FUZZY SOFT SUPER MATRIX

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ABSTRACT

The purpose of this paper is to put forward the notion soft super matrix theory[Based on [11]W.B.Vasantha Kandasami , Florentin Smarandche, K. Amal "Super Fuzzy Matrices and Super Fuzzy Models for Social Scientists", 2008] and some basic properties. In this paper, we define fuzzy soft super matrix and some new definitions based on fuzzy soft super matrix . Some properties are given with appropriate examples .Lastly, minor and major product of two fuzzy soft super matrices are defined .

Key words – Soft set, soft matrix , soft super matrices , fuzzy soft super matrices.

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1. INTRODUCTION

Soft set theory was firstly introduced by Molodsov in 1999[1] as a general mathematical tool for dealing with uncertain , fuzzy , not clearly defined objects. In classical mathematics, all mathematical tools for modeling , reasoning and computing are crisp, deterministic and precise in character .So , they can't solve those complex problems in real life situations. In recent years, researchers are interested in dealing with the complexity of uncertain data. There are wide range of theories such as probability theory , fuzzy set theory , vague set theory etc. which have inherent difficulties due to lack of parameter as pointed out by Molodsov[1] . Soft set theory is a novel concept having parameterization tools dealing successfully with uncertainties. Maji et al.[2] introduced the concept of fuzzy soft sets by combining soft sets and fuzzy sets. Maji et al.[3,4] introduced soft set into decision making problems . Cagman and Enginoglu [5] defined soft matrices which were matrix representation of the soft sets and constructed a soft max-min decision making method. Cagman and Enginoglu [6] defined fuzzy soft matrices and constructed a decision making problem. Borah et al.[7] extended fuzzy soft matrix theory and its application. Maji and Roy [8] presented a novel method of object from an imprecise multi-observer data to deal with decision making based on fuzzy soft sets. Majumdar and Samanta[9] generalized the concept of fuzzy soft sets. M.J.I.Mondal and T.K.Roy[10] introduced some operators of fuzzy soft matrix and decision making on the basis of t-norm operators .

2. Soft Matrices

2.1 Soft Set ([1]). Let U be an initial universe , P(U) be the power set of U , E be the set of all parameters and A⊆ E. A soft set (f_A,E) on the universe U is defined by the set of order pairs

(f_A,E) = {(e , f_A(e)) : e∈ E , f_A(e) ∈ P(U) }

where f_A : E → P(U) such that f_A(e) = ϕ if e ∉ A.

Here f_A is called an approximate function of the soft set (f_A,E). The set f_A(e) is called e-approximate value set or e-approximate set which consists of related objects of the parameter e ∈ E .

2.2 Soft Matrix[2] Let (f_A,E) be a soft set over U. Then a subset of U x E is uniquely defined by

R_A = { (u , e) : e∈ A , u ∈ f_A(e) }

which is called relation form of (f_A,E) . The characteristic function of R_A is written by

χ_{R_A} : U x E → [0 , 1] ,

$$\chi_{R_A}(u,e) = \begin{cases} 1, & (u,e) \in R_A \\ 0, & (u,e) \notin R_A \end{cases}$$

If U = { u_1 , u_2 , …………u_m } , E = { e_1 , e_2 , …………e_n } and A ⊆ E , then R_A can be represented by a table in the following form :

R_A	e_1	e_2	…………	e_n
u_1	$\chi_{R_A}(u_1,e_1)$	$\chi_{R_A}(u_1,e_2)$	…………	$\chi_{R_A}(u_1,e_n)$
u_2	$\chi_{R_A}(u_2,e_1)$	$\chi_{R_A}(u_2,e_2)$	…………	$\chi_{R_A}(u_1,e_n)$

………….				
u_m	$\chi_{R_A}(u_m,e_1)$	$\chi_{R_A}(u_m,e_2)$	………….	$\chi_{R_A}(u_m,e_n)$

If $a_{ij} = \chi_{R_A}(u_i,e_j)$, we can define a matrix

$$[a_{ij}]_{mXn} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{21} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

which is called an m x n soft matrix of the soft set f_A,E) over U.

