Studies*, Vol. 7, special issue, pp. 57-68, 2014.

[13] Jharotia, P. and Sharma, D. C., "Machine repair problem with spare and three repairmen with partial server vacation policy," *International Journal of Engineering, Management Sciences*, Vol. 2, No. 3, pp. 44-49, 2015.

[14] Jharotia, P. and Sharma D. C., "Mathematical Analysis of Machine repair problem with common cause failure, hot spares and multiple repairmen," *Asian Journal of Multidisciplinary Studies*, Vol. 4, No. 2, pp. 43-51, 2016.

[15] Jharotia, P. and Sharma D. C., "Mathematical analysis of machining system with warm spares and variable service rate," *Int. J. Process Management and Benchmarking*, Vol. 7, No. 2, pp. 141-157, 2017.

[16] Ke, J. C., Liu, T. H. and Yang, D. Y., "Machine repairing systems with standby switching failure," *Computer & Industrial Engineering*, Vol. 99, No. 3, pp. 223-228, 2016.

[17] Kuo, W. and Zuo, M. J., "*Optimum reliability modeling: principles and applications*," Wiley, New York, 2003.

[18] Kumar, K. and Jain, M., "Threshold N-policy for (M, m) degraded machining system with K-heterogeneous servers, standby switching failure and multiple vacations," *International Journal of Mathematics in Operational Research* , Vol. 5, No. 4, pp. 423-445, 2013.

[19] Kvam, P. H. and Miller, J. G., "Common cause failure predication using data mapping," *Reliability engineering & System Safety*, Vol. 76, No. 3, pp. 273-278, 2002.

[20] Lewis, E. E., "*Introduction to reliability engineering*," 2nd Ed., Wiley, New York, 1996.

[21] Wang, K. H., Ke, J. B. and Ke, J. C., "Profit analysis of M/M/R machine repair problem with balking, reneging and standby switching failure," *Computer and Operation Research*, Vol. 34, No. 3, pp. 835-847, 2005.

[22] Wang, K. H., Dong, W. L. and Ke, J. B., "Comparing of reliability and the availability between four systems with warm standby, switching failures," *Applied Mathematics and Computation*, Vol. 183, No. 2, pp. 1310-1322, 2006.

[23] Sharma, D. C., "Machine repair problem with spares and N-policy vacation," *Research Journal of Recent Sciences*, Vol. 1, No. 4, pp. 72-78, 2012.

[24] Shree, L., Singh, P., Sharma, D. C. and Jharotia, P., "Mathematical modeling and performance analysis of machine repairable system with hot spares," *Proceedings of the National Academy of Sciences, India Section A: Physical Science*, Vol. 85, No. 1, pp. 127-135, 2015.

[25] Shekhar, C., Jain, M. and Bhatia, S., "Fuzzy analysis of machine repair problem with switching failure and reboot," *Journal of Reliability and Statistical Studies*, Vol. 7, Special issue, pp. 41-55, 2014.

[26] Shekhar, C., Raina, A.A., Jain, M. and Mishra, R. P., "Sensitivity analysis of repairable redundant system with switching failure and geometric reneging," *Decision Science Letters,* Vol.6, No. 1, 337-350, 2017.

[27] Shekhar, C., Jain, M. and Raina, A.A., "Transient analysis of machining system with spares provisioning and geometric reneging," *International Journal of Mathematics in Operational Research*, Vol.11, No. 1, xxxx (In press), 2017. DOI:10.1504/IJMOR.2017.10002701

[28] Sivazlian, B. D. and Wang, K. H., "Economic analysis of the M/M/R machine repair problem with warm standbys," *Microelectronic Reliability*, Vol. 29, No. 1, pp. 25-35, 1989.