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MESH CONVERGENCE ANALYSIS IN SIMULATIONS FINITE ELEMENT:

AN APPLICATION TO THE POISSON EQUATION

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KEYWORDS:

Finite element; Simulation; Convergence rate; Python FEniCs package

ABSTRACT

This research paper the effect of mesh convergence on finite element simulation accuracy is examined. We perform a case study on the Poisson equation, examining various mesh sizes and evaluating the error and convergence rates using the Python FEniCS package. Our findings highlight the significance of mesh refinement in obtaining precise numerical results.

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1. INTRODUCTION

Numerical simulations are anchored by the Finite Element Method (FEM), which provides a flexible and effective means of resolving a broad range of challenging issues in science and engineering. Making sure that numerical solutions are accurate and dependable is one of the main obstacles to using FEM efficiently. An essential component of the FEM, mesh convergence analysis, is crucial to achieving this goal.

This research begins with a thorough examination of mesh convergence procedures and approaches as provided in the seminal work of Zienkiewicz and Taylor (2005)[10]. It is crucial to comprehend the nuances of mesh convergence analysis to guarantee that FEM simulations yield accurate and dependable outcomes. The need for adaptive mesh approaches to solve space fractional differential equations with singular or finite-time blowup solutions was the main topic of Jingtang Ma and colleagues' work [5]. Using the L2-norm, they examined the convergence theories of various techniques and supported their theoretical conclusions with numerical evidence. The research derived error estimations for the projection under a changing mesh framework and introduced a fractional Ritz projection operator to ease the analysis. The authors of the cited study [9] discussed the convergence of a finite element approximation for the Freidlin–Wentzell (F–W) action functional minimizer. This approximation applied to dynamical systems that are non-gradient and perturbed by modest amounts of noise. We conducted a thorough analysis of small-noise-induced transitions in dynamical systems using the F-W theory of big deviations. Finding the minimizer and minimal of the F-W action functional was the main goal. By applying linear finite elements to discretize this action functional, the authors were able to prove the approximation's convergence using the notion of Γ -convergence. The mesh convergence test for a two-dimensional high-pressure turbine disc rim was the main objective of the cited study [7], which also focused on the use of the energy norm as a substitute method. Through the discretization of time and space variables, numerical methods were utilised to solve complex problems governed by partial differential equations in real-time. Additionally, for second-order elliptic interface problems, a novel and stable Petrov–Galerkin (PG) immersed finite element method (IFEM) was created and examined [1]. In order to solve the absence of local positivity in the traditional PG-IFEM, this approach added stabilization terms. Standard finite element functions were employed for the test space and submerged finite element functions for the trial space in this method. Both a prior and a posterior error estimates were presented in the paper. Stability and convergence analyses were carried out for the domain decomposition finite element/finite difference (FE/FD) method by the authors Mohammad and at el [6]. These analyses were specifically created for time-dependent Maxwell's equations using a semi-discrete finite element scheme.

The paper presented a domain decomposition algorithm and explored the creation of explicit finite element schemes in several geographical domain settings. The authors offered multiple numerical examples that validated the study's convergence rates in order to bolster their theoretical conclusions. Finite element analysis (FEA) and process parameter optimization for Nimonic 90 formability in sheet hydroforming were examined in the cited paper [3].

2. Methodology

2.1 Mesh Generation

Key feature of the finite element simulation is the finite element mesh. In our study, we looked at a variety of mesh size from 4×4 to 32×32 points, in order to investigate the impact of mesh refinement. The fe. Unit square mesh function was used to create a uniform mesh at first, and an adaptive technique was used to enhance it even further.

2.2 Boundary Conditions

To guarantee that the issues were well-posed, homogeneous Dirichlet boundary conditions were used. Singularities are avoided by fixing the solution to zero on the domain boundary by these constraints.

2.3 Weak Formulation

In our investigation, we used the Poisson equation, a popular PDE in scientific and engineering simulations. Trial and test functions were used to build the weak version of the Poisson equation, and a constant forcing term was added. The expressions on the left and the right were deduced.

2.4 Finite Element Assembly and Solution

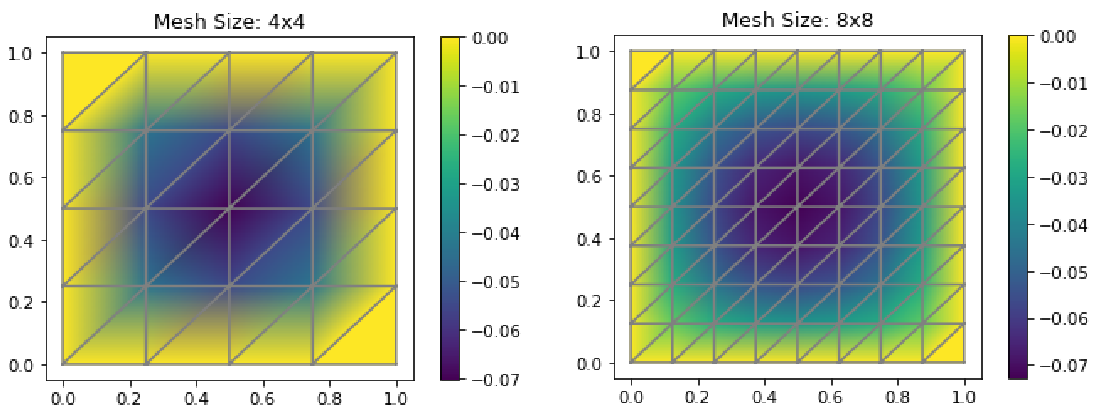
The linear system was solved using the finite element assembly method. The fe.solve function was utilized to acquire the answer, and the outcomes were saved in a function space.