According to the definition a soft set (f_A,E) is uniquely characterized by the matrix $[a_{ij}]_{m\,X\,n}$. It means that a soft set (f_A,E) is formally equal to its soft matrix $[a_{ij}]_{m\,X\,n}$ .Therefore , we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

The set of all m x n soft matrices over U will be denoted by SM$_{m\,X\,n}$. From now we shall use $[a_{ij}]$ in place of $[a_{ij}]_{m\,X\,n}$,by omitting the subscripts m x n of $[a_{ij}]_{m\,X\,n}$ .Since $[a_{ij}] \in$ SM$_{m\,X\,n}$, it means that $[a_{ij}]$ is an m x n soft matrix for I = 1,2,…..n and j = 1,2,……..m.

Example1. Assume that U = { u_1 , u_2 , u_3 , u_4 , u_5 } is a universal set and E = { e_1 , e_2 , e_3 , e_4 } is a set of all parameters. If A ={e_1 , e_2 , e_3} and $f_A(e_1)$ = { u_1 , u_3 } , $f_A(e_1) = \phi$,

$f_A(e_3)$= U , then we write a soft set

(f_A,E) = {(e_1, { u_1 , u_3 }), (e_3,U} and then the relation form of (f_A,E) is written by

R_A ={ (u_1, e_1), (u_3, e_1), (u_1, e_3), (u_2, e_3), (u_3, e_3), (u_4, e_3), (u_5, e_3) }

Hence the soft matrix $[a_{ij}]$ is written as

$$[a_{ij}] = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

2.3. Soft Super Matrix (SSM) [11 : A soft super matrix A = $[a_{ij}]$ is a matrix whose elements are themselves matrices with elements that can either be 0 or 1 or other matrices .

2.4 : Soft Super Sub Matrix : Soft super sub matrices are those matrices considered as elements of soft super matrix .

Example2. From the example 1 , we can write

$$[a_{ij}] = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \end{bmatrix} \text{, where } a_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} , a_{12} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$a_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} , \quad a_{22} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

Here , $[a_{ij}]$ is a soft super matrix and a_{11} , a_{12} , a_{21} , a_{22} are soft super sub matrices of $[a_{ij}]$.

2.5 Order of Soft Super Matrix: Order of soft super matrix is nothing but the order of simple matrix from which it is obtained.

2.6 [3] Soft Partitioned matrix: A soft matrix can be partitioned by dividing or separating the matrix between certain specified rows or certain specified columns . The matrix so obtained is known as Soft Partitioned Matrix

2.7 Zero Soft Super Matrix : Let $[a_{ij}] \in \mathrm{SSM}_{m\,X\,n}$. Then $[a_{ij}]$ is called a zero soft super matrix denoted by [0], if a_{ij} = 0 for all i and j.

2.8 Universal Soft Super Matrix : Let $[a_{ij}] \in \mathrm{SSM}_{m\,X\,n}$. Then $[a_{ij}]$ is called a universal soft super matrix denoted by [1] , if a_{ij} = 1 for all i and j .

Example3. Assume that U = { u_1 , u_2 , u_3 , u_4 , u_5 } is a universal set and E = { e_1 , e_2 , e_3 , e_4 } is a set of all parameters , and

$[a_{ij}]$, $[b_{ij}]$, $[c_{ij}] \in \mathrm{SM}_{5\,X\,4}$.

If A = { e_1 , e_2 } , $f_A(e_1) = \phi$, $f_A(e_2) = \phi$, then $[a_{ij}]$ = [0] is a zero matrix written by

$$[0] = \begin{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$$

If B =E and $f_B(e_i)$ = U , $e_i \in$ B ,i = 1,2 ,3 ,4 then $[b_{ij}]$ = [1] is a universal soft matrix written by

$$[1] = \begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{bmatrix}$$

2.9 Soft Super Sub Matrix : Let $[a_{ij}]$, $[b_{ij}]$ $\in \mathrm{SSM}_{m\,X\,n}$. Then $[a_{ij}]$ is called a soft super sub matrix of $[b_{ij}]$denoted by $[a_{ij}] \mathrel{\tilde{\subseteq}} [b_{ij}]$, if $[a_{ij}] \leq [b_{ij}]$ for all i and j.

3.0 Soft Super Sub Matrix : Let $[a_{ij}]$, $[b_{ij}]$ $\in \mathrm{SSM}_{m\,X\,n}$. Then $[a_{ij}]$ is said to be equal to $[b_{ij}]$ denoted by $[a_{ij}] = [b_{ij}]$, if $a_{ij} = b_{ij}$ for all i and j.

3.1 Union of Soft Super Matrices : Let $[a_{ij}]$, $[b_{ij}]$ $\in \mathrm{SSM}_{m\,X\,n}$. Then the soft super matrix $[c_{ij}]$ $\in$ $\mathrm{SSM}_{m\,X\,n}$ is called union of $[a_{ij}]$and $[b_{ij}]$ denoted by$[a_{ij}] \mathbin{\tilde{\cup}} [b_{ij}]$ if c_{ij} = max { a_{ij} , b_{ij}} for all i and j.

3.2 Intersection of Soft Super Matrices : Let $[a_{ij}]$, $[b_{ij}]$ $\in \mathrm{SSM}_{m\,X\,n}$. then the soft super matrix $[c_{ij}]$ $\in$ $\mathrm{SSM}_{m\,X\,n}$ is called intersection of $[a_{ij}]$and $[b_{ij}]$ denoted by$[a_{ij}] \mathbin{\tilde{\cap}} [b_{ij}]$ if c_{ij} = min { a_{ij} , b_{ij}} for all i and j.

3.3 Compliment of Soft Super Matrix: Let $[a_{ij}] \in \mathrm{SSM}_{m\,X\,n}$. Then $[c_{ij}]$ $\in \mathrm{SSM}_{m\,X\,n}$ is called complement of $[a_{ij}]$ denoted by $[a_{ij}]^0$, if $c_{ij} = 1 - a_{ij}$ for all i and j.

3.4 Disjoint Soft Super Matrix: Let $[a_{ij}] \in \mathrm{SSM}_{m\,X\,n}$. Then $[a_{ij}]$and $[b_{ij}]$ are disjoint if $[a_{ij}]\ \tilde{\cap}\ [b_{ij}] = [0]$ for all i and j.

$$[a_{ij}] = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}, \quad [b_{ij}] = \begin{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$$

Then $[a_{ij}]\ \tilde{\cup}\ [b_{ij}] = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$,

$[a_{ij}]\ \tilde{\cap}\ [b_{ij}] = [0]$

and $[a_{ij}]^0 = \begin{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \end{bmatrix}$

Proposition 1. Let $[a_{ij}] \in \mathrm{SSM}_{m\,X\,n}$.Then

i. $[[a_{ij}]^0]^0 = [a_{ij}]$

ii. $[0]^0 = [1]$

iii. $[a_{ij}]\ \tilde{\subseteq}\ [1]$

iv. $[0]\ \tilde{\subseteq}\ [1]$

v. $[a_{ij}]\ \tilde{\subseteq}\ [a_{ij}]$

vi. $[a_{ij}]\ \tilde{\cup}\ [a_{ij}] = [a_{ij}]$

vii. $[a_{ij}]\ \tilde{\cup}\ [0] = [a_{ij}]$

viii. $[a_{ij}]\ \tilde{\cup}\ [1] = [1]$

ix. $[a_{ij}]\ \tilde{\cap}\ [a_{ij}] = [a_{ij}]$

x. $[a_{ij}]\ \tilde{\cap}\ [0] = [0]$

xi. $[a_{ij}]\ \tilde{\cap}\ [1] = [1]$

xii. $[a_{ij}]\ \tilde{\cap}\ [a_{ij}]^0 = [0]$

Proposition 2. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}] \in \mathrm{SSM}_{m\,X\,n}$.Then

i. $[a_{ij}]\ \tilde{\subseteq}\ [b_{ij}]$ and $[b_{ij}]\ \tilde{\subseteq}\ [c_{ij}] \Rightarrow [a_{ij}]\ \tilde{\subseteq}\ [c_{ij}]$

ii. $[a_{ij}] = [b_{ij}]$ and $[b_{ij}] = [c_{ij}] \Rightarrow [a_{ij}] = [c_{ij}]$

iii. $[a_{ij}]\ \tilde{\subseteq}\ [b_{ij}]$ and $[b_{ij}]\ \tilde{\subseteq}\ [a_{ij}] \Rightarrow [a_{ij}] = [b_{ij}]$