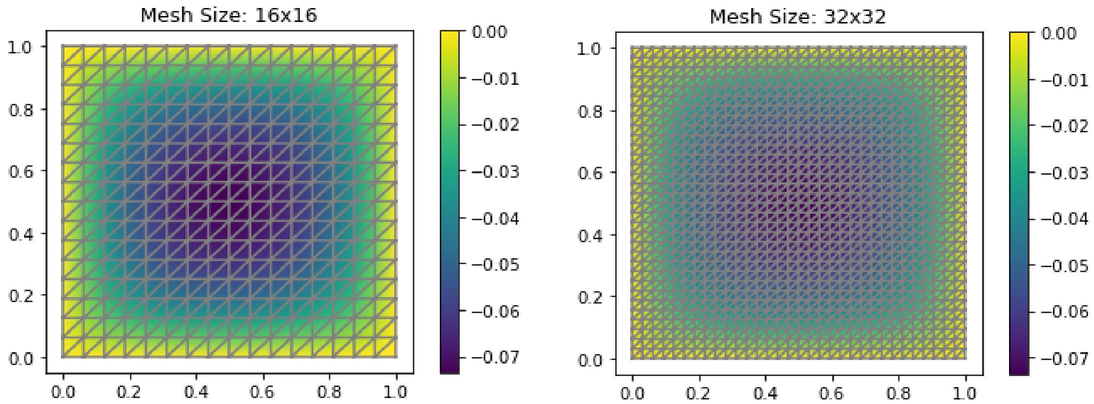
3. Mesh Convergence Analysis

The mesh convergence analysis findings for the numerical solution of the 2D Poisson problem are shown in this section. We compute the L2 norm of the error, examine the rate of convergence, and look into how mesh refinement affects the accuracy of the solution. We also offer numerical solutions for visual verification at particular nodes.

3.1 Solution Visualization

Visual representations of the numerical solutions for various mesh sizes are shown in Figure 1. The variation in the solution domain is displayed by the color plots.





3.2 Numerical solution at Specific nodes

We analyze the solution at particular nodes in order to visually confirm the correctness of our numerical results. As an illustration, we look at the solution at $(0.3, 0.3)$, $(0.5, 0.5)$, and $(0.7, 0.7)$. Table-1 displays the numerical solution values at these places.

Table 1: Numerical Solutions at Specific Nodes

Mesh Size	Node	Coordinates	FE Solutions
4×4	1	$(0.3, 0.3)$	-0.04843
	2	$(0.5, 0.5)$	-0.07031
	3	$(0.7, 0.7)$	-0.04843
8×8	1	$(0.3, 0.3)$	-0.05288
	2	$(0.5, 0.5)$	-0.07278
	3	$(0.7, 0.7)$	-0.05288
16×16	1	$(0.3, 0.3)$	-0.05447
	2	$(0.5, 0.5)$	-0.07344
	3	$(0.7, 0.7)$	-0.05447
32×32	2	$(0.3, 0.3)$	-0.05472
	1	$(0.5, 0.5)$	-0.07361
	3	$(0.7, 0.7)$	-0.05472

3.3 L2 Norm of the Error

The difference between the current and previous numerical grid solution is measured by the L2 norm of the error. This is how it is computed:

$$E = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(u_i^c - u_i^p\right)^2}$$

.

Where u_i^c represents displacement at node i for the current mesh and u_i^p represents displacement at node i for the previous mesh, and E is the L2 error norm.

3.4 Rate of Convergence

The r rate of convergence tells us how fast the error goes down as the mesh gets more precise. It's computed in this way:

$$r = \left| \frac{ln\left(\frac{E_i}{E_{i-1}}\right)}{ln\left(\frac{N_{i-1}}{N_i}\right)} \right|$$

Where r : Rate of convergence, E_i : L2 error norm for mesh i and N_i : Total number of nodes for mesh i. The convergence behavior is evaluated by computing the rate of convergence between successive mesh refinements.

4. Result and Discussion

Table 2 displays the L2 error norms and rates of convergence for various mesh sizes. Table 1 displays the numerical solutions at specific nodes.

Table 2: Error Norms and Convergence Rates

Mesh Size	L2 Error Norm	Rate of Convergence
4×4	0.033	-
8×8	0.032	0.044
16×16	0.030	0.093
32×32	0.027	0.152

The results show a distinct pattern of error decrease at smaller mesh resolutions. The convergence rate offers important information about how well mesh refinement increases solution correctness. Our examination of the outcomes emphasizes how important mesh convergence is to producing accurate simulations. The L2 error norm decreases as the mesh density increases, as Table 1 illustrates. Further demonstrating how finer meshes result in smoother and more accurate solutions are the visualizations in Figure 1. Table 2's computed convergence rates show how mesh refinement gets less and less beneficial over time.

5. Conclusion

With an emphasis on the Poisson equation, we have investigated the function of mesh convergence in finite element simulations in this work. Our results highlight how crucial it is to choose the right mesh size in order to strike a compromise between computing speed and correctness of the answer. Finite element simulation accuracy is greatly increased by mesh refining, while there is a diminishing return.

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