iv. $[a_{ij}]\ \tilde{\cup}\ [b_{ij}] = [b_{ij}]\ \tilde{\cup}\ [a_{ij}]$

v. $[a_{ij}]\ \tilde{\cap}\ [b_{ij}] = [b_{ij}]\ \tilde{\cap}\ [a_{ij}]$

vi. $(\,[a_{ij}]\ \tilde{\cup}\ [b_{ij}]\,)\ \tilde{\cup}\ [c_{ij}] = [a_{ij}]\ \tilde{\cup}\ (\,[b_{ij}]\ \tilde{\cup}\ [c_{ij}]\,)$

vii. $(\,[a_{ij}]\ \tilde{\cap}\ [b_{ij}]\,)\ \tilde{\cap}\ [c_{ij}] = [a_{ij}]\ \tilde{\cap}\ (\,[b_{ij}]\ \tilde{\cap}\ [c_{ij}]\,)$

Proposition 3. Let $[a_{ij}]$, $[b_{ij}]$ $\in SSM_{m\,X\,n}$.Then De Morgan's laws are valid.

i. $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0$

ii. $([a_{ij}] \tilde{\cap} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cup} [b_{ij}]^0$

Proof. For all i and j

i. $([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [\max\{a_{ij}, b_{ij}\}]^0$

$= [1 - \max\{a_{ij}, b_{ij}\}]$

$= [\min\{1 - a_{ij}, 1 - b_{ij}\}]$

$= [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0$

ii. $([a_{ij}] \tilde{\cap} [b_{ij}])^0 = [\min\{a_{ij}, b_{ij}\}]^0$

$= [1 - \min\{a_{ij}, b_{ij}\}]$

$= [\max\{1 - a_{ij}, 1 - b_{ij}\}]$

$= [a_{ij}]^0 \tilde{\cup} [b_{ij}]^0$

Example4. Let $[a_{ij}]$, $[b_{ij}]$ $\in SM_{5\,X\,4}$ as given in example3. Then

$$([a_{ij}] \tilde{\cup} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cap} [b_{ij}]^0 = \begin{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \end{bmatrix}$$

$$([a_{ij}] \tilde{\cap} [b_{ij}])^0 = [a_{ij}]^0 \tilde{\cup} [b_{ij}]^0 = \begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{bmatrix}$$

Proposition 4. Let $[a_{ij}]$, $[b_{ij}]$, $[c_{ij}]$ $\in SSM_{m\,X\,n}$.Then

i. $[a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cap} [c_{ij}]) = ([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cap} ([a_{ij}] \tilde{\cup} [c_{ij}])$

ii. $[a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cup} [c_{ij}]) = ([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cup} ([a_{ij}] \tilde{\cap} [c_{ij}])$

3.5 Soft Super Square Matrix : Let $[a_{ij}] \in SSM_{m\,X\,n}$. Then $[a_{ij}]$ is said to be soft super square matrix if m = n for all i and j.

3.6 Soft Super Symmetric Partitioning Matrix : Let $[a_{ij}] \in SSM_{m\,X\,n}$ be a soft super square matrix .Then $[a_{ij}]$ is said to be soft super symmetric partitioning matrix if it is partitioned symmetrically between rows and columns.

Example4. Let a = $[a_{ij}] \in SSM_{4\,X\,4}$

$$a = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \hline 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

This matrix is symmetrically partitioned between second and third rows and second and third columns.

It can also be partitioned between rows one and two and rows three and four and columns one and two and columns three and four in the following way:

$$a = \left[\begin{array}{c|cc|c} 1 & 0 & 1 & 0 \\ \hline 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ \hline 0 & 1 & 0 & 1 \end{array}\right]$$

Definition 3.7. Fuzzy Set : A Fuzzy Set (FS) in the universal set U is defined as A= { (x , μ_A(x)) | x ∈ U } where μ_A : U → [0,1] is a mapping called membership function of the fuzzy set A .

Definition 3.8 Fuzzy Matrix : A Fuzzy Matrix of order m x n is defined as A= $[a_{ij}]$ where a_{ij} is the membership value of ij –th element in A .

Definition 3.9[11] Fuzzy Soft Super Matrix (FSSM) : A Fuzzy Soft Super Matrix A = $[a_{ij}]$ is a matrix whose elements are themselves matrices with elements ∈ [0 ,1] or other matrices .

Definition 3.10 Addition of Fuzzy Soft Super Matrices: Let A= $[a_{ij}]$, B= $[b_{ij}] \in FSSM_{mxn}$. Then Addition of Fuzzy Soft Super Matrices A and B is defined by A + B = [max $\{a_{ij} , b_{ij}\}$] for all i and j .

Definition 3.11 Subtraction of Fuzzy Soft Super Matrices: Let A= $[a_{ij}]$, B= $[b_{ij}] \in FSSM_{mxn}$. Then Subtraction of Fuzzy Soft Super Matrices A and B is defined by A − B = $[a_{ij} - b_{ij}]$, $a_{ij} > b_{ij}$

= $[b_{ij} - a_{ij}]$, $a_{ij} < b_{ij}$

= [0] , otherwise , for all i and j .

Definition 3.12 Multiplication of Fuzzy Soft Super Matrices: Let A= $[a_{ij}]$,B = $[b_{ij}]$ $\in FSSM_{mxn}$. Then Multiplication of Fuzzy Soft Super Matrices A and B is defined by A B = [min { a_{ij} , b_{ij} }] for all i and j .

Definition 3.13 Fuzzy Soft Super Submatrix: The Fuzzy Soft Super Submatrix of a Fuzzy Soft Super Matrix of order ≥ 1 is obtained by deleting some rows or some columns or both or neither .

A Fuzzy Soft Super Matrix is itself a Fuzzy Soft Super Submatrix .

Definition 3.14 Fuzzy Soft Principal Super Submatrix: The Fuzzy Soft Principal Super Submatrix of order (n – r) obtained by deleting r rows and columns of an n-th order Square Fuzzy Soft Super Matrix is called Fuzzy Soft Principal Super Submatrix .

Example5. Let A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$ be a fourth order Square Fuzzy Soft Super Matrix . Then first order fuzzy soft principal super submatrices obtained from A are $[a_{11}]$, $[a_{12}]$, $[a_{13}]$, $[a_{14}]$.

The second order fuzzy soft principal super submatrices are $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $\begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$ etc .

The third order fuzzy soft principal super sub matrices are $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$,

$\begin{bmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{bmatrix}$ etc .

The fourth order fuzzy soft principal super sub matrix is the given matrix A .

Definition 3.15 Fuzzy Soft Super Matrix: The Fuzzy Soft Super Matrix A = $[a_{ij}]$ is a matrix whose elements are themselves matrices with elements ∈ [0 ,1].

Definition 3.16 Fuzzy Soft Super Partitioning Matrix: If a FSSM is divided or partitioned into smaller FSSMs called blocks with consecutive rows and columns separated by dotted horizontal lines between rows and vertical lines between columns , then it is called Fuzzy Soft Super Partitioning Matrix.

It is also called Fuzzy Soft Super Block Matrix.

Definition 3.17 Fuzzy Soft Super Block Matrix: The Fuzzy Soft Super Block Matrix A = $[a_{ij}]$ is obtained by dividing or separating the matrix between certain specified rows and columns .

Then A = $\begin{bmatrix} a_{11} & \vdots & a_{12} & a_{13} & a_{14} \\ \dots & \vdots & \dots & \dots & \dots \\ a_{21} & \vdots & a_{22} & a_{23} & a_{24} \\ a_{31} & \vdots & a_{32} & a_{33} & a_{34} \\ a_{41} & \vdots & a_{42} & a_{43} & a_{44} \end{bmatrix}$ = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, where

$A_{11} = [a_{11}]$, $A_{12} = [a_{12} \quad a_{13} \quad a_{14}]$, $A_{21} = \begin{bmatrix} a_{21} \\ a_{31} \\ a_{41} \end{bmatrix}$, $A_{22} = \begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix}$

is an example fuzzy soft super block matrix .

Definition 3.18 Transpose of Fuzzy Soft Super Block Matrix: Transpose of Fuzzy Soft Super Block Matrix is the transpose of both the blocks and its constituent blocks .

$A^T = \begin{bmatrix} {A_{11}}^T & {A_{21}}^T \\ {A_{12}}^T & {A_{22}}^T \end{bmatrix}$

Definition 3.19 Fuzzy Soft Super Square Block Matrix: If the number of rows and number of columns of blocks are equal , then the matrix is said to be Fuzzy Soft Super Square Block Matrix .

A = $\begin{bmatrix} a_{11} & a_{12} & \vdots & a_{13} & a_{14} \\ a_{21} & a_{22} & \vdots & a_{23} & a_{24} \\ \dots & \dots & \dots & \dots & \dots \\ a_{31} & a_{32} & \vdots & a_{32} & a_{33} \\ a_{41} & a_{42} & \vdots & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ is a fuzzy soft super square block matrix , since A_{ij} is a square block .

3.20 Addition of Fuzzy Soft Super Block Matrices : Fuzzy Soft Super Block Matrices can be added if corresponding block are of same order.

Example5. Let A = $\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \dots & \dots & \dots & \dots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix}$ and B = $\begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \dots & \dots & \dots & \dots \\ B_{m1} & B_{m2} & \dots & B_{mn} \end{bmatrix}$ be two fuzzy soft super block matrices. Then

A + B = $\begin{bmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \dots & A_{1n} + B_{1n} \\ A_{21} + B_{21} & A_{22} + B_{22} & \dots & a_{2n} + B_{2n} \\ \dots & \dots & \dots & \dots \\ A_{m1} + B_{m1} & A_{m2} + B_{m2} & \dots & A_{mn} + B_{mn} \end{bmatrix}$

3.21 Scalar Multiplication: Scalar multiplication of a fuzzy soft super block matrix means each block of the matrix is multiplied by the matrix .

Thus , α A = $\begin{bmatrix} \alpha A_{11} & \alpha A_{12} & \dots & \alpha A_{1n} \\ \alpha A_{21} & \alpha A_{22} & \dots & \alpha A_{2n} \\ \dots & \dots & \dots & \dots \\ \alpha A_{m1} & \alpha A_{m2} & \dots & \alpha A_{mn} \end{bmatrix}$

Definition 3.22Fuzzy Soft Super Column Matrix of Type I: If A = $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{n1} \end{bmatrix}$ be a super matrix of order n x 1 , then A is said to be Fuzzy Soft Super Column Matrix of Type I where each a_{i1} ,i = 1, 2, … , n is sub matrix.

Definition 3.23 Fuzzy Soft Super Row Matrix of Type I: If A = $[a_{11} \quad a_{12} \quad a_{13} \quad \dots \quad a_{1n}]$ be a super matrix of order 1 x n , then A is said to be Fuzzy Soft Super Row Matrix of Type I where each a_{1i} ,i = 1, 2, … , n is sub matrix.

Definition 3.24 Fuzzy Soft Super Column Matrix of Type II : If A = $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ be a matrix of order m x n , then A = $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$ is said to be Fuzzy Soft Super Column Matrix of Type II where

$a_1 = [a_{11} \quad a_{12} \quad \dots \quad a_{1n}]$

$a_2 = [a_{21} \quad a_{22} \quad \dots \quad a_{2n}]$

....

$a_m = [a_{m1} \quad a_{m2} \quad \dots \quad a_{mn}]$

Definition 3.25 Fuzzy Soft Super Row Matrix of Type II : If A = $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ be a super matrix of order m x n , then A = [$a^1 \quad a^2 \quad \dots \quad a^m$] is said to be Fuzzy Soft Super Row Matrix of Type II where $a^1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$, $a^2 = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{mn} \end{bmatrix}$, … , $a^m = \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$

Note: Obviously , $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$ = [$a^1 \quad a^2 \quad \dots \quad a^m$]

Definition 3.26 Transpose of Fuzzy Soft Super Matrix : If A = $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ be a fuzzy soft super matrix of order m x n , then the transpose of A denoted by A^T = $\begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix}$ is a n x m fuzzy soft super matrix obtained by taking the transpose of each element i.e., the sub matrices of A .

Definition 3.27 Transpose of Fuzzy Soft Super Symmetric Matrix : If A = $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix}$ be a fuzzy soft super symmetric matrix of order m x m , then the transpose of A denoted by A^T = $\begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \dots & \dots & \dots & \dots \\ a_{1m} & a_{2m} & \dots & a_{mm} \end{bmatrix}$.

Note: The diagonal matrices are symmetric matrices unaltered by transposition .

Definition 3.28 Fuzzy Soft Super Diagonal Matrix : If D = $\begin{bmatrix} D_1 & 0 & \dots & 0 \\ 0 & D_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & D_m \end{bmatrix}$, then D is said to be Fuzzy Soft Super Diagonal Matrix where zeros denote matrices with zero as all entries .

Example 6 :

$$\left[\begin{array}{ccc|cc} .5 & .4 & .8 & 0 & 0 \\ .3 & .6 & .7 & 0 & 0 \\ \hline 0 & 0 & 0 & .5 & .8 \\ 0 & 0 & 0 & .3 & .4 \\ 0 & 0 & 0 & .7 & .5 \end{array}\right]$$

is an example of fuzzy soft super diagonal matrix .

Definition 3.29 Fuzzy Soft Super Identity Matrix : If I = $\begin{bmatrix} I_1 & 0 & \dots & 0 \\ 0 & I_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_m \end{bmatrix}$, then D is said to be Fuzzy Soft Super Identity Matrix .

Definition 3.30 Minor Product of Two Fuzzy Soft Super Matrices : If A = $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{n1} \end{bmatrix}$ and B = $\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{bmatrix}$ be two fuzzy soft super matrices of type I , then the minor product of A and B is given by

A^T B

$$= [a_{11} \quad a_{21} \quad a_{31} \quad \dots \quad a_{n1}] \,.\, \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{bmatrix}$$

$$= a_{11}\, b_{11} + a_{21}\, b_{21} + a_{31}\, b_{31} + \dots + a_{n1}\, b_{n1}$$

Definition 3.31 Major Product of Two Fuzzy Soft Super Matrices : If A = $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{n1} \end{bmatrix}$ and B = $\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{bmatrix}$ be two fuzzy soft super matrices of type I , then the major product of A and B is given by

$$A\, B^T = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{n1} \end{bmatrix} . [b_{11} \quad b_{21} \quad b_{31} \quad \dots \quad b_{n1}]$$

$$= \begin{bmatrix} a_{11}b_{11} & a_{11}b_{21} & \dots & a_{11}b_{n1} \\ a_{21}b_{11} & a_{21}b_{21} & \dots & a_{21}b_{n1} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1}b_{11} & a_{n1}b_{21} & \dots & a_{n1}b_{n1} \end{bmatrix}$$

Proposition 4: If A = $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{n1} \end{bmatrix}$ and B = $\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{bmatrix}$ be two fuzzy soft super matrices of type I , then

i) A^T B = B^T A

ii) (A $B^T)^T$ = B A^T

Example7 : Let A = $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$ and B = $\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}$ be two fuzzy soft super matrices , where a_{11} = $\begin{bmatrix} .1 \\ .2 \\ .3 \end{bmatrix}$, a_{21} = $\begin{bmatrix} .4 \\ .3 \\ .2 \\ .6 \end{bmatrix}$ and a_{31}= $\begin{bmatrix} .1 \\ .2 \end{bmatrix}$ and b_{11} = $\begin{bmatrix} .5 \\ .2 \\ .4 \end{bmatrix}$, b_{21} = $\begin{bmatrix} .3 \\ .8 \\ .4 \\ .2 \end{bmatrix}$, a_{31}= $\begin{bmatrix} .7 \\ .5 \end{bmatrix}$. Then

$$A^T \text{ B} = [.1 \quad .2 \quad .3] \begin{bmatrix} .5 \\ .2 \\ .4 \end{bmatrix} + [.4 \quad .3 \quad .2 \quad .6] \begin{bmatrix} .3 \\ .8 \\ .4 \\ .2 \end{bmatrix} + [.1 \quad .2] \begin{bmatrix} .7 \\ .5 \end{bmatrix}$$

= .05 + .04 + .8 + .12 + .24 + .08 + .12 + .07 + .1

=1.62

= B^T A □

CONCLUSION

In this paper, we define soft super matrix and some properties with examples. We define fuzzy soft super matrix and some new definitions based on fuzzy soft super matrix . Some properties are given with appropriate examples .Lastly, minor and major product of two fuzzy soft super matrices are defined .

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